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Shear viscosity of a pion gas due to $\rho\pi\pi$ and $\sigma\pi\pi$ interactions

Sabyasachi Ghosh∗ †
Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271 - Bloco II, 01140-070 São Paulo, SP, Brazil.
E-mail: sabyaphy@gmail.com

Gastão Krein
Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271 - Bloco II, 01140-070 São Paulo, SP, Brazil.
E-mail: gkrein@ift.unesp.br

Sourav Sarkar
Theoretical Physics Division, Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700064, India
E-mail: sourav@vecc.gov.in

We have calculated the shear viscosity of a medium containing pions where the propagating pion has a finite thermal width due to interaction with the low mass resonances, σ and ρ. With the help of standard thermal field theoretical technique, the thermal width of the pion has been calculated from the pion self-energy diagram for $\pi\sigma$ and $\pi\rho$ loops, where an effective Lagrangian density has been used for interaction part. We have found a very small value of shear viscosity by entropy density ratio (η/s), which is very close to the KSS bound.

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1. Introduction

The new state of matter known as quark gluon plasma which is expected to be produced in high energy nucleus-nucleus collisions is likely to have a very small ratio of shear viscosity to entropy density, η/s. This conclusion has been made by different hydrodynamical and transport simulations to explain the elliptic flow parameter, v_2, extracted from data collected at RHIC (Relativistic Heavy Ion Collider). Such a small η/s is not really compatible with standard finite temperature calculations of Quantum Chromo Dynamics (QCD), which exhibits weakly interacting gas due to the asymptotic freedom of QCD at high temperature. Therefore, several investigations on microscopic and model dependent calculation of η/s for quark matter as well as hadronic matter have been done in recent times. The latter has attracted some importance in recent times when Niemi et al. [1] showed that the extracted transverse momentum p_T dependence of the elliptic flow parameter, $v_2(p_T)$, obtained from RHIC data is highly sensitive to the temperature dependent η/s in hadronic matter. In this context, we have calculated η as well as η/s of a pion gas using an effective Lagrangian for $\pi\pi\sigma$ and $\pi\pi\rho$ interactions and then we have also compared our results to others of the recent literature.

2. Formalism

Let us start with the standard expression of shear viscosity for pion gas, obtained from relaxation time approximation (RTA) [5, 6, 4]:

$$\eta = \frac{\beta}{10\pi^2}\int \frac{d^3\vec{k}\,\vec{k}^6}{\Gamma_\pi(\vec{k},T)\,\omega_k^2}\,n(\omega_k)\,[1+n(\omega_k)]\,, \tag{2.1}$$

where $n(\omega_k) = 1/\{e^{\beta\omega_k}-1\}$ is Bose-Einstein (BE) distribution function for a temperature $T=1/\beta$, with $\omega_k = (\vec{k}^2+m_\pi^2)^{1/2}$, and $\Gamma_\pi(\vec{k},T)$ is identified as the thermal width of π mesons in the medium.

As σ and ρ resonances play a dominant role in π-π scattering, therefore we have adopted an effective interaction Lagrangian density,

$$\mathscr{L} = g_\rho\,\rho_\mu\cdot\pi\times\partial^\mu\pi + \frac{g_\sigma}{2}m_\sigma\pi\cdot\pi\,\sigma, \tag{2.2}$$

to handle the dynamics of pion interaction with mesonic medium. We have taken coupling constants $g_\rho = 6$ and $g_\sigma = 5.82$, which are fixed from experimental decay widths of ρ and σ respectively.

The thermal width of the pion Γ_π has been obtained from the imaginary part of pion self-energy $\Pi_{\pi(\pi R)}$ for πR loops, where (resonance) R stands for σ and ρ mesons. The self-energy diagram is shown in the left panel of Fig. (1). We obtain from a thermal field theoretical evaluation [7]

$$\begin{aligned}\Gamma_\pi(\vec{k},T) &= \sum_{R=\sigma,\rho}\Gamma_{\pi(\pi R)} = \sum_{R=\sigma,\rho}\frac{\mathrm{Im}\Pi_{\pi(\pi R)}(k_0=\omega_k,\vec{k},T)}{m_\pi}\\ &= \sum_{R=\sigma,\rho}\frac{1}{m_\pi}\int\frac{d^3\vec{l}}{32\pi^2\omega_l\omega_u}L(k,l)|_{l_0=-\omega_l,k_0=\omega_k}\{n(\omega_l)-n(\omega_u)\}\delta(\omega_k+\omega_l-\omega_u)\,,\end{aligned} \tag{2.3}$$

where $n(\omega_l)$ and $n(\omega_u)$ are BE distribution functions of intermediate π and R mesons for $\omega_l = \{\vec{l}^2 + m_\pi^2\}^{1/2}$ and $\omega_u = \{|\vec{k} - \vec{l}|^2 + m_R^2\}^{1/2}$ respectively. The vertex factors -

$$
\begin{aligned}
L(k,l) &= -\frac{g_\sigma^2 m_\sigma^2}{4}, \qquad\qquad \text{for } R = \sigma\ , \\
&= -\frac{g_\rho^2}{m_\rho^2}\,[k^2\left(k^2 - m_\rho^2\right) + l^2\left(l^2 - m_\rho^2\right) - 2\{(k\cdot l)\,m_\rho^2 + k^2\,l^2\}],\ \text{for}\quad R = \rho \qquad (2.4)
\end{aligned}
$$

can be obtained from the effective Lagrangian density, given in Eq. 2.2.

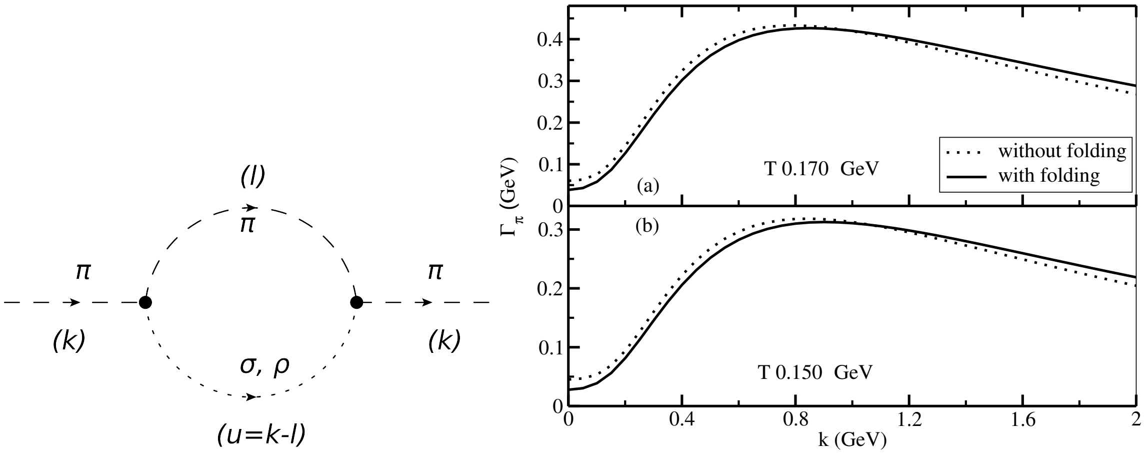

Figure 1: Left : General representation of pion self-energy diagram for $\pi\sigma$ and $\pi\rho$ loops. Right : Momentum dependence of the thermal width at temperature $T = 0.170$ GeV (upper panel) and $T = 0.150$ GeV (lower panel).

Next, to take into account the widths of the resonances, at first their masses m_R are taken as free parameters denoted by M. Then the modified $\Gamma_{\pi(\pi R)}(\vec{k}, T; M)$ of Eq. (2.3) are folded by vacuum spectral functions of the corresponding resonances whose general form is

$$
\rho_R(M) = \frac{1}{\pi}\mathrm{Im}\left[\frac{1}{M^2 - m_R^2 + iM\Gamma_R(M)}\right]\ . \qquad (2.5)
$$

With the help of the Lagrangian density 2.2, the vacuum decay width $\Gamma_R(M)$ of the $R = \sigma, \rho$ mesons can be obtained as

$$
\Gamma_\sigma(M) = \frac{3g_\sigma^2 m_\sigma^2}{32\pi M}\left(1 - \frac{4m_\pi^2}{M^2}\right)^{1/2}, \qquad (2.6)
$$

$$
\Gamma_\rho(M) = \frac{g_\rho^2 M}{48\pi}\left(1 - \frac{4m_\pi^2}{M^2}\right)^{3/2}\ . \qquad (2.7)
$$

The normalized relation for this folding is given by

$$
\Gamma_{\pi(\pi R)}(\vec{k}, T, m_R) = \frac{\int dM^2\,\rho_R(M)\,\Gamma_{\pi(\pi R)}(\vec{k}, T; M)}{\int dM^2\,\rho_R(M)}\ . \qquad (2.8)
$$

3. Results and Discussion

Using Eq. (2.3) and (2.8), without (dotted line) and with (solid line) the results of thermal width after folding as a function of momentum are respectively estimated and displayed in the right panel of Fig. 1 at $T = 0.170$ GeV (upper panel) and $T = 0.150$ GeV (lower panel). We see that due to folding effect, modification of $\Gamma_\pi(\vec{k})$ at low momentum becomes larger than that at high momentum and this modification increases with the increase of temperature. This momentum distribution of Γ_π is integrated out to obtain shear viscosity η of pion gas, as expressed in Eq. (2.1). The temperature dependence of η originates from two sources - the Bose-Einstein distribution of pion and its thermal width Γ_π. In the left panel of Fig. 2 results for the temperature dependence of η is presented without (upper panel) and with (lower panel) folding of Γ_π. The shear viscosity due to $\pi\rho$ (dotted line) and $\pi\sigma$ (dashed line) loops become divergent in the higher ($T > 0.100$ GeV) and lower ($T < 0.100$ GeV) temperature regions respectively. This complementary feature of these two loops indicates that consideration of both resonances in $\pi - \pi$ scattering is strictly necessary to obtain a smooth, non divergent η for temperatures below the critical, $T_c \simeq 0.175$ GeV. Though results of η without folding at very low temperatures ($T < 0.020$ GeV) tends to diverge, but after folding this trend disappears.

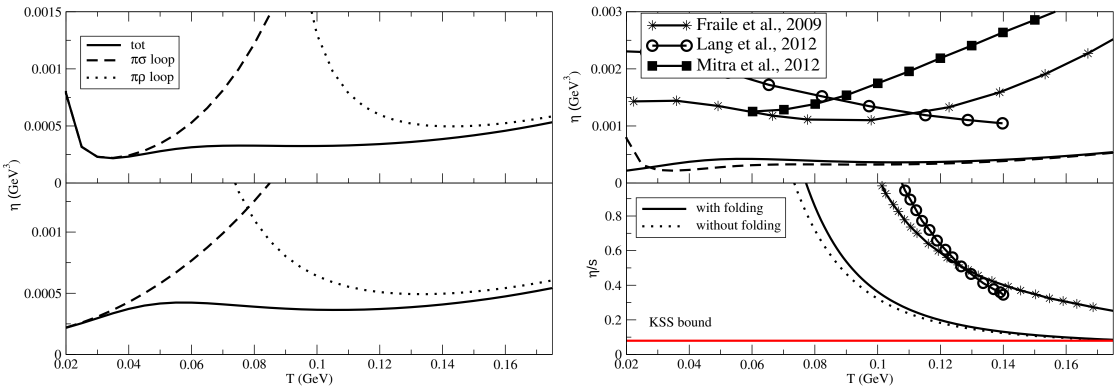

Figure 2: (Color online) Left : With (lower panel) and without (upper panel) folding results of η vs T due to $\pi\sigma$ (dashed lines), $\pi\rho$ (dotted lines) loops and their total (solid line). Right : Our estimation of $\eta(T)$ (upper panel) $\eta/s(T)$ (lower panel) have been compared to some other results. Horizontal red line indicates the KSS bound of η/s.

Our results have been compared with the earlier results obtained by Fraile et al. [2], Lang et al. [3] and Mitra et al. [4], which are attached in the upper and right panel of Fig. (2). Shear viscosity of pion gas, obtained by Refs. [6, 5, 4] in the kinetic theory approach, has been found to be a monotonically increasing function of temperature in the hadronic temperature range (0.100 GeV $< T <$ 0.175 GeV) for vanishing baryon chemical potential ($\mu = 0$) whereas Lang et al. [3] in the Kubo approach have shown η as a decreasing function of temperature. Similar decreasing nature of $\eta(T)$ is obtained in the Kubo-approach calculation by Fraile et al. [2] but it turns to an increasing function after dynamically generating low mass resonances (ρ and σ) via unitarization technique. This change in nature of η vs T may be because of the transformation from the scenario of pionic medium without resonances to one with resonances. Using the effective Lagrangian density, we have approximately considered a similar kind of scenario, where probabilities of pion scattering with the low mass resonances ρ and σ have been found from the finite temperature calculation of pion self-energy for $\pi\rho$ and $\pi\sigma$ loops. Similar to the results obtained by Fraile et al. [2] after unitarization, we have observed an increasing $\eta(T)$ after $T = 0.100$ GeV but lower in magnitude and increasing slope.

Using ideal expression of entropy density s for pion, we have shown the variation of the ratio η/s with respect to T in the lower panel of Fig. (2), on the right side. Within the hadronic temperature domain, our results of η/s respect the KSS bound $\frac{1}{4\pi}$ as shown by horizontal red line. Since Niemi et al. [1] have taken a similar range of $\eta/s(T)$, close to the KSS bound and found its important role to fit $v_2(P_T)$ of RHIC data, therefore our estimation has a good association with the phenomenological side also.

4. Summary and Perspectives

By considering the interaction of pion with the low mass resonances σ and ρ, we have calculated the shear viscosity coefficient of hot pion gas. In the framework of thermal field theoretical technique, the thermal width Γ_π has been calculated from one-loop pion self-energy at finite temperature. For the interaction part, we have taken an effective Lagrangian density, by which pion self-energy for $\pi\sigma$ and $\pi\rho$ loops have been evaluated at finite temperature. To treat σ and ρ resonances as two-pion states, we have folded the self-energy of $\pi\sigma$ and $\pi\rho$ loops with Breit-Wigner type spectral functions of σ and ρ with their vacuum width in $\pi\pi$ channel respectively. We have observed a complementary role of $\pi\sigma$ and $\pi\rho$ loops to make η non-divergent in the lower ($T < 0.100$ GeV) and higher ($T > 0.100$ GeV) temperature regions respectively. From the investigations of Niemi et al. [1], we see that $v_2(P_T)$ of RHIC data prefers these small values of $\eta/s(T)$ for hadronic matter. This provides experimental justification to the microscopic calculation of shear viscosity due to $\sigma\pi\pi$ and $\rho\pi\pi$ interaction performed in this work.

References

[1] H. Niemi, G.S. Denicol, P. Huovinen, E. Molnár, D.H. Rischke, *Influence of the shear viscosity of the quark-gluon plasma on elliptic flow in ultra-relativistic heavy-ion collisions*, *Phys. Rev. Lett.* **106** (2011) 212302 [`nucl-th/11012442`].

[2] D. Fernandez-Fraile and A. Gomez Nicola, *Transport coefficients and resonances for a meson gas in chiral perturbation theory*, *Eur. Phys. J. C* **62** (2009) 37 [`hep-ph/09024829`].

[3] R. Lang, N. Kaiser, and W. Weise, *Shear viscosity of a hot pion gas*, *Eur. Phys. J. A* **48** (2012) 109 [`hep-ph/12056648`].

[4] S. Mitra, S. Ghosh, and S. Sarkar, *Effect of a spectral modification of the ρ meson on the shear viscosity of a pion gas*, *Phys. Rev. C* **85** (2012) 064917 [`nucl-th/12042388`].

[5] S. Gavin, *transport coefficients in ultra-relativistic heavy ion collisions*, *Nucl. Phys. A* **435** (1985) 826.

[6] M. Prakash, M. Prakash, R. Venugopalan, and G. Welke, *Non-equilibrium properties of hadronic mixtures*, *Phys. Rep.* **227** (1993) 321.

[7] S. Ghosh, G. Krein, S. Sarkar, *Shear viscosity of a pion gas resulting from $\rho\pi\pi$ and $\sigma\pi\pi$ interactions*, *Phys. Rev. C* **89** (2014) 045201 [`nucl-th/14015392`].