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Phonon contribution to the thermal conductivity in neutron star core

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We explore contribution of phonon to the thermal conductivity coefficient (κ) in the core of neutron stars. We formulate phonon contribution with the help of the effective field theory which furnishes phonon scattering rates in terms of equation of state. We also compute phonon dispersion relation beyond leading order which is function of the gap. With phonon scattering rates and next-to leading order dispersion relation we evaluate the coefficient solving Boltzmann equation. It is seen that κ is dominated by small-large angle combined collisions and at low temperature can be written as $\kappa \propto 1/\Delta^6$. We show that thermal conductivity in the core is dominated by phonon-phonon collisions when phonon physics is governed by pure hydrodynamics.

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1. Introduction

Neutron stars can be thought of as a wonderful astrophysical laboratory for studying extremely dense matter. Our present understanding suggests that compact stars are composed of well-defined layers and expected to have supranuclear densities in their cores. The crustal part consists of lattice of ions presents in a liquid of electrons and in the inner region nuclei remain in a liquid of both electrons and superfluid neutrons. In the inner most regime of the star both neutrons and protons are expected to form BCS-like superfluids as observed by Migdal [1].

Neutron superfluidity is a low temperature phenomena which arises when quantum condensate appears because of neutron pairing. Global $U(1)$ symmetry related to baryon number conservation is spontaneously broken by the condensate. In this scenario according to goldstone theorem there exists a low energy mode, superfluid phonon, with linear dispersion law.

Study of superfluidity in the core of neutron star has been a matter of serious research since past few years. But its effect on different transport coefficients has been largely ignored [2, 3, 4]. Recently the effect of phonon scatterings on shear viscosity and bulk viscosity have been observed in Refs. [5, 6, 7]. It has been revealed in the calculation that hydrodynamics of compact star is essentially modified by the presence of superfluidity. In the present manuscript we evaluate phonon contribution to the thermal conductivity coefficient (κ) [8]. κ is an important coefficient to study because the cooling curve of a neutron star depends on the photon emission rate, neutrino emission rate, the specific heat and the thermal conductivity [9, 10]. Since all these quantities depend on the microscopic processes occurring inside the star, the computation of the cooling rate of a neutron star could give idea about its composition. The coefficient also determine the relaxation time of produced heat flux inside the star. All the previous calculation of κ suggests that it is dominated by electron scatterings and at large densities muon scatterings [2, 11, 12]. We employ effective field theory techniques to calculate phonon scattering rates and at leading order it is determined by the equation of state (EoS) of the system. For the present study we use causal parametrization of APR EoS which describes β-stable nuclear matter inside the star.

2. Scatterings of superfluid phonon

Theory of superfluid phonon is governed by microscopic physics once the heavy degrees of freedom are integrated out. Scatterings between superfluid phonons can be derived using effective field theory technique. The Lagrangian for the phonon field is written in terms of derivatives of the phonon field with respect to either momentum or energy,

$$\begin{aligned}\mathscr{L}_{LO} &= \frac{1}{2}\left[(\partial_t\phi)^2 - v_{ph}^2(\nabla\phi)^2\right] - g\left[(\partial_t\phi)^3 - 3\eta_g\partial_t\phi\,(\nabla\phi)^2\right] \\ &+\lambda\left[(\partial_t\phi)^4 - \eta_{\lambda,1}(\partial_t\phi)^2(\nabla\phi)^2 + \eta_{\lambda,2}(\nabla\phi)^4\right] + \cdots \end{aligned} \tag{2.1}$$

Self couplings at leading order can be expressed in terms of speed of sound as well as its derivatives at $T = 0$ [13]. Hence, phonon physics at leading order (LO) is solely dependent on the EoS [5, 6, 7, 8].

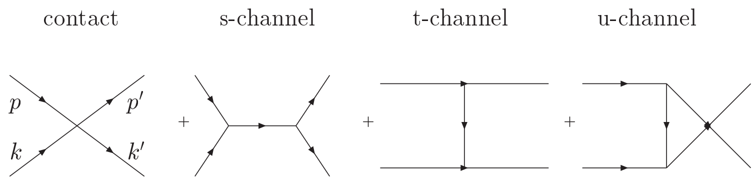

Figure 1: Binary phonon scattering processes contributing in κ

3. Formalism

Evaluation of thermal conductivity requires the knowledge of hydrodynamics as well as kinetic theory. In hydrodynamics heat flux (q) is related to the temperature gradient and the proportionality constant is κ [14, 15],

$$\mathbf{q} = -\kappa \nabla T \, . \tag{3.1}$$

In kinetic theory the heat flow can be expressed as,

$$\mathbf{q} = \int \frac{d^3 p}{(2\pi)^3} \mathbf{v}_p E_p \delta f_p \, . \tag{3.2}$$

In the above equation $\mathbf{v}_p = \partial E_P / \partial \mathbf{p}$ is the particle velocity, and δf_p $(f_p - f_p^0)$ is the non-equilibrium part of the distribution function. The non-equilibrium term depends on the thermal gradient and is given by,

$$\delta f_p = -\frac{f_p^0 (1 + f_p^0)}{T^3} g(p) \boldsymbol{p} \cdot \boldsymbol{\nabla} T \, , \tag{3.3}$$

where, $g(p)$ is the function to be evaluated by solving Boltzmann equation. In the present formalism we use the technique of variational principle, where $g(p)$ can be expressed in terms of orthogonal polynomials,

$$g(p) = \sum_{s=0}^{\infty} b_s B_s(p^2) \, , \tag{3.4}$$

$B_s(p^2)$ in the above equation is a polynomial of order s. The coefficient of B_0 is set to one. The polynomials obey orthogonality relation. To evaluate thermal conductivity we now appeal to Boltzmann equation. In the presence of a weak stationary temperature gradient and absence of any external force the Boltzmann equation takes the following form,

$$\mathbf{v_p} . \nabla_{\mathbf{r}} f_p = \mathscr{C}[f_p]. \tag{3.5}$$

The collision-integral on the right-hand-side (RHS) is given by the rate of scattering of phonons. Solution of the Boltzmann equation, δf_p, obey following constraints of both energy and momentum conservation,

$$\int d\Gamma E_p \delta f_p = \int d\Gamma \boldsymbol{p} \delta f_p = 0 \, . \tag{3.6}$$

After some algebraic calculation with the help of the above equations κ turns out,

$$\kappa = \left(\frac{4a_1^2}{3T^2}\right) A_1^2 M_{11}^{-1}, \tag{3.7}$$

where M_{11}^{-1} is the (1,1) element of the inverse of the following matrix,

$$\begin{aligned} M_{st} &= \int_{pkp'k'} (2\pi)^4 \delta(P+K-K'-P') |\mathscr{M}|^2 f_p^0 f_k^0 (1+f_{k'}^0)(1+f_{p'}^0) \boldsymbol{Q}_s \cdot \boldsymbol{Q}_t \\ \boldsymbol{Q}_s &= B_s(p^2)\boldsymbol{p} + B_s(k^2)\boldsymbol{k} - B_s(k'^2)\boldsymbol{k}' - B_s(p'^2)\boldsymbol{p}' \ . \end{aligned} \tag{3.8}$$

The matrix M is of infinite dimensions but the variational treatment is performed by considering the dimension of the matrix to be $N \times N$. Then one can write,

$$\kappa \geq \left(\frac{4a_1^2}{3T^2}\right) A_1^2 M_{11}^{-1}. \tag{3.9}$$

The number N is the variational parameter in the Vegas Monte Carlo algorithm [16]. For a_1 and A_1 one needs to have the information of phonon dispersion law. Using Eq.(3.6) it can be easily shown that phonons with linear dispersion relation gives vanishing contribution to the coefficient. Hence, one needs to go beyond leading order. The next to leading order (NLO) phonon dispersion relation can be obtained by performing phonon one loop calculation with the expansion parameters kv_F/Δ, $k_0 v_F/\Delta$ [17] and becomes [8],

$$E_k = c_s k \left(1 + \gamma k^2\right) + \mathscr{O}(k^5) \ , \tag{3.10}$$

where,

$$c_s = \frac{v_F}{\sqrt{3}} \ , \qquad \gamma = -\frac{v_F^2}{45\Delta^2} \ . \tag{3.11}$$

In the above equation c_s is the speed of sound of the system and v_F is the Fermi velocity. For the superfluid phonons with NLO dispersion law thus we obtain [8],

$$a_1 = \frac{4c_s^4}{15\Delta^2} \ , \qquad A_1 = \frac{256\pi^6}{245c_s^9} T^9 \ . \tag{3.12}$$

From Eq.(3.11) it is seen that the non-linear piece of the dispersion relation is negative, hence, processes like one phonon decaying into two or three are kinematically forbidden. Then leading order processes relevant for the thermal conductivity are $2 \to 2$ collisions of phonons, see Fig. 1 [8]. Expressions for the scattering amplitudes of these collisions are explicitly given in Ref. [5].

4. Results

In this section we present numerical study of κ. At first we present temperature and gap dependence of the coefficient analytically using a dimensionless variable $x = c_s p/T$. Eq.(3.7) then yields $\kappa \propto \frac{T^{16}}{\Delta^4} M_{11}^{-1}$. Assuming $N = 1$ for large angle collisions one attains $|\mathscr{M}|^2 \propto T^8$ and thermal conductivity becomes $\kappa \propto \frac{1}{T^2}\frac{1}{\Delta^4}$. For small angle collisions T dependence takes the form $\kappa \propto T^2 \frac{1}{\Delta^8}$.

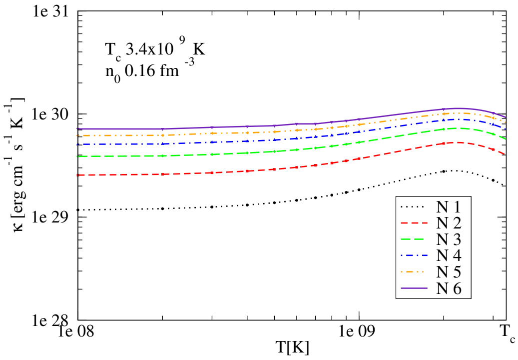

Figure 2: Numerical results for κ as a function of temperature for various basis size N. The calculation is performed using ${}^1S_0(A)+{}^3P_2(i)$ model of the energy gap. End temperature is critical temperature $T_c = 0.57\Delta(n_0) = 3.4\times10^9$ K [8].

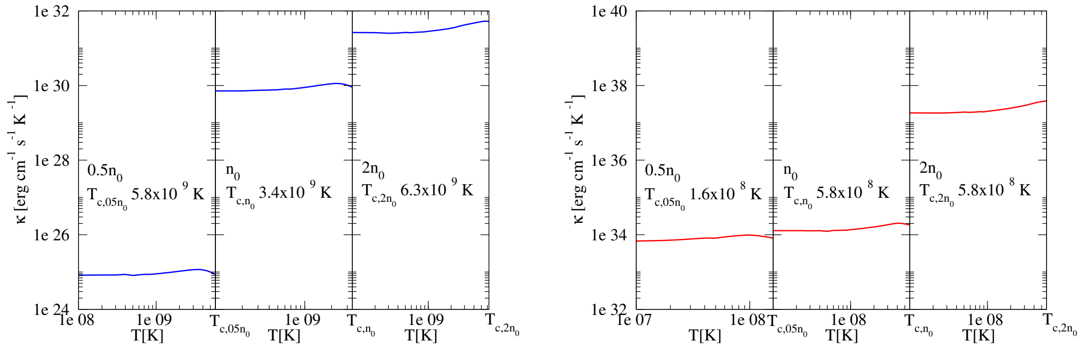

Figure 3: Variation of phonon thermal conductivity with temperature for different values of density. Calculation has been done considering two models for the energy gap ${}^1S_0(A)+{}^3P_2(i)$(left panel) and ${}^1S_0(c)+{}^3P_2(k)$(right panel)[8].

For combined large-small angle collisions behavior of $|\mathscr{M}|^2$ is determined by $\mathscr{M}$ and $\mathscr{M}^*$ where, these corresponds to either small and large angle dominated collisions, respectively, or vice versa. In this regime κ becomes $\kappa \propto \frac{1}{\Delta^6}$ [8]. In color-flavor-locked phase also κ shows similar temperature dependence [14].

In order to study the phonon contribution numerically one needs to have the EoS for the β equilibrated neutron star matter as well as the value of the gap. In the current analysis we use causal parametrization of APR equation of state [18, 19]. The impact of neutron pairing is neglected in the calculation since it does not have a big effect on the EoS. The EoS is given by,

$$\mathscr{E}(n,x_p) = (m + E/A(n,x_p)) \, , \tag{4.1}$$

where, m is the mass of the nucleon. The binding energy per nucleon (E/A) in nuclear matter is,

$$E/A = \mathscr{E}_0 y \frac{y-2-\delta}{1+\delta y} + S_0 y^{\beta}(1-2x_p)^2. \tag{4.2}$$

Values of the parameters of the above equation can be found in Ref.[5, 7, 18] in great detail.

For the energy gap we have taken the parametrization of the gap equation from Table I of Ref.[20]. The gap equation at the Fermi surface is given by the following phenomenological formula,

$$\Delta(k_F) = \Delta_0 \frac{(k_F - k_1)^2}{(k_F - k_1)^2 + k_2} \frac{(k_F - k_3)^2}{(k_F - k_3)^2 + k_4}, \tag{4.3}$$

here, k_F is the Fermi momentum and discussions on other parameters can be found in [20]. In the present calculation superfluid gap in both ${}^1S_0$ and ${}^3P_2$ channels are relevant. We employ two models for the gap equation ${}^1S_0(A)+{}^3P_2(i)$ and ${}^1S_0(c)+{}^3P_2(k)$ as described in Ref.[20].

In Fig.(2) we present temperature dependence of κ for various values of N using Vegas Monte Carlo algorithm. From the plot it is evident that the value converges at $N = 6$. Convergence is tested by imposing the condition that deviation from previous result should be $\lesssim 10\%$.

From Fig.(2) it is evident that for $T \lesssim 10^9$ K, κ is almost temperature independent and close to T_c the temperature correction is of the nature T and T^2. This is the region where we expect higher order corrections in the energy and momentum expansion in the phonon dispersion law and self-interactions should be considered. For $N = 6$ in Fig. 2 fit to the numerical results gives $\kappa \sim (7.02\times10^{29} + 9.28\times10^{19}\ T + 9.08\times10^{10}\ T^2)$ erg cm^{-1} s^{-1} K^{-1}, with T is in Kelvin. In fact, comparison of the numerical study with the theoretical one reveals that the dominant contribution to the phonon contribution to the thermal conductivity are those which corresponds to an almost temperature independent behaviour in small-large angle collisions combined regime, as elaborated previously in the present section (Fig.(3)). Comparison between our result and results of [12] divulges the fact that thermal conductivity in the neutron star core is dominated by phonon-phonon collisions when phonons are in a pure hydrodynamical regime, from the density 0.5 n_0 to 2 n_0 for temperatures up to T_c.

5. Summary

In the present manuscript we have formulated the derivation of phonon contribution to the thermal conductivity coefficient. It is evident from the calculation that phonon dispersion relation beyond leading order is necessary to evaluate the coefficient. Next to leading order phonon dispersion relation has been derived by expanding the phonon loop in kv_F/Δ and k_0v_F/Δ. The NLO term turns out to be negative, hence, only binary phonon scatterings have been considered and other scatterings have been ignored. The calculation has been computed using effective field theory technique to derive phonon scattering rates in terms of APR equation of state for β stable neutron matter. Thermal conductivity coefficient becomes temperature dependent as well as angles of collisions dependent. Phonon contribution becomes dominant over electron, muon collisions for the density 0.5 n_0 to $2n_0$ for temperatures up to T_c.

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