PROCEEDINGS OF SCIENCE

The QCD equation of state to $\mathcal{O}\left(\mu_B^4\right)$

Prasad Hegde* **(for the Bielefeld-BNL-CCNU collaboration)**
Key Laboratory of Quark & Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China.
E-mail: phegde@mail.ccnu.edu.cn

The QCD equation of state is necessary for modelling the hydrodynamical expansion of the quark-gluon plasma created in a heavy-ion collision. Here we present preliminary results for a finite-density equation of state that should be useful in the context of the Beam Energy Scan program at RHIC. We Taylor-expand the partition function and calculate all the coefficients upto sixth order in the quark chemical potentials. By comparing our second, fourth and sixth order expansions for different values of μ_B where μ_B is the baryon chemical potential, we conclude that our fourth-order equation of state should be valid upto $\mu_B/T \sim 2$ or equivalently, down to RHIC beam energies of approximately 20 GeV.

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*Speaker.

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1. Introduction

With the advent of the Beam Energy Scan (BES) program at RHIC, attention has turned towards the finite-μ_B region of the QCD phase diagram. In this connection there are questions which can only be answered through *ab initio* QCD calculations, such as the location of the conjectured critical point or the equation of state at finite density. Unfortunately lattice QCD, which is the only known technique to extract predictions directly from QCD in its non-perturbative regime, suffers from the infamous sign problem at $\mu_B \neq 0$[1]. As of today, no complete solution to this problem is known. Nevertheless, several partial solutions, such as reweighting, Taylor expansion, analytic continuation, etc. exist, among which the method of Taylor expansions [1, 2] is perhaps the most straightforward. In this one expands the pressure, which is just the logarithm of the partition function, in a Taylor series in μ viz.

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}}{i!\,j!\,k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k. \tag{1.1}$$

With three flavors of quarks, one has three chemical potentials (μ_u, μ_d, μ_s). Through a change of basis, we may work with a more physical set of chemical potentials corresponding to the conserved charges, namely baryon number, electric charge and net strangeness (μ_B, μ_Q, μ_S). Of these, the baryochemical potential μ_B is the one that is most relevant for our purposes. In this section, we shall present results with μ_Q and μ_S set to zero. In the next section, we will fix μ_Q and μ_S from the initial conditions relevant to Pb-Pb collisions.

Since the Taylor coefficients χ_{ijk} are defined at zero chemical potential, they can be evaluated using standard lattice QCD techniques. However the signal-to-noise ratio deteriorates rapidly as one goes to higher orders. This makes these calculations quite challenging and a significant amount of computer time is required to obtain good results.

The BNL-Bielefeld-CCNU collaboration is in the process of calculating every Taylor coefficient upto sixth order. By doing this we hope to obtain, among other things, an equation of state that is valid at all but the lowest beam energies of BES. We are currently working at two lattice spacings viz. $N_\tau = 6$ and 8, and with a nearly physical pion at $m_\pi \approx 160$ MeV ($m_l = m_s/20$). Our results for the 2$^{\text{nd}}$ and 4$^{\text{th}}$ order coefficients are shown in Fig. 1.

Our 6$^{\text{th}}$ order results are currently quite noisy despite the significant statistics. They can nevertheless be used to obtain qualitative bounds on the maximum value of μ_B/T up to which a 4$^{\text{th}}$ order extrapolation may be trusted. The sixth-order expansion of the pressure may be written as

$$\frac{p}{T^4} = c_0 + c_2 \left(\frac{\mu_B}{T}\right)^2 \left\{1 + \frac{c_4}{c_2}\left(\frac{\mu_B}{T}\right)^2 \left\{1 + \frac{c_6}{c_4}\left(\frac{\mu_B}{T}\right)^2\right\}\right\}, \qquad c_n = \frac{\chi_n^B}{n!}. \tag{1.2}$$

Let us focus on the important transition region $T \approx 155$ MeV. From elsewhere [3] we know that the zeroth-order contribution $c_0 \approx 1$ at this temperature. Similarly from Figs. 1 and 2 we get $c_2 \approx 0.05$, $c_4/c_2 \approx 1/24$ and $c_6/c_4 \approx 0.1$. At $\mu_B/T = 2$, these values lead to the 2$^{\text{nd}}$, 4$^{\text{th}}$ and 6$^{\text{th}}$ order corrections being roughly 20, 23 and 25% of the zeroth-order result respectively. Thus

[1] Lattice QCD relies on the quark determinant being positive so that it can be interpreted as a probability distribution and Monte Carlo techniques applied. At $\mu > 0$ however, the determinant becomes complex. This means that the probabilistic interpretation is lost and Monte Carlo techniques no longer apply.

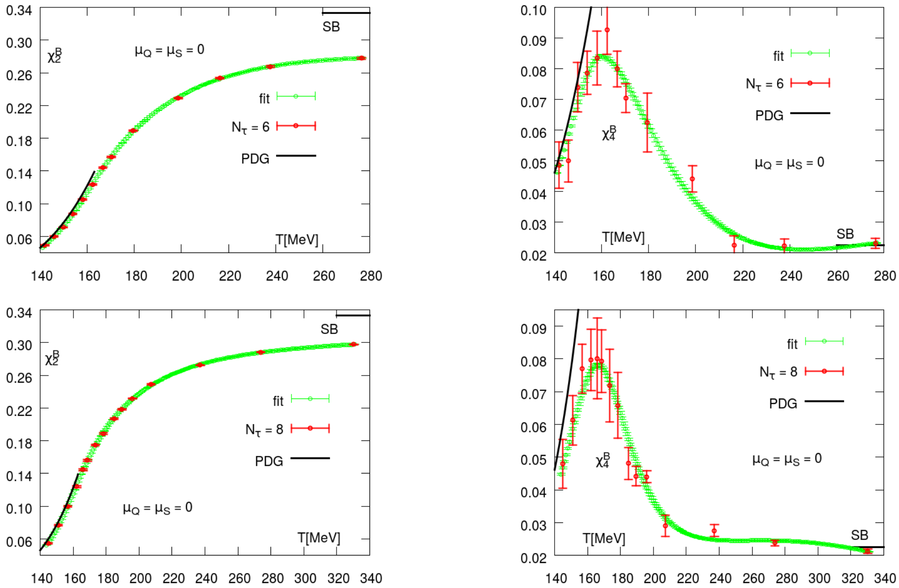

Figure 1: The first two Taylor coefficients with respect to μ_B (All odd coefficients are zero by CP symmetry) for $N_\tau = 6$ (top) and 8 (bottom). In all the figures, PDG refers to the Hadron Resonance Gas model curves, constructed by including all Particle Data Group resonances up to 3 GeV and towards which our lattice data tend at temperatures below the crossover region. The opposite limit, namely the free quark gas limit at $T = \infty$, is marked "SB" in the figures. The green band in all three curves are our fits to the data (see text).

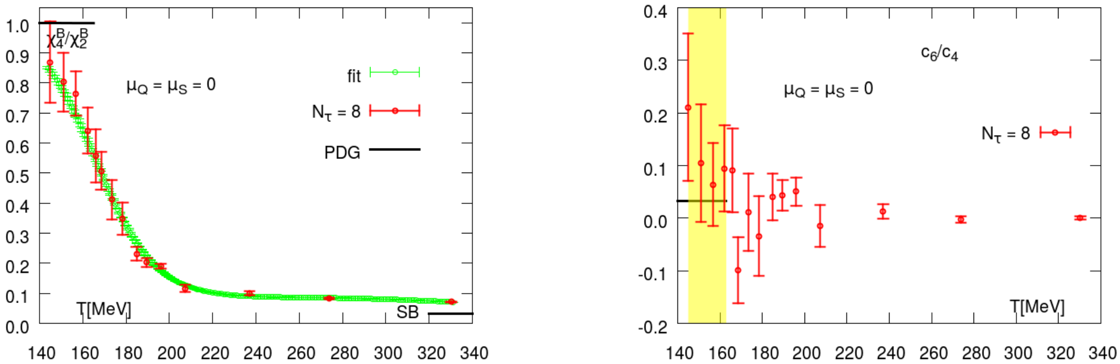

Figure 2: The ratios c_4/c_2 (left) and c_6/c_4 (right) for $N_\tau = 8$.

the bulk of the finite-μ_B contribution come from the leading *i.e.* 2$^{\text{nd}}$ order, with higher orders contributing only small corrections to this. This is a sign that the expansion is under control. By contrast, the corresponding corrections at $\mu_B/T = 3$ are 45, 63 and 77% respectively. Thus, not only is each contribution significant by itself, but successive orders too alter the lower-order results considerably. This is a sign that our sixth-order expansion is insufficient and that higher orders are needed. Thus a conservative estimate for the range of applicability of our fourth-order equation of state would be $0 \leqslant \mu_B/T \leqslant 2$[2].

[2]It must be remembered that this is for $N_\tau = 8$. The continuum values for c_4/c_2, etc. could be different, leading to a different estimate for the validity range of a fourth-order equation of state. However since our cutoff effects do not seem

2. Physics on the freeze-out curve

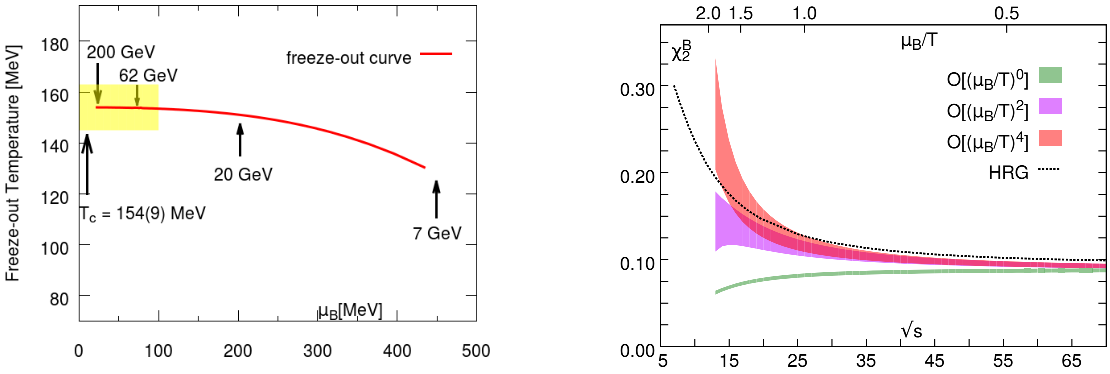

Figure 3: (Left) The freezeout curve according to the parametrization of Ref. (Right) χ_2^B, which is the baryon number variance, expanded to different orders and shown as a function of the beam energy and μ_B/T.

The multiplicities of various hadrons produced in a heavy-ion collision are found to be well-described by a statistical bootstrap model in which hadrons are in thermal equilibrium at temperature and baryochemical potential T^f and μ_B^f respectively. These freezeout parameters, when extracted for various collision energies, fall on a curve known as the freezeout curve (Fig. 3 (left)). There exist different parametrizations of this curve in the literature [4, 5, 6]. By plugging any one of these parametrizations into our Taylor expansions, we can construct a 4$^{\text{th}}$ order equation of state along this curve.

Before doing this however, we must fix the chemical potentials μ_Q and μ_S, which we had set to zero in the previous section. To do this, we take into account the initial conditions in heavy-ion collisions viz.

$$S = 0 \qquad \text{and} \qquad Q = rB. \tag{2.1}$$

That is, the net strangeness is zero and the ratio of electric charge to baryon number is a constant that is determined by the atomic and mass numbers of the colliding nuclei. By expanding these conditions with respect to (μ_B, μ_Q, μ_S) and solving them order by order, we can fix μ_Q/μ_B and μ_S/μ_B [7]. For the lead isotope that is collided at RHIC, one has $Z = 82$ and $A = 205$. We will therefore use $r = 0.4$ in this section.

Fig. 3 (right) shows χ_2^B, which is also the variance of the baryon number density, expanded to different orders along the freezeout curve. The chemical potential increases as the beam energy decreases. We see that a 2$^{\text{nd}}$ order expansion starts to break down below $s_{NN}^{1/2} \sim 60$ GeV, and a 4$^{\text{th}}$ order expansion is needed. Similarly, a 4$^{\text{th}}$ order expansion too would be insufficient below $s_{NN}^{1/2} \sim 20$ GeV. In the context of BES therefore, our 4$^{\text{th}}$ order equation of state can provide a good description of heavy-ion collisions down to $s_{NN}^{1/2} \sim 20$ GeV. In the future, we similarly hope to provide an equation of state that will be valid for the entire range of BES i.e. down to $s_{NN}^{1/2} \sim 5.5$ GeV.

to be very large, we do not expect this estimate to change by much in going from $N_\tau = 8$ to $N_\tau = \infty$.

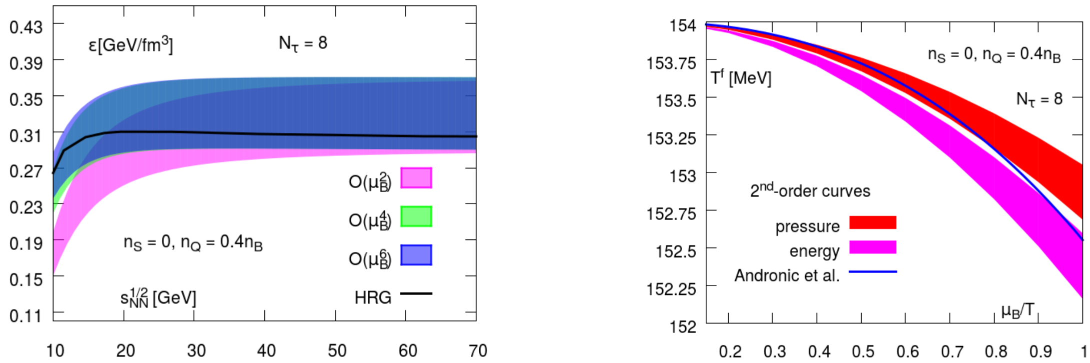

Figure 4: (Left) Energy density along the freezeout curve, calculated according to the parametrization of Ref. [5] (Right) Line of constant pressure and energy density, calculated using our 2$^{\text{nd}}$ order Taylor expansions.

Fig. 4 (left) also shows the energy density ε along the freezeout curve. There has been some discussion in the literature [4] about the energy density having a constant value along the freezeout curve. We verify this by calculating ε in our Taylor expansion. From Fig. 4 (right), we see that the energy density does indeed remain constant at all but the lowest beam energies. We also see that without the fourth-order term, this would seem to break down around $s_{NN}^{1/2} \sim$ GeV. Conversely, we may also construct "lines (or contours) of constant energy density," and compare these to the experimental freezeout curve. We do this in Fig. 4. The red and blue curves are our 2$^{\text{nd}}$ order curves keeping the energy density fixed at its $T = 154$ MeV, $\mu_B = 0$ value. We see that the agreement between theory and experiment, while not perfect, is nevertheless quite reasonable. From our construction, we are also able to extract a value for the curvature of the line κ_2^f viz. $\kappa_2^f = 0.0073(12)$ for the pressure and $\kappa_2^f = 0.0105(14)$ for the energy density. These numbers may be compared with an upper bound on the curvature of the freezeout line obtained recently by our collaboration viz. $\kappa_2^f < 0.011$ [8]. Of course, all of this is subject to the validity of the original conjecture, namely that freezeout occurs at a constant value of the pressure or energy density.

3. Conclusions

The phase diagram of QCD at nonzero density is currently unknown. The ongoing BES program at RHIC, as well as several upcoming experiments, aim to explore this part of the phase diagram in greater detail. A knowledgde of the QCD equation of state at moderately large densities therefore would be very useful. At this conference, we presented preliminary results for an equation of state which we believe should be valid for a baryochemical potential $\mu_B/T \lesssim 2$. We used the method of Taylor expansions and carried out an expansion upto sixth order. Unfortunately, our sixth order results are currently noisy; we therefore used them to put an upper bound on the maximum value of μ_B upto which a fourth order equation may be trusted.

We found that a second order expansion proved to be inadequate below beam energies of approximately 50 GeV. We also demonstrated that our fourth-order equation was valid down to beam energies of roughly 20 GeV. We hope to improve on this in two ways in the future: (i) by taking the continuum limit and (ii) by extending our calculations to lower beam energies.

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