Neutrino Oscillation: Past, Present and Future

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After a first part devoted to the description of the main steps in the discovery of neutrino oscillations, in the second part the results of a recent global analysis of solar, reactor, accelerator and atmospheric neutrino oscillation experiments is reported, searching for the first indications about the still unknown parameters, the mass hierarchy, the $\theta_{23}$ octant and the CP-violating phase $\delta$. Concerning the hierarchy, no significant difference emerges between normal and inverted mass ordering. A slight overall preference is found for $\theta_{23}$ in the first octant and for nonzero CP violation with $\sin \delta < 0$. In the third part the prospects of the future experimental searches for $\theta_{13}$, $\theta_{13}$ and its octant, $\delta_{CP}$ and the neutrino mass hierarchy: $\text{sign}(\Delta m^2)$ are briefly discussed.

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1. The Past

1.1 Going deeply back in time ...

Two protagonists are seen to emerge, Bruno Pontecorvo, with his vivid and firm intuition, and Raymond Davis Jr., for the perseverance in his challenging experimental quest.

Bruno Pontecorvo mentions for the first time neutrino oscillations in 1957, in his paper about muonium ↔ antimuonium transition [1], the same year in which parity violation is discovered [2] and the two-component theory of massless neutrino is proposed [3].

He writes his first paper on neutrino oscillations one year later, in 1958, when the famous reactor experiment of F. Reines and C. Cowan was just finished, with the discovery of the electron antineutrino through the observation of the inverse $\beta$ decay [4]. Since at the same time Raymond Davis is testing lepton number violation with reactor antineutrinos [5], this induces Pontecorvo to study in detail for the first time the consequences of possible antineutrino ↔ neutrino transitions [6]. In his paper he considers oscillations of active right-handed antineutrino in right-handed neutrino, the only possibility in the case of only one type of neutrino.

Pontecorvo comes back to neutrino oscillations several years later, in 1967, when the phenomenological V-A theory is well established [7], it has been shown that neutrino is left-handed [8], and the Brookhaven experiment has revealed that at least two types of neutrinos, $\nu_e$ and $\nu_\mu$, exist [9]. In his paper [10], he fixes the conditions at which neutrino oscillations are possible. Two years later, in 1969, Gribov and Pontecorvo consider explicitly a model in which the left-handed components of $\nu_e$ and $\nu_\mu$ are given as linear combinations of two left-handed neutrinos, $\nu_1$ and $\nu_2$, in terms of a mixing angle $\theta$ [11]. In the same paper the oscillations of solar neutrinos in vacuum are discussed and the survival probability of the $\nu_e$ is explicitly derived.

Quite independently, in 1962 Ziro Maki, Masami Nakagawa and Shoichi Sakata, in the context of a model of the elementary particle structure, also introduce the mixing of two neutrinos, called “weak neutrinos” and identified as $\nu_e$ and $\nu_\mu$, in terms of two neutrinos, $\nu_1$ and $\nu_2$, called “true” neutrinos [12].

This is the origin of what is now called PMNS (Pontecorvo, Maki, Nagakawa, Sakata) neutrino mixing matrix. More details about this first period of the neutrino oscillations story can be found in the very interesting review paper of Wanda Alberico and Samoil Bilenky [13] and in the study of the Bruno Pontecorvo life written by Luisa Bonolis [14].

1.2 The long standing “Solar Neutrino Problem”

In the meantime, Raymond Davis is preparing his famous Homestake experiment on the detection of solar neutrinos, based on the radiochemical method proposed by Pontecorvo in 1946 [15]. John N. Bahcall did the theoretical calculations of the expected solar fluxes. The experiment becomes operative in 1967.

The Davis experiment operates continuously until 1994 [16]. The flux measured is about 1/3 of the expected flux calculated by Bahcall. Further experiments (Super-Kamiokande [17], SAGE [18], GALLEX/GNO [19], SNO [20], and more recently BOREXINO [21]) also found a deficit, but, at that time, the first 90’s, it was not clear if the problem was related to particle physics (oscillations?) or to astrophysics (solar model?).
On the other hand, since the 80’s oscillations in matter, due to the MSW (Mikheyev, Smirnov, Wolfenstein) effect [22], were theoretically able to provide a quite attractive particle physics solution. But at that time data were not precise enough to allow a conclusive analysis. For example, in the paper [23] a comprehensive analysis is performed of solar, atmospheric accelerator and reactor neutrino experiments in a hierarchical three-generation scheme (the first analysis of this type), searching for the MSW solutions. It is easily seen that both small and large mixing angle solutions (SMA and LMA) are allowed, so that no a definite solution to the “solar neutrino problem” is found.

1.3 The first breakthrough: the “Atmospheric Neutrino Anomaly”

The so-called “Atmospheric Neutrino Anomaly” is the unexpected difference between measured and predicted muon-to-electron flavor composition of the atmospheric neutrino flux, appeared in the first 90’s. Once again: is it a problem of particle physics (oscillations?) or of astrophysics (primary cosmic ray fluxes?).

But in 1998 ... the breakthrough! The atmospheric neutrino flux shows a strong zenith angle dependence! At the Neutrino ’98 conference at Takayama the Super-Kamiokande Collaboration shows a $6.2\sigma$ asymmetry effect as the evidence of the oscillation of the $\nu_\mu$ neutrinos. Indeed, the zenith angle dependence is inconsistent with expectations based on calculations of the atmospheric neutrino flux, and are consistent with a two-flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. No relevant effect is seen as far as $\nu_e$ neutrinos are concerned. Similar effects, but with less statistics, are seen also by the MACRO Collaboration [25].

A detailed analysis of the atmospheric neutrino oscillations in a three-flavor approach can be found in [26].

1.4 2002: the “Annus Mirabilis” of the neutrino oscillation physics

In the meantime, what about the “solar neutrino problem”? As in the best western movies, a new experiment come to rescue: the SNO (Sudbury Neutrino Observatory) experiment, using a heavy-water target. Why deuterium? Since in deuterium one can separate $CC$ events (induced by $\nu_e$ only) from $NC$ events (induced by $\nu_e, \nu_\mu, \nu_\tau$) and make a double check via $ES$ (Elastic Scattering), due to both $NC$ and $CC$. We obtain the experimental breakthrough, since the result $CC/NC < 1$ represents the smoking gun proof of flavor change. The solar model is then OK! In particular it is measured $CC/NC \sim P_{ec} \sim \sin^2\theta_{12}$ (LMA) $\sim 1/3$, which gives evidence of mixing in the first octant and also of matter effects in solar neutrino oscillations.

In the April of 2002 a direct and highly significant evidence for $\nu_e$ flavor change into active states is definitely announced by the SNO experiment [27], crowning a four-decade long series of observations of the solar $\nu_e$ deficit. But the year 2002 is likely to be remembered as the Annus Mirabilis of solar neutrino physics, since in the same year the role of solar neutrino physics is recognized through the Nobel Prize jointly awarded to Raymond Davis Jr. and Masatoshi Koshiba for their pioneering contributions to the detection of cosmic neutrinos [28].

A further success justifies to consider the 2002 as the “Annus mirabilis” of the solar neutrino physics. The KamLAND experiment, projected to reproduce “solar neutrino oscillations” in laboratory, announces the observation of electron antineutrino disappearance [29], so confirming the
interpretation of solar neutrino data in terms of $\nu_e \rightarrow \nu_{\mu,\tau}$ oscillations induced by neutrino masses and mixing. In particular the experiment is able to restrict the corresponding parameter space ($\delta m^2, \theta_{12}$) within the so-called large mixing angle (LMA) region.

In conclusion, the solar neutrino problem is solved. In the following years, by combining the solar experiments data (dominated by SNO) and the KamLAND results, the estimate of the solar parameters has been refined. Moreover, it has been possible to verify that the solar neutrino oscillations proceed through the MSW effect. A first indication is obtained in 2006 [30], from the analysis of the available data making use of a convenient parametrization of the effect. The effect is definitively seen experimentally by BOREXINO in 2008 [31].

1.5 Long Baseline Neutrino expts: aim of reproducing “atmospheric neutrino oscillations” in laboratory

Several experiments have been projected and realized with the aim of reproducing “atmospheric $\nu$ oscillations” in labs, with a proper choice of neutrino beam energy and baseline.

The first is the KEK-to-Kamioka (K2K) experiment, aimed at testing disappearance of accelerator $\nu_\mu$ in the range probed by atmospheric $\nu$: $(L/E)_{K2K} \sim (250 \text{ km}/1.3 \text{ GeV}) \sim (L/E)_{\text{ATM}}$. In 2002 muon disappearance is observed at 99% C.L., without electron appearance [32].

MINOS (Main Injector Neutrino Oscillation Search) is another experiment designed to study “atmospheric neutrino oscillations” in laboratory. Neutrinos are produced by the NuMI (neutrinos at Main Injector) beamline at Fermilab and are observed at both the near detector, close to where the beam is produced, and the far detector, 735 km away in northern Minnesota. The MINOS experiment started detecting neutrinos from the NuMI beam in February 2005. About one year later the collaboration announced [33] that the analysis of the initial data was consistent with neutrino oscillations, with the oscillation parameters in agreement with the results obtained by SuperKamiokande.

K2K and MINOS agree about muon flavor disappearance and no electron appearance, but there is still a missing piece ... the direct observation of a $\nu_\tau$ in a $\nu_\mu$ beam! This is the goal of the experiment OPERA (Oscillation Project with Emulsion-tRacking Apparatus). The beam is tuned to relatively high energy, with some suppression of the oscillation (small $L/E$), but with an enhancement of the $\tau$ production. OPERA has been able to see the first event of oscillation in appearance mode. With 4 $\nu_\tau$ candidates with 0.23 of background the non oscillation hypothesis is excluded at 4.2$\sigma$ [34]. Very recently, a fifth event has been seen, an observation that allows for the first time to claim the discovery of $\nu_\mu \rightarrow \nu_\tau$ oscillation in the appearance mode with a significance larger that 5$\sigma$.

1.6 The hunt to $\theta_{13}$, the last mixing angle

The hunt to $\theta_{13}$ is crucial in neutrino research activity, in order to plan future CP violation searches! In 2006 the upper bound still comes from the first reactor experiment, CHOOZ [35], in which no indications in favor of neutrino oscillations are observed. Comparing with the data of Super-Kamiokande, the limit coming from CHOOZ is consistent with $\sin^2 \theta_{13} < \text{few\%}$.

But in the meantime, some weak hints of lower bounds are seen to appear ... An old, but persisting, hint for $\theta_{13} > 0$ comes from a 3$\nu$ analysis of atmospheric data, including also long
baseline accelerator experiments (LBL) and CHOOZ. It appears mainly due to subleading “solar terms” effects, which help fitting atmospheric electron event data (especially sub.GeV), with a statistics significance of about 1σ [30].

A second indication comes from an accurate comparison within a 3ν approach of solar (SNO dominated) and KamLAND data, considered assuming θ_{13} = 0. Indeed, the disagreement is reduced if θ_{13} > 0 is assumed, thanks to the different correlation between θ_{12} and θ_{13} in KamLand and SNO data. The effect, seen independently also by Balantekin and Yilmaz [36], has been presented for the first time at the conference NO-VE 2008 [37].

Taken together, the two hints (which corresponds to combine solar+ KamLAND data with atmospheric + CHOOZ + LBL data) provide a possible indication in favor of θ_{13} > 0 at the level of 1.6σ [38]. Not so bad! In the same year a first global analysis of neutrino oscillation data is published [39] in which all the mass-mixing parameters are estimated, including θ_{13}.

In the next years, the θ_{13} hints discussed before are debated at length, reaching but not exceeding the statistical level of about 2σ. But once again, new experimental results come to rescue! In 2011 T2K and MINOS found some electron event excess when running in appearance mode ... Both experiments favor sin^2 θ_{13} ~ few%! It make sense to combine these with all the other oscillation data ... In 2011 a new analysis is published [40], in which the hints become evidence in favor of θ_{13} > 0, with the ATM + LBL + CHOOZ sector of data now more significant than the sector Solar + KamLAND. There is an astonishing conspiracy of the two totally independent sets of data, with sin^2 θ_{13} = 0.021 ± 0.007, i.e. evidence of θ_{13} > 0 at ~ 3σ.

1.7 The Short Baseline Reactor experiments

In 2012 the Short Baseline Reactor (SBR) experiments close the hunt to θ_{13}. In China Daya Bay [41] and in Korea RENO [42], making use of near and far detector, show a clear νe disappearance at the FD (far detector) with respect to the approximatively unoscillated signal at ND (near detector), and definitely establish that θ_{13} > 0 at ~ 5σ. In particular, Daya Bay and RENO measure sin^2 θ_{13} ≃ 0.023 ± 0.003 [43] and sin^2 θ_{13} ≃ 0.029 ± 0.006 [44], respectively. Consistent indications are also found in the Double Chooz reactor experiment with far detector only (sin^2 θ_{13} ≃ 0.028 ± 0.010) [43, 46]. All these reactor data are in good agreement with the results of our global analysis of oscillation data in [40], mentioned before.

In the recent years, the improvement of the SBR data has been impressive, as it can seen from the spectra presented by these experiments at the Neutrino 2014 conference in Boston [47].

2. The Present

2.1 Introduction

Current neutrino oscillation experiments (except for a few anomalous results) can be interpreted within a three-neutrino framework, where the three flavor states ν_{α} = (ν_e, ν_μ, ν_τ) are quantum superpositions of three light mass states ν_i = (ν_1, ν_3, ν_3) via a unitary mixing matrix U_{αi}, depending on three mixing angles (θ_{12}, θ_{13}, θ_{23}) and one possible CP-violating phase δ [48].

In neutrino oscillations, CP violation is a genuine 3ν effect which may be observed (provided that δ ≠ 0, π) only if all the mixings θ_{ij} and the squared mass differences m_i^2 − m_j^2 are nonzero
The latter condition is experimentally established, and can be expressed in terms of the two independent parameters \( \delta m^2 = m_2^2 - m_1^2 > 0 \) and \( \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2 \), where \( \Delta m^2 > 0 \) and \( < 0 \) correspond to normal (NH) and inverted (IH) mass spectrum hierarchy, respectively.

At present we know five oscillation parameters, each one with an accuracy largely dominated by a specific class of experiments, namely: \( \theta_{12} \) by solar data, \( \theta_{13} \) by short baseline (SBL) reactor data, \( \theta_{23} \) by atmospheric data, mainly from Super.Kamiokande (SK), \( \delta m^2 \) by long-baseline reactor data from KamLAND (KL), and \( \Delta m^2 \) by long-baseline (LBL) accelerator data, mainly from MINOS and T2K. However, the available data are not yet able to determine the mass hierarchy, to discriminate the \( \theta_{23} \) octant, or to discover CP violation effects. A worldwide research program is underway to address such open questions and the related experimental and theoretical issues.

In this context, the global neutrino data analysis performed in [50] (for alternative analyses see [51, 52]) has been useful to get the most restrictive bounds on the known parameters, via the synergic combination of results from different classes of oscillation searches, providing, at the same time, some guidance about the unknown oscillation parameters.

With \( \sin^2 \theta_{13} \) as large as \( 2 - 3 \times 10^{-2} \), the door is open to CP violation searches in the neutrino sector, although the road ahead appears to be long and difficult [53, 54]. In particular, it makes sense to update the analysis in [50] by including the most recent data from the different experiments. Accordingly, with respect to [50], we include in our updated analysis, for the first time reported in [55], the recent SBL reactor data from Daya Bay [56] and RENO [57], which reduce significantly the range of \( \theta_{13} \). We also include the latest appearance and disappearance event spectra published in 2013 and at the beginning of 2014 by the LBL accelerator experiments T2K [58, 59, 60] and MINOS [63, 64, 65], which not only constraint the known parameters (\( \Delta m^2, \theta_{23}, \theta_{13} \)), but, in combination with other data, provide some guidance on the \( \theta_{23} \) octant and on the leptonic CP violation.

More explicitly, we find a slight overall preference for \( \theta_{23} < \pi/4 \) and for nonzero CP violation with \( \sin \delta > 0 \); however, for both parameters, such hints exceed 1\( \sigma \) only for normal hierarchy. No significant preference emerges for normal versus inverted hierarchy. Among the various results which can be of interest, we find it useful to report the preferred \( N\sigma \) ranges of each oscillation parameter and covariance plots of selected couples of parameters, as well as to discuss their stability and the role of different data sets in the global analysis. More details about the present analysis can be found in [55].

### 2.2 Global 3\( \nu \) analyses: some methodological issues

No single oscillation experiment can sensitively probe, at present, the full parameter space spanned by \( (\delta m^2, \pm \Delta m^2, \theta_{12}, \theta_{13}, \theta_{23}, \delta) \). Therefore, it is necessary to group in some way the experimental data, in order to study their impact on the oscillation parameters. For instance, in [40] we showed that consistent indications in favor of nonzero \( \theta_{13} \) emerged from two different datasets, one mainly sensitive to \( \delta m^2 \) (solar plus KamLAND experiments) and another mainly sensitive to \( \Delta m^2 \) (CHOOZ plus atmospheric and LBL accelerator experiments). In this work we adopt an alternative grouping of datasets, which is more appropriate to discuss interesting features of the current data analysis, such as the covariance among the parameters \( (\sin^2 \theta_{13}, \sin^2 \theta_{23}, \delta) \) in both mass hierarchies.
**LBL + solar + KamLAND data.** We remind that LBL accelerator data (from the K2K, T2K, and MINOS experiments) in the $\nu_\mu \rightarrow \nu_\mu$ disappearance channel probe dominantly the $\Delta m^2$-driven amplitude

$$|U_{\mu3}|^2(1 - |U_{\mu3}|^2) = \cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23}) ,$$

(2.1)

which is slightly octant-asymmetric in $\theta_{23}$ for $\theta_{13} \neq 0$. In the $\nu_\mu \rightarrow \nu_e$ appearance channel, the dominant $\Delta m^2$-driven amplitude is

$$|U_{\mu3}|^2|U_{e3}|^2 = \cos^2 \theta_{13} \sin^2 \theta_{13} \sin^2 \theta_{23} ,$$

(2.2)

which is definitely octant-asymmetric in $\theta_{23}$ for $\theta_{13} \neq 0$. In both the appearance and the disappearance channels, subdominant terms driven by $\delta m^2$ and by matter effects can also contribute to lift the octant symmetry and to provide some weak sensitivity to sign($\Delta m^2$) and to $\delta$. As already noted in [40], the T2K and MINOS indications in favor of $\nu_\mu \rightarrow \nu_e$ appearance induce an anti-correlation, via Eq. (2.2), between the preferred values of $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$. This covariance is relevant in the analysis of the $\theta_{23}$ octant degeneracy [66] and has an indirect impact also on the preferred ranges of $\delta$ via subdominant effects.

In order to make the best use of LBL accelerator data, it is thus useful to: (1) analyze both disappearance and appearance data at the same time and in a full $3\nu$ approach; (2) combine LBL with solar and KamLAND data, which provide independent constraints on ($\delta m^2$, $\theta_{12}$, $\theta_{13}$) and thus on the subdominant $3\nu$ oscillation terms. As discussed below, once the (relatively well known) oscillation parameters $\sin^2 \theta_{12}$, $\delta m^2$ and $\Delta m^2$ are marginalized away, interesting correlations emerge among the remaining parameters ($\sin^2 \theta_{13}, \sin^2 \theta_{23}, \delta$).

**Adding SBL reactor data.** After grouping LBL accelerator plus solar plus KamLAND data (LBL + solar + KamLAND), it is important to add the independent and “clean” constraints on $\theta_{13}$ coming from SBL reactor experiments in the $\nu_e \rightarrow \nu_e$ disappearance channel, which probe dominantly the $\Delta m^2$-driven amplitude

$$|U_{e3}|^2(1 - |U_{e3}|^2) = \sin^2 \theta_{13} \cos^2 \theta_{13} .$$

(2.3)

In the reactor dataset, subdominant terms are slightly sensitive to ($\delta m^2$, $\theta_{12}$) and, as noted in [67] and discussed in [68], probe also the neutrino mass hierarchy. We include far-detector data from CHOOZ [69] and Double Chooz [46] and near-to-far detector constraints from Daya Bay [43] and RENO [42, 44]. We do not include data from pre-CHOOZ reactor experiments, which mainly affect normalization issues.

Indeed, the analysis of reactor experiments without near detectors depends, to some extent, on the absolute normalization of the neutrino fluxes, which we choose to be the “old” (or “low”) one, in the terminology of [40]. We shall also comment on the effect of adopting the “new” (or “high”) normalization recently proposed in [70, 71]. Constraints from Daya Bay and RENO are basically independent of such normalization, which is left free in the official analyses and is largely canceled by comparing near and far rates of events [41, 42]. At present, it is not possible to reproduce, from published information, the official Daya Bay and RENO data analyses with the permill accuracy appropriate to deal with the small systematics affecting near/far ratios. We think that, for the purposes of this work, it is sufficient to take their measurements of $\sin^2 2\theta_{13}$ at face value, as gaussian constraints on such parameter. Luckily, such constraints appear to depend very
little on the $\Delta m^2$ parameter within its currently allowed range; see the $(\Delta m^2, \sin^2 2\theta_{13})$ prospective sensitivity plots in [72] (Daya Bay) and [73] (RENO).

As shown in [66], LBL data in disappearance and appearance mode generally select [via Eqs. (2.1) and (2.2)], two degenerate $(\theta_{23}, \theta_{13})$ solutions, characterized by nearly octant-symmetric values of $\theta_{23}$ and by slightly different values of $\theta_{13}$. By selecting a narrow range of $\theta_{13}$, precise reactor data can thus (partly) lift the $\theta_{23}$ octant degeneracy [66] (see also [74]). Amusingly, the fit results in Subsection 2.3 resembles the hypothetical, qualitative 3$\nu$ scenario studied in [66].

Adding atmospheric neutrino data. After combining the (LBL + solar + KamLAND) and (SBL reactor) datasets, we finally add the Super-Kamiokande atmospheric neutrino data (SK atm.), as reported for the joint SK phases I–IV in [75, 76]. The SK data span several decades in neutrino and antineutrino energy and pathlengths, both in vacuum and in matter, in all appearance and disappearance channels involving $\nu_\mu$ and $\nu_e$, and thus they embed an extremely rich 3$\nu$ oscillation physics.

In practice, it is difficult to infer —from atmospheric data— clean 3$\nu$ information beyond the dominant parameters $(\Delta m^2, \theta_{23})$. Subdominant oscillation effects are often smeared out over wide energy-angle spectra of events, and can be partly mimicked by systematic effects. For this reason, “hints” coming from current atmospheric data should be taken with a grain of salt, and should be possibly supported by independent datasets. For instance, we have attributed some importance to a weak preference for $\theta_{13} > 0$ found from atmospheric SK data in [30], only after it was independently supported by solar+KamLAND data [38] and, later, by LBL accelerator data [40]. Similarly, we have typically found a preference of atmospheric SK data for $\theta_{23} < \pi/4$ [30, 40].

In this work, the analysis of SK atmospheric neutrino data (phases I-IV) [75, 76] is essentially unchanged with respect to [50]. We remind that such data involve a very rich oscillation phenomenology which is sensitive, in principle, also to subleading effects related to the mass hierarchy, the $\theta_{23}$ octant and the CP phase $\delta$. However, within the current experimental and theoretical uncertainties, it remains difficult to disentangle and probe such small effects at a level exceeding $1\sigma - 2\sigma$ [30]. Moreover, independent 3$\nu$ fits of SK I-IV data [50, 51, 76] converge on some but not all the hints about subleading effects. Therefore, as also argued in [50], we prefer to add these data only in the final “LBL Acc. + Solar + KL + SBL Reac. + SK Atm.” combination, in order to separately gauge their effects on the various 3$\nu$ parameters.

Finally, we shall also report the relative preference of the data for either NH or IH, as measured by the quantity $\chi^2_{\text{min}}(\text{IH}) - \chi^2_{\text{min}}(\text{NH})$. This quantity cannot immediately be translated into “$N\sigma$” by taking the square root of its absolute value, because it refers to two discrete hypotheses, not connected by variations of a physical parameter. We shall not enter into the current debate about the statistical interpretation of $\Delta\chi^2_{N-I}$ because, as shown in the next Subsection, its numerical values are not yet significant enough to warrant a dedicated discussion.

2.3 Results on single oscillation parameters

In this Subsection we graphically report the results of our global analysis for each single oscillation parameter, making use of an of increasingly richer data sets, grouped in accordance with the methodology discussed before.
Figures 1, 2 and 3 show the $N\sigma$ curves for the data sets defined in the previous Subsection. In each figure, the solid (dashed) curves refer to NH (IH); the two curves basically coincide for $\delta m^2$ and $\theta_{12}$, since they are determined by Solar+KL data, which are largely insensitive to the hierarchy.

Figure 1 refers to the combination LBL Acc. + Solar + KL, which, by itself, sets highly significant lower and upper bounds on all the oscillation parameters but $\delta$. In the figure, the relatively strong appearance signal in T2K [59] dominates the lower bound on $\theta_{13}$, and also drives the slight but intriguing preference for $\delta \sim 1.5\pi$: indeed, for $\sin \delta \sim 1$, the CP-odd term in the $\nu_{\mu} \rightarrow \nu_e$ appearance probability [61, 62] is maximized [59]. It should be noted that current MINOS appearance data generally prefer $\sin \delta > 0$ [64, 65]; however, the stronger T2K appearance signal largely dominates in the global fit. On the other hand, MINOS disappearance data [64, 65] drive the slight preference for non-maximal $\theta_{23}$, as compared with nearly maximal $\theta_{23}$ in T2K [58, 60]. The (even slighter) preference for the second $\theta_{23}$ octant is due to the interplay of LBL accelerator and Solar + KL data, as discussed in the next Subsection.

Figure 2 shows the results obtained by adding the SBL reactor data, which strongly reduce the $\theta_{13}$ uncertainty. Further effects of these data include: (i) a slightly more pronounced preference for $\delta \sim 1.5\pi$ and $\sin \delta < 0$, and (ii) a swap of the preferred $\theta_{23}$ octant with the hierarchy ($\theta_{23} < \pi/4$ in NH and $\theta_{23} > \pi/4$ in IH).
Figure 3 shows the results obtained by adding the SK atmospheric data, thus obtaining the most complete data set. The main differences with respect to Fig. 2 include: (i) an even more pronounced preference for \( \sin \delta < 0 \), with a slightly lower best fit at \( \delta \sim 1.4\pi \); (ii) a slight reduction of the errors on \( \Delta m^2 \) and a relatively larger variation of its best-fit value with the hierarchy; (iii) a preference for \( \theta_{23} \) in the first octant for both NH and IH, which is a persisting feature of our analyses. The effects (ii) and (iii) show that atmospheric neutrino data have the potential to probe subleading hierarchy effects, although they do not yet emerge in a stable or a significant way.

In the three figures an intriguing feature is the increasingly pronounced preference for nonzero CP violation with increasing data sets, although the two CP conserving cases (\( \delta = 0, \pi \)) remain allowed at \(< 2\sigma \) in both NH and IH, even when all data are combined (see Fig. 3). It is worth noticing that the two maximally CP-violating cases (\( \sin \delta = \pm 1 \)) have opposite likelihood: while the range around \( \delta \sim 1.5\pi \) (\( \sin \delta = -1 \)) is consistently preferred, small ranges around \( \delta \sim 0.5\pi \) (\( \sin \delta = +1 \)) appear to be disfavored (at more than \( 2\sigma \) in Fig. 3). In the next few years, the appearance channel in LBL accelerator experiments will provide crucial data to investigate these hints about \( \nu \) CP violation, with relevant implications for models of leptogenesis.

From the comparison of the three figures one can also notice a generic preference for non-maximal mixing (\( \theta_{23} \neq 0 \)), although it appears to be weaker than in our previous analyses, essentially because the most recent T2K data [58, 60] prefer nearly maximal mixing, and thus “dilute” the opposite preference coming from MINOS [63, 65] and atmospheric data [30]. Moreover, the
indications about the octant appear to be somewhat unstable in different combinations of data. In the present analysis, only atmospheric data consistently prefer the first octant in both hierarchies, but the overall significance remains at the level of $\sim 2\sigma$ in NH and is much lower in IH. These fluctuations show how difficult is to reduce the allowed range of $\theta_{23}$. In this context, the disappearance channel in LBL accelerator experiments will provide crucial data to address the issue of non-maximal $\theta_{23}$ in the next few years.

2.4 Global $3\nu$ analysis: correlations between $\theta_{13}$, $\theta_{23}$ and $\delta$

In this Subsection we show the allowed regions for selected couples of oscillation parameters, and discuss some interesting correlation effects.

Figure 4 shows the results of the analysis in the plane $(\sin^2 \theta_{13}, \sin^2 \theta_{23})$, for both normal hierarchy (NH, upper panels) and inverted hierarchy (IH, lower panels). It is understood that all the other parameters are marginalized away. From left to right, the panels refer to increasingly rich datasets: LBL accelerator + Solar + KamLAND data (left), plus SBL reactor data (middle), plus SK atmospheric data (right).

In the left panels, a slight negative correlation emerges from LBL appearance data, since the dominant oscillation amplitude contains a factor $\sin^2 \theta_{23} \sin^2 \theta_{13}$ via Eq. (2.2). The contours extend towards relatively large values of $\theta_{13}$, in particular for IH, in order to accommodate the relatively
Figure 4: Results of the analysis in the plane charted by \((\sin^2 \theta_{23}, \sin^2 \theta_{13})\), all other parameters being marginalized away. From left to right, the regions allowed at 1, 2 and 3\(\sigma\) refer to increasingly rich datasets: LBL+solar+KamLAND data (left panels), plus SBL reactor data (middle panels), plus SK atmospheric data (right panels). Best fits are marked by dots. The three upper (lower) panels refer to normal (inverted) hierarchy.

strong T2K appearance signal \[59\]. However, Solar + KamLAND data provide independent (although weaker) constraints on \(\theta_{13}\) and, in particular, prefer \(\sin^2 \theta_{13} \sim 0.02\) in our analysis. This value is on the “low” side of the allowed regions and thus responsible for the relatively high value of \(\theta_{23}\) at best fit, namely, for the second octant preference in both NH and IH. However, when current SBL reactor data are included (middle panels), a slightly higher value of \(\theta_{13}\) (\(\sin^2 \theta_{13} \simeq 0.023\)) is preferred with very small uncertainties: this value is high enough to shift the best-fit value of \(\theta_{23}\) from the second to the first octant in NH, but not in IH. Finally, the inclusion of SK atmospheric data (right panels) provides in our analysis an overall preference for the first octant, which is however quite weak in IH. Unfortunately, as previously mentioned, the current hints about the \(\theta_{23}\) octant do not appear particularly stable or convergent.

Figure 5 shows the results of the analysis in the plane \((\sin^2 \theta_{13}, \delta/\pi)\). The conventions used are the same as in Fig. 4. Since the boundary values \(\delta/\pi = 0\) and 2 are physically equivalent, each panel could be ideally “curled” by smoothly joining the upper and lower boundaries.

The behavior of the CP violating phase \(\delta\) is at the focus of current research in neutrino physics. In the left panels of Fig. 5 there is a remarkable preference for \(\delta \sim 1.5\pi\), with a compromise reached between the relatively high values of \(\theta_{13}\) preferred by the T2K appearance signal and the relatively low values preferred by Solar + KL data. In the middle panel, SBL reactor data strengthen this
Figure 5: Results of the analysis in the plane charted by \((\sin^2 \theta_{13}, \delta)\), all other parameters being marginalized away. From left to right, the regions allowed at 1, 2 and 3\(\sigma\) refer to increasingly rich datasets: LBL+solar+KamLAND data (left panels), plus SBL reactor data (middle panels), plus SK atmospheric data (right panels). A preference emerges for \(\delta\) values around \(\pi\) in both normal hierarchy (NH, upper panels) and inverted hierarchy (IH, lower panels).

In the light of the recent results coming from reactor and accelerator experiments, and of their interplay with solar and atmospheric data, we have estimated \(N\sigma\) ranges of the known 3\(\nu\) parameters, \(\Delta m^2\), \(\delta m^2\), \(\theta_{12}\), \(\theta_{23}\), \(\theta_{13}\), and we have revisited the status of the current unknowns, \(\text{sign}(\Delta m^2)\), \(\text{sign}(\theta_{23} - \pi/4)\) and CP violation phase \(\delta\).

In order to understand how the various constraints and hints emerge from the analysis, and to appreciate their (in)stability, we have considered increasingly rich data set, starting from the combination of LBL accelerator + Solar plus KamLAND data, then adding SBL reactor data, and finally including atmospheric data. We have discussed the results both on single parameters and on selected couples of correlated parameters.

The results of the global analysis of all data are shown in Fig. 3, from which one can derive the ranges of the known parameters. One can appreciate the high accuracy reached in the determination...
of the known oscillation parameters; in particular, as compared with a previous analysis [50], one can appreciate a significant reduction of the $\theta_{13}$ uncertainties, and some changes in the $(\Delta m^2, \theta_{23})$ ranges.

We have also discussed in some detail the status of the unknown parameters. It turns out that the hints about $\theta_{23}$ octant appear somewhat unstable at present, while those about $\delta$ (despite being statistically weaker) seem to arise from an intriguing convergence of several pieces of data. Concerning the hierarchy, i.e. sign$(\Delta m^2)$, we find no significant difference between normal and inverted mass ordering. However, assuming normal hierarchy, we find possible hints about the other two unknowns, namely: a slight preference for the first $\theta_{23}$ octant, and a possible indication for non-zero CP violation (with $\sin \delta < 0$), although at the level below 2$\sigma$ for both the two cases. Note that the second hint appears also in inverted hierarchy, but with even lower statistical significance.

3. The Future

3.1 $\theta_{13}$

This parameter is already well measured, but it is important to improve its estimate, since the measurements of the phase $\delta_{\text{CP}}$ and of the mass hierarchy sign$(\Delta m^2)$ are strongly sensitive to the precise determination of $\theta_{13}$. The estimate of Daya Bay has been recently improved: $\sin^2 2\theta_{13} = 0.084 \pm 0.005$, with an impressive improvement in the last year [77]. However, the total uncertainty is still dominated by statistics, so that the measurement can be further improved.

Prospects for precision measurements of $\sin^2 2\theta_{13}$ with reactor antineutrinos at Daya Bay are given in [78]. Accordingly, the total uncertainty can be reduced to 0.003 in 2 or 3 years. With this uncertainty, the significance is evaluated with which $\delta_{\text{CP}}$ can be measured at NOvA + T2K and at NOvA + T2K and LBNE [78].

3.2 $\theta_{23}$ and its octant

From the $\nu_{\mu}$ disappearance at LBL accelerators one can estimate $\Delta m^2$ and $\sin^2 \theta_{23}$, using $\sin^2 \theta_{13}$ measured at reactors (and then independent of $\delta_{\text{CP}}$). The most recent measurement has been presented by T2K only a few days ago [79]: the 68% and 90% regions of $\Delta m^2$ vs. $\sin^2 \theta_{23}$ are shown. It is interesting to note that these regions are smaller than the expected sensitivity, since the best-fit point is near to the boundary of maximal disappearance. With the best point so near to the maximal disappearance, the $\theta_{23}$ octant degeneracy seems rather difficult to be solved.

3.3 $\delta_{\text{CP}}$

The $\nu_{\mu}$ appearance measurements at LBL accelerators are particularly sensitive to $\delta_{\text{CP}}$. Again T2K reports on a very recent estimate based on joint $\nu_{\mu}$ disappearance and $\nu_e$ appearance analysis, with $\Delta m^2$, $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$ and $\delta_{\text{CP}}$ unknown. However a similar analysis has little power to constrain $\delta_{\text{CP}}$ without the reactor measurement of $\sin^2 \theta_{13}$. In other words, in order to constrain $\delta_{\text{CP}}$, it is mandatory to include the estimate of $\sin^2 \theta_{13}$ from reactors. The effect of including the “ultimate” $\sin^2 \theta_{13}$ as measured at reactors in the analysis of the T2K data is well described in the last T2K paper [79], in particular by identifying the regions excluded at the 90% of C.L.: more precisely, it is estimated $\delta_{\text{CP}}/\pi = [-0.08, 1.09]$ excluded at 90% of C.L. in the case of inverted
Figure 6: The figure is taken from ref. [81]. The left (right) panel shows, for the different experiments as a function of the time in years, the median sensitivity in number of $\sigma$’s for rejecting the IH (NH) hypothesis if the NH (IH) is true. The width of each band corresponds to different true values of $\delta_{CP}$ for NOvA [82] and LBNE [83], different true values of $\theta_{23}$ between 40° and 50° for INO [85] and PINGU [84], and energy resolution between 3% $\sqrt{1 \text{MeV}/E}$ and 3.5% $\sqrt{1 \text{MeV}/E}$ for JUNO [86]. For LBL experiments, the bands with solid (dashed) contours correspond to a true value for $\theta_{23}$ of 40° (50°).

The ability of T2K of measuring $\delta_{CP}$ would be greatly enhanced by the knowledge of the mass hierarchy, with a consequent breaking of the degeneracy. Unfortunately, T2K does not have sufficient sensitivity to determine the mass hierarchy by itself. But a similar sensitivity is achieved by NOvA, which has a longer baseline (10 km.) and a higher peak neutrino energy ($\sim 2 \text{GeV}$), which means a larger impact of matter effects and a greater sensitivity to the mass hierarchy. A comparison of the $\nu_e$ appearance data of the two experiment is discussed in [80], for specific ranges of values of $\delta_{CP}$, $\sin^2 \theta_{23}$ and both the two mass hierarchies.

3.4 The neutrino mass hierarchy: sign($\Delta m^2$)

Maybe the most fascinating item of neutrino physics. No indications so far from the current experiments, but we hope to solve the dilemma within the next ten years. The following three types of experiments are expected to compete in determining the neutrino mass ordering:

- Long Baseline (LBL) accelerator neutrino experiments, as NOvA [82] and LBNE [83], studying matter effects in $\nu_\mu \rightarrow \nu_e$ appearance.

- Atmospheric neutrino experiments, as PINGU [84] and INO [85], studying matter effects in atmospheric neutrino experiments.
Medium baseline experiments (MBL) reactor neutrino experiments, specifically JUNO [86], studying in vacuum the interference between solar and atmospheric oscillation amplitudes.

A detailed study of sensitivity and discovery potential of these experiments is beyond the scopes of this talk. Many studies exist in literature, based on the available details of each experimental apparatus, taking into account efficiencies, energy resolution, angular resolution, systematics, etc. We close by reporting in Fig. 6, taken from ref. [81], a detailed comparison of the sensitivity of each of the cited experiments, in terms of number of σ’s, plotted in terms of the time-scale. Due to the dichotomous character of the neutrino mass ordering, the sensitivity is plotted on the left for rejecting IH if NH is true, and vice versa on the right.

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