PROCEEDINGS OF SCIENCE

CP violation

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The most recent understanding about CP violation in the quark sector is presented, mainly mixing-induced CP violation in B_d and B_s meson decays are described. In order to go further than testing the Kobayashi-Maskawa theory, interesting attempts have been made to constrain possible penguin contribution in the CP violation angles $\phi_1 = \beta$ and ϕ_s to establish the firm Standard Model (SM) anchor-point to search for the new physics. In addition, the CP violation measurements to constrain the angles $\phi_2 = \alpha$ and $\phi_3 = \gamma$ as well as in charm meson decays are also mentioned.

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1. Introduction

CP violation is one of the key ingredients to understand the today matter-dominant universe [1]. In the Standard Model (SM), an irreducible complex phase in the quark-mixing matrix, Kobayashi-Maskawa (KM) matrix, causes *CP* violation [2] in the quark sector. *CP* violation was first observed in 1964 as the $K_L^0 \to \pi\pi$ decay [3], after that it took more than 30 years to realize through comprehensive experimental test by the B meson system.

Because of the unitarity of the KM-matrix, the terms related to the B_d and $B^\pm$ decays are expected to hold following relation:

$$V_{td}V_{tb}^* + V_{cd}V_{cb}^* + V_{ud}V_{ub}^* = 0. \tag{1.1}$$

It can be expressed as a closed triangle in the complex plane and we call it the unitarity triangle. In the Wolfenstein parameterization [4], which express the KM matrix components by an expansion of $\lambda = \sin\theta_c$, where θ_c is the Cabbibo angle, all the three side lengths of this triangle are $\mathscr{O}(\lambda^2)$. Consequently the *CP* asymmetries in the proper B_d and $B^\pm$ decays are expected to be $\mathscr{O}(0.1)$ that give rich variety of opportunities for studying *CP* violation phenomena to reach a comprehensive understanding. The $B_d - \overline{B}_d$ mixing involves the V_{td} containing the complex phase denoted as $\bar{\eta}$ thus it gives rise to *CP*-violating asymmetries in the time-dependent rates of B_d and $\overline{B}_d$ decays into a common *CP* eigenstate [5]. Among the B_d decays into a *CP* eigenstate, the ones caused by $b \to c$ transition are the suitable processes to measure the time-dependent *CP* violation to get the angle $\phi_1 = \beta \equiv \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$. In the B_d decays governed by $b \to u$ transition are sensitive to determine the angle $\phi_2 = \alpha \equiv \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$. The direct *CP* asymmetries in $B \to D^{(*)0}K^{(*)}$ decays are the proper quantities to obtain the angle $\phi_3 = \gamma \equiv \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$.

In the B_s system, the corresponding unitarity triangle relation is;

$$V_{ub}^*V_{us} + V_{cb}^*V_{cs} + V_{tb}^*V_{ts} = 0. \tag{1.2}$$

Here, the first term is $\mathscr{O}(\lambda^4)$ while the second and third terms are $\mathscr{O}(\lambda^2)$, thus the mixing induced *CP* violation parameter in B_s system, $\phi_s = -2\arg(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*)$ is $\mathscr{O}(0.01)$ quantity. Similar to the B_d case, the $b \to c$ transition induced B_s decays into f_{CP} are the suitable process to measure the *CP* violation angle ϕ_s. For the B_s system, because of the large production rate as well as the higher boost to resolve its fast oscillation, high energy pp (or $\bar{p}p$) colliding beam experiments are carrying out the time-dependent *CP* violation measurements.

All the *CP* violation measurements mentioned above are using the B_d, $B^\pm$ and B_s decays governed by the tree diagrams, thus those are suitable to determine *CP* violation parameters in the SM. Now it becomes important to constrain the potential penguin sub-leading contributions in these decays, as the reference point to discuss the effects coming from new physics (NP) to the *CP* violation in $b \to s$ and $b \to d$ mediated rare B decay modes at the coming higher statistics experiments. *CP* violation in charm provides a complementary role as a search for NP. Recent these activities are reviewed in this text.

2. Time-dependent *CP* violation in *B* meson system at *B*-factories and LHCb

In order to perform time-dependent *CP* violation measurements, the experiments have to satisfy following conditions; (i) producing enough number of B_d or B_s mesons and recording their decays, (ii) reconstruction of the B_d or B_s decays into the CP eigenstate f_{CP}, (iii) tagging its b-flavor and (iv) measuring the time evolution from the B decay vertices. Taking the branching fractions of B_d or B_s decays into the CP eigenstates, an excellent performance accelerator is necessary to satisfy (i), 10^{34} cm^{-2}s^{-1} in the e^+e^- colliders at the $\Upsilon(4S)$ and 10^{32} cm^{-2}s^{-1} in the LHCb environment. For (ii) and (iii), an enough large acceptance high resolution spectrometer with particle identification capability is required. Superb B decay vertices detection is apparently indispensable to realize (iv), the asymmetric-energy e^+e^- collision is the key condition at B-factories in addition. In both e^+e^- B-factories and LHCb experiments, all these conditions have been satisfied, but there are differences in the selected technical solutions and resultant features.

At the asymmetric-energy e^+e^- B-factories, very high efficiency for the B meson pair production events are generically obtained, nearly 100%. In the event of interest, one of B_d mesons decays into a CP eigenstate, f_{CP}, at the time t_{CP} while the accompanying B meson decays into the b-flavor specific final state, $f_{\rm tag}$ at the time $t_{\rm tag}$, and the time evolution is to be measured as a function of the proper time difference, $\Delta t \equiv t_{CP} - t_{\rm tag}$. The time-dependent CP asymmetry is expressed as

$$A_{CP}(\Delta t) = \frac{\Gamma(\overline{B}_d(\Delta t) \to f_{CP}) - \Gamma(B_d(\Delta t) \to f_{CP})}{\Gamma(\overline{B}_d(\Delta t) \to f_{CP}) + \Gamma(B_d(\Delta t) \to f_{CP})} = \mathscr{S}_{f_{CP}} \sin(\Delta m_d \Delta t) + \mathscr{A}_{f_{CP}} \cos(\Delta m_d \Delta t), \quad (2.1)$$

where $\Gamma(\overline{B}_d(\Delta t) \to f_{CP})$ ($\Gamma(B_d(\Delta t) \to f_{CP})$), Δm_d, $\mathscr{S}_{f_{CP}}$ and $\mathscr{A}_{f_{CP}}$ ($\mathscr{C}_{f_{CP}} = -\mathscr{A}_{f_{CP}}$ is also used in some literature) denote the corresponding time-dependent $\overline{B}_d$ (B_d) decay rate, $B_d - \overline{B}_d$ mixing frequency, mixing induced and direct CP asymmetries, respectively. In the $b \to c$ transition induced B_d decay to f_{CP} cases, the mixing induced CP violation, $\mathscr{S}_{f_{CP}} = -\eta_f \sin 2\phi_1 = -\eta_f \sin 2\beta$ and $\mathscr{A}_{f_{CP}} = 0$ are predicted in the SM, where η_f is the CP eigenvalue of the final state f_{CP}. In the case of two-body B_d decays where both dauthters are vector mesons, generally such final states are admixture of CP-even and CP-odd states and it is necessary to determine CP-even (or CP-odd) fraction in the signal events from the angular distribution of decay products. Because of the clean environment to produce only one B meson pair in an event, flagger tagging performance is excellent to get 30% of the effective tagging efficiency, $\varepsilon(1-2w)^2$, where ε and w are the flavor tagging efficiency and wrong tag fraction, respectively. While since the Δt resolution (~ 500 fs) is approximately one third of the B_d lifetime, its resolution function has to be precisely estimated.

At the LHCb experiment, the single-arm spectrometer detects the b-hadrons produced in forward direction in p-p collisions provided by the LHC accelerator. Since there are extra particles production in addition to the target b-hadron creation, its time evolution is measured from its production point. So the Δt in eq. (2.1) is replaced by t. Wrong tag fraction is higher thus effective tagging efficiency is approximately 3%. To bring competitive measurements, the lower flavor tagging effective efficiency is compensated by the large b-hadron production rate, 1000 or 2000 times as large the B decay signal yields as the e^+e^- collider-based B-factories per unit integrated luminosity in ideal cases. Note that actual B meson signal yields depends on the decay modes because the trigger and reconstruction efficiencies vary. On the other hand, because of the larger boost of the created b-hadrons, much better proper time resolution (~ 50 fs) can be achieved.

Exploiting these features, the B-factories and the LHCb are providing interesting and important measurements of the time-dependent CP violation in B_d and B_s decays. Some of them are competitive and some other are complementary each other. Recent results are based on the integrated luminosities of 433 fb^{-1} and 711 fb^{-1} at BaBar and Belle experiments, respectively. While LHCb accumulated 3 fb^{-1} during Run-1 period.

3. Measurements of $\sin 2\phi_1 = \sin 2\beta$

The time-dependent *CP* violation in the $B_d \to f_{CP}$ decays induced by $b \to c\bar{c}s$ transition such as $B_d \to J/\psi K^0_S$ gives the *CP* violation angle $\phi_1 = \beta$. In addition to the precision measurements carried out by BaBar and Belle collaborations, the LHCb experiment has brought their result. The resultant *CP* violation parameters are found to be $\mathscr{S}_{f_{CP}} = 0.731 \pm 0.035(\mathrm{stat}) \pm 0.020(\mathrm{syst})$ and $\mathscr{A}_{f_{CP}} = -\mathscr{C}_{f_{CP}} = 0.038 \pm 0.032(\mathrm{stat}) \pm 0.005(\mathrm{syst})$ [6].

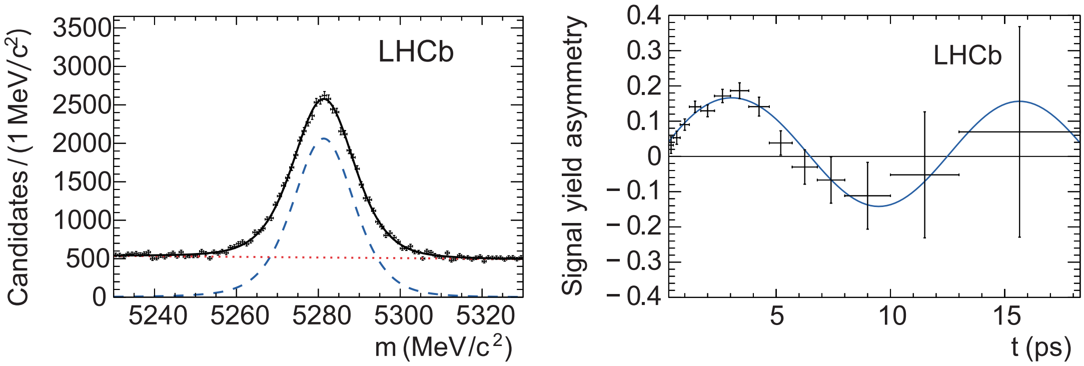

Figure 1: Distribution of the reconstructed mass (left) and time evolution of the asymmetry (right) in tagged $B_d \to J/\psi K^0_S$ candidates [6].

Now the world average is given to be $\sin 2\phi_1 = 0.69 \pm 0.02$ [7] by combining with the last measurements brought by BaBar [8] and Belle [9] collaborations, $\sin 2\beta = 0.687 \pm 0.028(\mathrm{stat}) \pm 0.012(\mathrm{syst})$ and $\sin 2\phi_1 = 0.668 \pm 0.023(\mathrm{stat}) \pm 0.013(\mathrm{syst})$, respectively. Now the LHCb experiment's capability has been demonstrated by bringing competitive results with the ones in *B*-factory experiments.

4. Penguin-free decay of $B_d \to D^{(*)0}_{CP} h^0$

Though the leading term of the $b \to c\bar{c}s$ transition is the tree diagram, the sub-leading one is the $b \to s$ penguin. In the SM, $b \to s$ penguin contains no additional complex phase and the additional $c\bar{c}$ pair formation is the OZI-suppressed diagram. Therefore the theoretical uncertainty in the time-dependent *CP* violation in $b \to c\bar{c}s$ is thought to be small. However, because of the one-loop transition nature, the effect coming from the NP might not be zero, therefore possible ways to constrain or avoid the potential penguin pollution are desired.

Using a "Penguin-free" B_d decays to f_{CP} is one possible approach. The leading term of $b \to c\bar{u}d$ transition is tree diagram and has no complex phase. This transition causes $B_d \to D^{(*)0} h^0$ decays, where h^0 denotes a light neutral hadron. The $B_d \to D^{(*)0} h^0$ decays' sub-leading term is also the tree diagram with the doubly Cabbibo-suppression. Therefore this B_d decay mode is regarded to be "Penguin-free". Though the sub-leading diagram has the V_{ub} with the complex phase, it is the tree diagram thus theoretically under control within the SM.

So far, the low branching fraction of the neutral $D^{(*)}$ meson decays to a *CP* eigenstate has been limiting the sensitivity. As the solution to overcome it, the joint analysis using BaBar and Belle data has been performed. Hereafter, D^0_{CP} denotes the neutral *D* meson decaying to the *CP* eigenstates of K^+K^-, $K^0_S\pi^0$ or $K^0_S\omega$ modes. D^{*0}_{CP} represents the neutral D^{*0} mesons reconstructed by the $D^0_{CP}\pi^0$ final state and thus $D^{(*)0}_{CP}$ is a general term for D^0_{CP} and D^{*0}_{CP}. π^0, η or ω are reconstructed as the h^0. $B_d \to D^{(*)0}_{CP} h^0$ decay signal yields are obtained to be 508 ± 31 events (BaBar) plus 757 ± 44 events (Belle) as shown in Figure 2. The resultant time-dependent *CP* violation parameters are found to be $-\eta_f \mathscr{S}_{fCP} = +0.66 \pm 0.10(\text{stat}) \pm 0.06(\text{syst})$ and $\mathscr{A}_{fCP} = -\mathscr{C}_{fCP} = 0.02 \pm 0.07(\text{stat}) \pm 0.03(\text{syst})$ [11]. The Δt distributions as well as time-dependent asymmetries for the BaBar and Belle combined sample is also shown in Figure 2.

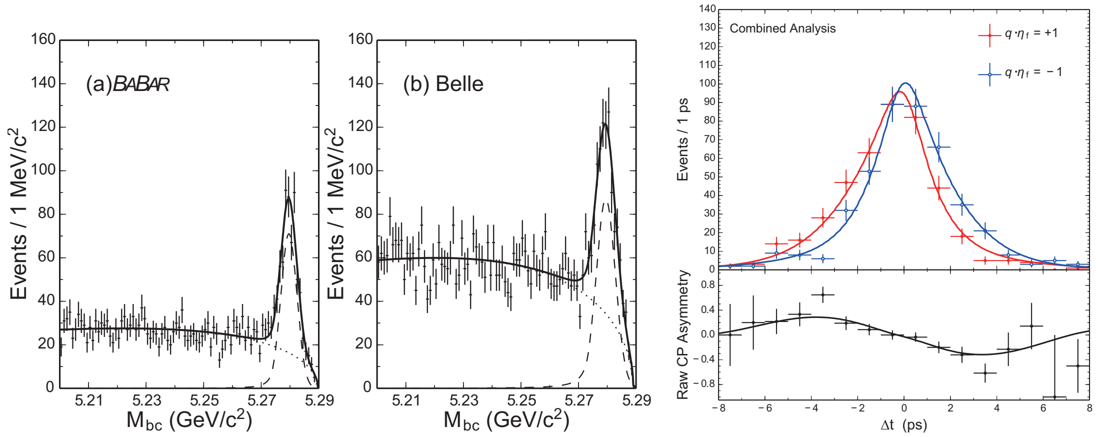

Figure 2: Distribution of the M_{bc} distributions for the reconstructed $B_d \to D^{(*)0}_{CP} h^0$ candidates in BaBar (left) and Belle (middle) data together with the Δt distributions (right upper) with classifying events according to $q \cdot \eta_f$ where q and η_f are *b* flavor and the *CP* eigenvalue of the final state. The raw *CP* asymmetry as a function of Δt (right lower) is also shown. [11].

Scaling this result to the full statistics of Belle II experiment at SuperKEKB, 50 ab^{-1}, we expect the uncertainty becomes down to $\sim \pm 0.015$ that is the same level as the current $\sin 2\phi_1$ determination by the $B_d \to (c\bar{c})K^0$ decays. Hence the time-dependent *CP* violation in $B_d \to D^{(*)0}_{CP} h^0$ can give us a new additional reference of the *CP* violation angle $\phi_1 = \beta$ in the Belle II era.

5. Studies of $B_d \to \rho\rho$ decays to constrain $\phi_2 = \alpha$

Measurements of the time-dependent *CP* violation in the $b \to u$ transition induced B_d decays to the flavor non-specific final states such as $\pi\pi$, $\rho\rho$ and $\rho\pi$ are sensitive to the angle $\phi_2 = \alpha$. In order to solve possible penguin pollution, relevant $B^\pm$ decay modes' branching fractions and direct *CP* asymmetries are also important to perform an isospin analysis [12] to extract $\phi 2 = \alpha$.

Belle come up with the final result of $B_d \to \rho^+\rho^-$ mode using its final $\Upsilon(4S)$ data sample [13]. The resultant branching fraction, longitudinal polarization fraction as well as the mixing-induced and direct *CP* violation parameters are found to be; $\mathscr{B}(B_d \to \rho^+\rho^-) = (28.3 \pm 1.5(\text{stat}) \pm 1.5(\text{syst})) \times 10^{-6}$, $f_L = 0.988 \pm 0.012(\text{stat}) \pm 0.023(\text{syst})$, $\mathscr{S}_{fCP} = -0.13 \pm 0.15(\text{stat}) \pm 0.05(\text{syst})$ and $\mathscr{A}_{fCP} = -\mathscr{C}_{fCP} = 0.00 \pm 0.10(\text{stat}) \pm 0.06(\text{syst})$. Precision of the resultant *CP* violation parameters is improved factor 2 with respect to the previously published result. This is due to not only increase of data but also simultaneous extraction of observables and analysis optimization for high signal yield. As a complementary measurement to constrain the angle $\phi_2 = \alpha$, LHCb has brought the result of $B_d \to \rho^0\rho^0$ mode [14]. The $B_d \to (\pi^+\pi^-)(\pi^+\pi^-)$ signal yield is found to be $634 \pm 28(\text{stat}) \pm 8(\text{syst})$ events. Out of this event sample, using di-pion mass, helicity and azimuthal angle distributions, the net $B_d \to \rho^0\rho^0$ contribution is extracted and the branching fraction is determined to be $\mathscr{B}(B_d \to \rho^0\rho^0) = (0.94 \pm 0.17(\text{stat}) \pm 0.09(\text{syst}) \pm 0.06(\text{BF})) \times 10^{-6}$ that is the most precise to date. The longitudinal polarization fraction f_L is found to be $0.745^{+0.048}_{-0.058}(\text{stat}) \pm 0.034(\text{syst})$, consistent with BaBar measurement while 2.3σ away from Belle. Using these newest information, the most updated value is $\phi_2 = \alpha = 90.6^{+3.9\circ}_{-1.1}$.

6. B_s decays to CP eigenstates to obtain ϕ_s

As already mentioned, the time-dependent measurement in the B_s decays to a CP eigenstates can access the angle $\phi_s = -2\beta_s + \Delta\phi_s^P + \delta^{\text{NP}}$, where $\Delta\phi_s$ and δ^{NP} are the possible SM penguin and NP contributions, respectively. Here, $\beta_s = \arg(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*)$ and including other potential contributions, ϕ_s is $\mathscr{O}(0.01)$ quantity. The time-dependent CP asymmetry can be expressed as

$$A_{CP}(t) = \frac{\Gamma(\overline{B}_s{}^0(t) \to f_{CP}) - \Gamma(B_s^0(t) \to f_{CP})}{\Gamma(\overline{B}_s{}^0(t) \to f_{CP}) + \Gamma(B_s^0(t) \to f_{CP})} = \frac{\mathscr{S}_{f_{CP}} \sin(\Delta m_s t) + \mathscr{A}_{f_{CP}} \cos(\Delta m_s t)}{\cosh(\Delta\Gamma t/2) + \mathscr{A}_{\Delta\Gamma} \sinh(\Delta\Gamma t/2)} \qquad (6.1)$$

where, Δm_s and $\Delta\Gamma$ are the $B_s - \overline{B_s}$ oscillation frequency and decay width difference between the two neutral B_s meson mass eigenstates, respectively.

So far, measurements of $B_s \to J/\psi K^+K^-$ mode have been carried out by ATLAS, CMS and LHCb experiments, LHCb experiment also has brought $B_s \to J/\psi\pi^+\pi^-$ and $B_s \to D_s^+D_s^-$ results. The most recent world average is obtained to be $\phi_s = -0.034 \pm 0.033$ rad [15].

7. Exploiting $SU(3)$ flavor symmetry relation to constrain potential penguin pollution in $b \to c$ decays

The $b \to c\bar{c}d$ transition induced B_d decays can play interesting role to constrain possible penguin pollution in the measurements to constrain the CP violation angles of ϕ_1 and ϕ_s. The sub-leading term of $b \to c\bar{c}d$ is the penguin and because of an additional involvement of V_{td} and its complex phase in the decay amplitude, its effect may become more sizable in the time-dependent CP violation. Therefore, with exploiting plausible assumption based on the $SU(3)$ flavor symmetry, the penguin effect in the $b \to c\bar{c}s$ modes are constrained [16]. Among relevant B to charmonium modes, $B_d \to J/\psi\pi^0$ gives a constraint on the $\phi_1 = \beta$ determination by $B_d \to J/\psi K^0$ mode, similarly $B_d \to J/\psi\rho^0$ and $B_s \to J/\psi K^*$ modes can constrain the ϕ_s determination by $B_s \to J/\psi\phi$.

In the most recent LHCb measurement, 17650 ± 200 $B_d \to J/\psi\pi^+\pi^-$ signal yield is observed and 65% of the signal is due to $J/\psi\rho^0$. Since this is a B meson decay to two vector mesons, it is generally an admixture of CP-even and CP-odd. Angular analysis for the decay daughter particles tells that CP-odd fraction is 19.8 ± 1.7 % thus it is mostly CP-even. The CP violation angle is obtained to be $2\beta^{\text{eff}} = (41.7 \pm 9.6^{+2.8}_{-6.3})^\circ$ and the shift in ϕ_s is limited within the interval from -1.05° upto $+1.18^\circ$ at 95% confidence level [17].

8. Other topics

For the *CP* violation angle $\phi_3 = \gamma$, new measurements such as $B^+ \to D^0 h^+$ followed by $D^0 \to K^0_S h^+ h^-$ [18], $B^- \to DK^-\pi^+\pi^-$ and $B^- \to D\pi^-\pi^+\pi^-$ [19] have been brought by LHCb experiment, but there is no major change in $\phi_3 = \gamma$ determination itself from last year, $\phi_3 = \gamma = 73.2^{+6.3\circ}_{-7.0}$. *CP* violation in charm is a good window to look for NP, but so far no significant asymmetry appears, including the recent direct *CP* asymmetry measurement for $D^0 \to K^0_S K^0_S$ mode by LHCb experiment, $A_{CP}(K^0_S K^0_S) = -(2.9 \pm 5.2(\text{stat}) \pm 2.2(\text{syst}))\%$. These topics are to be pursuit of with the coming LHCb Run2 data accumulation as well as the Belle II data.

9. Summary and prospect

Mixing induced *CP* violation in B_d and B_s mesons require very precise discussion to settle firm SM reference. It is thought to be necessary step to hunt NP in penguin induced B decays. As the possible solutions, the BaBar+Belle joint analysis on the penguin-free $B_d \to D^{(*)0} h^0$ mode has been performed. Exploiting the $SU(3)$ relation, *CP* violation measurements in the $b \to c\bar{c}d$ transition induced B_d decays can constrain potential penguin effects in ϕ_s as well as $\phi_1 = \beta$ determinations. With Belle $B_d \to \rho^+\rho^-$ and LHCb $B_d \to \rho^0\rho^0$ measurements, $\phi_2 = \alpha = 90.6^{+3.9\circ}_{-1.1}$.

LHCb runs has started, while Belle II physics run starts in 2018, therefore a well-matched game is anticipated around 2020. Even before, innovative efforts should be paid to realize novel ideas to keep continuous physics outputs, there is a very exciting competition ahead.

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