Positive parity D_s mesons

Luka Leskovec[*]

Jozef Stefan Institute, Ljubljana, Slovenia
Department of Physics, University of Arizona, Tucson, Arizona, USA
E-mail: leskovec@email.arizona.edu

C.B. Lang

Institut für Physik, University of Graz, Graz, Austira
E-mail: christian.lang@uni-graz.at

Daniel Mohler

Fermi National Accelerator Laboratory, Batavia, Illinois, USA
Helmholtz-Institut Mainz, Johannes Gutenberg-Universität Mainz, Germany
E-mail: dmohler@fnal.gov

Sasa Prelovsek

Jozef Stefan Institute, Ljubljana, Slovenia
Faculty of Mathematics and Physics, University of Ljubljana, Slovenia
E-mail: sasa.prelovsek@ijs.si

R.M. Woloshyn

TRIUMF, Vancouver, Canada
E-mail: rwww@triumf.ca

We study the positive parity charmed strange mesons using lattice QCD, the only reliable *ab initio* method to study QCD at low energies. Especially the experimentally observed $D_{s0}^*(2317)$ and $D_{s1}(2460)$ have challenged theory for quite some time. The dynamical lattice QCD simulations are performed at two distinct pion masses, $m_\pi = 266$ MeV and 156 MeV, using both $\bar{c}s$ as well as DK and D^*K scattering operators in the construction of the correlation matrix in order to take into the account threshold effects. While the $J^P = 0^+$ channel benefited most from the inclusion of scattering operators, it was also crucial for the case of the $D_{s1}(2460)$. Using the Lüscher method, which relates the finite volume spectrum to the infinite volume scattering matrix, we were able to determine the near threshold behavior of the scattering amplitude. From it we extracted the binding momenta and the masses of the below-threshold bound states $D_{s0}^*(2317)$ and $D_{s1}(2460)$ by determining the pole positions of the scattering amplitudes. Our results compare well with experiment, resolving a long standing discrepancy between theory and experiment.

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[*]Speaker.

1. Introduction

The positive parity D_s mesons, especially the $D_{s0}(2317)$ and $D_{s1}(2460)$, have until recently presented an unsolved puzzle in theoretical physics. Experimentally their masses lie below the *DK* and D^*K thresholds respectively [1], however neither quark models nor lattice QCD studies were able to reproduce this phenomenon uncontroversially (see Refs. [2, 3] and references therein). Early quenched lattice studies, that ignored sea quark contributions, only took into account $c\bar{s}$ operators and found results consistent with the quark model expectations: $D_{s0}(2317)$ and $D_{s1}(2460)$ masses appeared above the *DK* and D^*K thresholds respectively [4]. Recent dynamical studies investigated whether the issue might lie in the sea quark contributions (or lack thereof), however when the pion and kaon masses in dynamical lattice QCD studies were close to their physical values, the low lying positive parity D_s mesons again had masses above their respective thresholds [5].

In this work we used lattice QCD as an *ab initio* method to study hadronic physics, to investigate the masses of positive parity D_s mesons: $D_{s0}(2317)$ and $D_{s1}(2460)$ with $J^P = 0^+$ and $J^P = 1^+$ respectively. Lattice QCD is the theory of the strong interaction formulated in discrete Euclidean space-time and its main advantage is that the correlation functions of hadronic operators are evaluated in terms of fundamental QCD degrees of freedom: quarks and gluons. The calculations are performed in a finite sized box with periodic boundary conditions, rendering the discrete spectrum of correlation functions.

Lattice QCD studies of the positive parity mesons thus far have only included $c\bar{s}$ operators[1] in the construction of the correlation functions. In our study of the positive parity D_s mesons we include also the *DK* and D^*K scattering operators as well as $c\bar{s}$ operators to investigate whether the coupling of $D_{s0}^*(2317)$ to the *DK* threshold and the coupling of $D_{s1}(2460)$ to the D^*K threshold could affect their masses.

2. Details of the lattice setup

The simulation was performed on two ensembles of dynamic gauge field configurations [6, 7, 8] and some details are presented in Table 1.

quantity	Ensemble (1)[6, 7]	Ensemble (2)[8]	experiment [1]
$N_L^3 \times N_T$ (N_f)	$16^3 \times 32$ (2)	$32^3 \times 64$ (2+1)	
L[fm]	1.98(2)	2.90(4)	
m_π[MeV]	266(3)	156(7)	139.57018(35)
m_K[MeV]	552(1)	504(1)	493.677(16)

Table 1: Details of gauge configurations used in our study of the positive parity D_s mesons. N_L and N_T denote the number of spatial and temporal points on the lattice, while L lists the spatial length of a single side of the box. π and K masses for both ensembles are compared to their experimental counterparts.

Due to the large charm quark mass we treat it differently than the light and strange quarks. To minimize the discretization errors associated with the large charm quark mass we use the Fermilab

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[1]Often referred to as the single hadron approach.

method [9], where the discretization effects are minimal in the difference between the energy of a given state E and the D_s spin averaged mass: $E - \overline{m}_{D_s}$, where

$$\overline{m}_{D_s} = \tfrac{1}{4}(m_{D_s} + 3m_{D_s^*}). \tag{2.1}$$

3. Spectrum

The energy spectrum of the positive parity D_s mesons in our calculation is comprised of discrete energy levels related to multihadron (DK or D^*K scattering) states and additional resonances and bound states. The spectrum is discrete due to periodic boundary conditions in space, where the expected noninteracting energies of two particle states are:

$$E = E_{D,D^*}(\vec{p}_1) + E_K(\vec{p}_2), \text{ where } |\vec{p}_i| = \vec{n}_i \frac{2\pi}{L} \text{ and } \vec{n}_i \in Z^3. \tag{3.1}$$

The correlation matrices $C_{ij}(t)$ for both channels built from operators O_i listed in [2, 3] with desired quantum numbers and symmetries:

$$C_{ij}(t) = \langle 0|O_i(t_{source}+t)|O_j^\dagger(t_{source})|0\rangle = \sum_n Z_i^n Z_j^{*n} e^{-E_n t} \tag{3.2}$$

are evaluated using the distillation [10] and stochastic distillation methods [11]. The spectrum E_n of discrete scattering states and additional resonances/bound states is extracted from the correlation matrix by solving the generalized eigenvalue problem:

$$C(t)\vec{v}_n(t) = \lambda_n(t)C(t_0)\vec{v}_n(t), \tag{3.3}$$

where $\lambda_n(t) \propto e^{-E_n t}$ for $t \to \infty$.
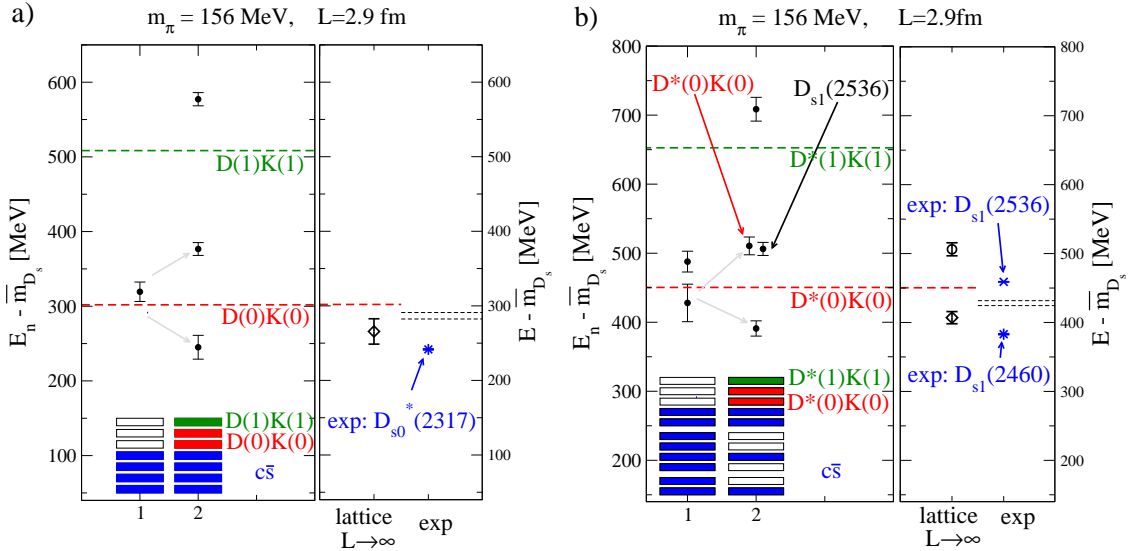
Figure 1: The spectrum for the $J^P = 0^+$ (left) and $J^P = 1^+$ (right) D_s meson channel on Ensemble (2).

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The $J^P = 0^+$ and $J^P = 1^+$ spectrum of the D_s mesons is presented in Fig. 1. In the $J^P = 0^+$ channel we first calculated the correlation matrix with only $c\bar{s}$ operators. The resulting spectrum is shown in the left panel of Fig. 1a. The energy level attributed to the $D_{s0}^*(2317)$ appears to be above the DK threshold indicated by the dashed red line, fully consistent with previous studies. In the middle panel of Fig. 1a we included the DK scattering operators in the correlation matrix. The ground state energy is split into two energy levels, one below threshold and one above. A similar scenario occurred in the $J^P = 1^+$ channel shown in Fig. 1b, where the ground state with $c\bar{s}$ operators only appeared to be consistent with the the threshold energy (left panel). The ground state decoupled into two distinct states with the addition of D^*K operators (middle panel).

To reliably disentangle the nature of the two states in both the $J^P = 0^+$ and $J^P = 1^+$ channels, we used the Lüscher analysis [12], which relates the finite volume spectrum to the infinite volume scattering matrix. This allowed us to determine the s-wave scattering lengths and effective ranges for both channels, which are presented in Table 2.

quantity	$a_0^{0^+}$ [fm]	$r_0^{0^+}$ [fm]	$a_0^{1^+}$ [fm]	$r_0^{1^+}$ [fm]	$m_D + m_K - m_{D_{s0}^*(2317)}^{BS}$ [MeV]	$m_{D^*} + m_K - m_{D_{s1}(2460)}^{BS}$ [MeV]
Ensemble (1)	-0.756(25)	-0.056(31)	-0.665(25)	-0.106(37)	78.9(5.4)	93.2(4.7)
Ensemble (2)	-1.33(20)	0.27(17)	-1.11(11)	0.10(10)	36.6(16.6)	44.2(9.9)
experiment	/	/	/	/	45.1	44.7

Table 2: Scattering lengths, effective ranges and the distance to the DK or D^*K threshold for the $J^P = 0^+$ and $J^P = 1^+$ D_s channels on Ensemble (1) and (2). For reference we also compare with experimental data from [1].

The negative scattering lengths presented in Table 2 demonstrate, that the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are indeed below threshold states. By plugging the effective range expansion in to the bound state pole condition $\cot\delta = i$, we find the masses of the two positive parity D_s mesons. Since the quark masses are unphysical, we present the mass difference between the bound states and their respective thresholds in Table 2. The discrepancy between the mass differences on Ensembles (1) and (2) in Table 2 is due to the unphysical pion mass of Ensemble (1), which is 266 MeV. The pion mass of Ensemble (2) is much closer to physical, 156 MeV, and is thus in better agreement with experiment.

4. Conclusions

The mesons $D_{s0}^*(2317)$ and $D_{s1}(2460)$ lie near DK and D^*K threshold in experiment, and reliable determination of their masses in lattice QCD necessitates that the effects of threshold are taken into account. The energy levels assigned to the positive parity D_s mesons in the single hadron approach were compatible with the DK and D^*K threshold energies. With the inclusion of multihadron operators, the ground states decoupled into two states mainly attributed to the $sD^{(*)}K$ and the other to the physical bound state. The analytical continuation of the scattering amplitude determined with the Lüscher method established the existence of poles below threshold related to $D_{s0}^*(2317)$ and $D_{s1}(2460)$. The resulting masses were found compatible with the experimental data for the first time and thus resolving a long standing puzzle in hadronic physics.

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