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DIS outlook: Into the woods with particle physics

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I provide some theoretical background to several of the topics discussed at the XXIII International Workshop on Deep-Inelastic Scattering. I also attempt to provide a personal outlook on how the subjects discussed at the workshop fit into the broader endeavor of particle physics.

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1. Introduction

With a little imagination, we can connect the ideas discussed at this conference with the Stephen Sondheim/James Lapine musical "Into the Woods," based on the familiar fairy tales collected by the Brothers Grimm. In the musical, a baker and his wife will have their wish for a child granted if they can meet a witch's four demands. This connects them with Jack (of beanstalks), Cinderella, Little Red Ridinghood, and Repunzel. What the witch wants is simple: the cow as white as milk, the cape as red as blood, the hair as yellow as corn, and the slipper as pure as gold.

For us, we can imagine going into the woods at the direction of Feynman and Fermi. Our heroes are particle physicists Alice and Bob. Their wish is to understand everything. The ghosts of Feynman and Fermi have five simple demands: find the Higgs boson; find dark matter; figure out how the Standard Model could be a low energy approximation to a more complete theory; make sense of the structure of QCD; and map the structure of hadrons.

2. Find the Higgs boson

In 1995 we discovered the top quark [1, 2]. This left the Higgs boson of the Standard model to be found. Recall that the theory with an elementary scalar Higgs field has some well known problems associated with understanding why the mass scale of the Higgs sector is not much bigger than 1 TeV. Thus one could easily have imagined that the simple elementary Higgs sector of the Standard Model was replaced by something else. If so, we would not find the simple Higgs boson of the Standard Model. But in 2012 we found the Higgs boson [3, 4].

It is, of course, not exactly true to say that we have found the Higgs boson. We have found something that looks like the Higgs boson. A major effort is now underway to see if the Higgs sector of the theory exhibits properties beyond what is expected in the Standard Model. Some of this effort was discussed at this conference in Working Group 3, *Electroweak physics and beyond the Standard Model*.

3. Find dark matter

We have good evidence that most of the mass of the universe consists of "dark matter." Presumably the dark matter is composed of one or more kinds of particle. I count it as a major failing of particle physics so far that we don't know what the dark matter is. It is not ruled out that dark matter particles don't interact with ordinary particles at all, except through gravity. But the clues that we have so far suggest that there is some interaction. The search goes on, using collisions of ordinary particles, particularly at the LHC; looking for collisions of dark matter particles with ordinary matter; and looking for collisions of dark matter particles producing ordinary particles in, for instance, the center of our galaxy. For a review, see ref. [5]. Dark matter was discussed at this conference in Working Group 3, *Electroweak physics and beyond the Standard Model*.

4. Find how the Standard Model can be a low energy approximate theory

It seems very plausible that the Standard Model and some quantum version of gravity is not all there is. Certainly, as just mentioned, we have good evidence that there are one or more kinds of

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dark matter particles. These particles are not part of the Standard Model. Thus many people expect new physics at a mass scale Λ that is much greater than 1 TeV. If this is the case, the Standard Model must be a low energy effective theory that emerges from a more comprehensive theory at a large mass scale.

Because of chiral symmetry, fermion masses can remain small in a low energy sector of a theory with also very heavy particles. However, the Higgs mass has no symmetry reason to be small compared to Λ. Furthermore, if the mass scale in the Higgs sector were large, this large mass would also be passed on to the W and Z bosons.

Thus, we need some reason, perhaps a new symmetry, that the mass scale of the Higgs sector is small compared to Λ. Maybe supersymmetry would help. Whatever helps, we need to find some traces of it at the LHC.

I should warn that in the woods of the Sondheim/Lapine musical, there can be problems that were not immediately apparent. In the musical, Little Red Ridinghood overcame the wolf, but didn't anticipate the problem of a giant that threatens everyone. In physics, we may figure out why the Higgs boson is light, but the problem of the small value of the energy density of the vacuum is bigger.

Again these issues were discussed Working Group 3, *Electroweak physics and beyond the Standard Model*.

5. Make sense of the structure of QCD

The QCD part of the Standard Model has been very well verified by experiment. With all of the experimental success, I am confident that we should not be tempted to fiddle with the QCD lagrangian (beyond adding new, heavy beyond-the-Standard-Model particles that carry color). However, we can ask to what extent we really understand QCD. This was a major topic at this conference, so I divide it into subtopics.

5.1 Lattice gauge theory

Long ago, pioneer theorists like Ken Wilson [6] and Michael Creutz [7] proposed to simulate QCD on a computer by making space-time discrete and using a lattice approximation to the QCD lagrangian. Lattice QCD can't do everything, but it is good for static properties of hadrons. It has turned into a very powerful tool.

Let me cite a recent example. It has long been a challenge to calculate the mass difference between the neutron and the proton, given the mass difference between the down quark and the up quark. Now there is a new lattice result [8]:

$$m(\mathrm{n}) - m(\mathrm{p}) = (1.51 \pm 0.28)\ \mathrm{MeV}\ . \tag{5.1}$$

This agrees within errors with the experimental value

$$m(\mathrm{n}) - m(\mathrm{p}) = 1.293\ \mathrm{MeV}\ . \tag{5.2}$$

The success of calculations like this tell me that we are are doing well in understanding QCD at least in the domain for which lattice calculations are possible.

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5.2 Perturbative calculations

The lattice method is non-perturbative, but we also have calculations based on the perturbative expansion in powers of the coupling α_s. For analyzing physics at a momentum scale μ, we use the running coupling $\alpha_s(\mu^2)$. We are helped by the fact that the $\alpha_s(\mu^2)$ becomes small for large μ^2.

Calculations beyond the leading order in $\alpha_s(\mu^2)$ are difficult, but we have been helped by the efforts of many very able theorists. We have results at next-to-leading order (NLO) and (NNLO) for a wide variety of important processes. Theoretical results and comparisons to experimental results were presented in Working Group 4 *QCD and hadronic final states* and also in Working Group 5 *Heavy flavors*.

5.3 Summing logarithms

Often simple perturbation theory is not sufficient. There can be large logarithms, $L = \log(Q^2/Q_0^2)$, where Q^2 and Q_0^2 are two momentum-squared scales in the physical problem. Depending on the specifics of the problem, we can get a single log series

$$\sigma = A_0 + \alpha_s[A_{11}L + A_{10}] + \alpha_s^2[A_{22}L^2 + A_{21}L + A_{20}] + \cdots \ . \tag{5.3}$$

Even when α_s is small, if $\alpha_s L$ is of order 1, one can't just use the first couple of terms in this series and expect to get a sensible result. Often there are two powers of L for each power of α_s:

$$\begin{aligned}\sigma = A_0 &+ \alpha_s[A_{12}L^2 + A_{11}L + A_{10}] \\ &+ \alpha_s^2[A_{24}L^4 + A_{23}L^3 + A_{22}L^2 + A_{21}L + A_{20}] + \cdots \ .\end{aligned} \tag{5.4}$$

Then we are even more sensitive to the large logarithms L. Since quite a number of talks in Working Group 4 *QCD and hadronic final states*, Working Group 5 *Heavy flavors*, and in Working Group 6 *Spin physics* were related to summing large logarithms, it may be helpful for me to provide a brief outline of some of the ideas involved.

Large logarithms can arise from having an integration like

$$\int_{Q_0}^{Q} \frac{dk_\perp}{k_\perp} \tag{5.5}$$

for each power of α_s. This gives one power of L for each power of α_s. The simplest example is the expansion of the running coupling $\alpha_s(Q^2)$ in powers of the coupling $\alpha_s(Q_0^2)$ at a fixed scale. A slightly more complicated example is the relation of parton distribution functions $f_{a/A}(x,Q^2)$ at a high scale Q^2 to $f_{a/A}(x,Q_0^2)$ at a lower scale Q_0^2. In cases like this, one can derive and then solve a first order differential equation, the *renormalization group equation*, to get the needed result.

Large logarithms can also arise because the amplitude to emit a gluon from a high energy particle is singular when the angle θ between the gluon and the emitting particle goes to zero. This can give us integrals like

$$\int_{\Delta}^{\pi/2} \frac{d\theta}{\theta} \ , \tag{5.6}$$

where Δ is a small quantity that comes from the exact kinematics. This gives one power of $\log\Delta$ for each power of α_s even in situations in which all of the transverse momenta involved are of roughly the same size.

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Sometimes the angle integration comes in the form of an integration over the rapidity $y = -\log(\tan(\theta/2))$ of an emitted gluon:

$$\int_{y_1}^{y_2} dy\,. \tag{5.7}$$

These logarithms are often called BFKL logs after Balitsky, Fadin, Kuraev, and Lipatov, who have studied their treatment extensively [9, 10, 11, 12]. They can be applied for small x parton distributions and also to situations in which there are two high $p_\perp$ jets that have a large separation in rapidity and one examines the effects of creating many smaller $p_\perp$ jets in the intervening rapidity region. My impression is that it has been difficult to isolate BFKL effects precisely and to connect them to experimental results. However, these logarithms are certainly part of QCD theory and need to be examined carefully. There was quite a lot of impressive, careful examination in Working Group 2 *Small-x, diffraction and vector mesons*.

The probability to emit a gluon is singular both when the energy or transverse momentum of the gluon tends to zero and when the angle between the gluon and the emitting parton goes to zero. Thus we get integrals like

$$\int_{Q_0}^{Q} \frac{dk_\perp}{k_\perp} \int_{\Delta}^{\pi/2} \frac{d\theta}{\theta}\,. \tag{5.8}$$

Now there are two logarithms per power of α_s. The classic example of this is in the production of a muon pair in $\mathrm{p}+\mathrm{p} \to \gamma^* + X \to \mu^+ + \mu^- + X$. When the dimuon mass Q is much larger than the dimuon transverse momentum $Q_\perp$, we get a power series expansion with two powers of $\log(Q^2/Q_\perp^2)$ for each power of α_s.

To sum the logarithms, one first breaks the cross section into parts. One represents a jet of partons that are approximately collinear to the first proton, one represents a jet of partons that are approximately collinear to the second proton, and one represents gluons that are soft but not collinear and can connect the two jets. Having broken the cross section into factors, it is helpful to take transverse Fourier transforms of everything, so that convolutions of functions of transverse momenta are turned into products of functions of transverse position [13]. This trick of taking an appropriate integral transform of the functions in the problem is quite generally useful for double-log problems. Finally, we need to derive and then solve two differential equations to get the result [14].

A modern alternative to the traditional derivation uses what is called *soft collinear effective theory*. This is based on breaking the cross section into collinear and soft parts, but typically avoids the Fourier transform. The needed differential equations come from requiring that the cross section cannot depend on the parameters that specify the boundaries among the parts. See, for example, ref. [15].

It is sometimes convenient to think of the cross section for this process as being the convolution of two transverse momentum dependent (TMD) parton distributions, one for each of the incoming protons. Then summing the logarithms $\log(Q^2/Q_\perp^2)$ amounts to summing the logarithms in the TMD parton distributions.

5.4 Parton shower generators

Much of the physics that we have discussed can be simulated by a parton shower event generator. Examples include PYTHIA, HERWIG, SHERPA, and a new one by Zoltan Nagy and me,

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DEDUCTOR. Parton splittings in HERWIG are ordered by angle, whereas the others are ordered by a hardness variable. I will concentrate on the showers ordered by hardness.

In the hardness ordered showers, the shower evolves with decreasing values of a hardness variable Q^2 that describe the splittings. For instance, Q^2 could be proportional to the virtuality of the splitting or it could be the squared transverse momentum of the daughters relative to the direction of the mother. For splittings of the initial state partons, generating splittings with decreasing Q^2 means starting at the hard interaction and working backwards in physical time.

At each Q^2 stage of the shower, there will be further splittings of each parton that has been generated so far, but these softer splittings are not resolved at scale Q^2. Thus for a final state parton, the parton really stands not for an on-shell parton but for a jet with jet squared mass scale smaller than Q^2. The jet can branch into subjets in many ways, with the probabilities for the branchings summing to 1.

Suppose that we use a parton shower generator to generate a cross section in which double logs as in eq. (5.4) appear. The parton shower generator will generate a result. If one were to expand the result in powers of α_s, it would likely have the form of eq. (5.4). That is, the parton shower will sum logarithms. However, if we have in mind all of the strange happenings in the musical "Into the Woods," we may want to ask whether the parton shower sums the logarithms correctly.

The answer depends on what parton shower algorithm we use and on what logarithms we wish to sum. In the case of the $Q_\perp$ distribution in the Drell-Yan process, Zoltan Nagy and I showed that for a suitable parton shower algorithm, the logs of $Q_\perp^2/Q^2$ are correctly summed [16]. This result, however, depends on some fairly obscure details of the organization of the parton shower.

6. Map the structure of hadrons

In the musical "Into the Woods," there are some surprises. For physicists Alice and Bob also, the woods used to seem simple, before we looked closely. The proton and neutron were each made of three quarks. Their masses were around 300 MeV. Simple predictions based on this model for the spins and magnetic moments of the hadrons worked pretty well.

However, this constituent quark model wasn't quite right. We now understand that nucleons are really made of lots of quarks, antiquarks, and gluons. We can find a description of this in the nucleon rest frame, but for this, we need the full power of a strongly interacting quantum field theory. For this, we can turn to lattice QCD.

However, the rest frame doesn't help much for high energy collisions. For this, we need a different picture.

6.1 Parton distribution functions

There is a simple picture that applies to a nucleon that has very high energy: the parton picture as realized in full QCD. This picture is useful for high energy collisions at the LHC, for deeply inelastic scattering experiments, and for experiments at RHIC and Jefferson Lab. In this picture, there is a large scale Q^2 and we are willing to neglect contributions to cross sections that are suppressed by a factor $(1\ \mathrm{GeV}^2)/Q^2$.

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In the parton picture, a nucleon is made of many quarks and gluons. That's not so simple. But the quarks and gluons don't interact much with each other. More precisely, they do interact, but the interactions are rather small and are under theoretical control.

In this picture, we have functions $f_{a/A}(x,\mu^2)$ that tell how the partons are distributed in the nucleon when measured at scale μ^2. Here a parton of type a carries a fraction x of the momentum of the nucleon. That, I think, is pretty intuitive. We don't do very well calculating these functions, although lattice gauge theory can help. However, we have determined them from experiments, especially from deeply inelastic scattering. The parton distribution functions and their determination were discussed in Working Group 1 *Structure functions and parton densities*.

Once we have determined the parton distribution functions, we can make predictions for many different kinds of experiments. These include the experiments at the LHC: without knowing parton distribution functions, we would be at a loss to understand anything that we see at the LHC. Without knowing parton distribution functions, we could not have found the Higgs boson. Thus the DIS experiments on which the parton distribution functions are largely based are like a keystone in the arch that supports the edifice of particle physics.

In order to claim that we have some understanding of hadron structure, we need to look beyond the standard parton distribution functions. That is, there is more to know about the structure of hadrons than is contained in the ordinary parton distribution functions.

6.2 Spin of the proton

The simplest extensions of the ordinary parton distribution functions $f_{a/A}(x,\mu^2)$ are the spin dependent functions giving the distribution of quark and gluon spins in a polarized proton. These functions were discussed in Working Group 6 *Spin physics*.

For a proton with helicity $+1/2$ we can ask what is the probability $f_{a/A}(x,\lambda;\mu^2)$ to find in the proton a parton of flavor a having helicity λ. For quarks or antiquarks, one often defines $\Delta f_{q/A}(x;\mu^2) = f_{q/A}(x,+1/2;\mu^2) - f_{q/A}(x,-1/2;\mu^2)$. For gluons, one can define $\Delta f_{g/A}(x;\mu^2) = f_{g/A}(x,+1;\mu^2) - f_{g/A}(x,-1;\mu^2)$. For the total quark contribution, the DSSV [17] fit gives

$$\sum_q \int_{0.001}^{1} \Delta f_{q/A}(x;\mu^2) = 0.366, \qquad \mu^2 = 10\ \text{GeV}\ . \tag{6.1}$$

If there were no gluons in the protons and the partons had no orbital angular momentum, this would be 1. For the gluon, including results from the RHIC 2009 run, DSSV [18] find

$$\int_{0.001}^{1} \Delta f_{g/A}(x;\mu^2) \approx 0.4, \qquad \mu^2 = 10\ \text{GeV}\ . \tag{6.2}$$

This is a very large number, considering that $\lambda = \pm 1$ for gluons, twice the value $\lambda = \pm 1/2$ for quarks. The error in the fit is large, but with the new fit the net spin carried by gluons is significantly non-zero.

We should not think that all of the proton spin is carried by the spin of the quarks and gluons in the proton. The vertices that describe one parton splitting into two contain factors $p_x \pm i p_y$, so that the new partons orbit each other with one unit of orbital angular momentum. We can thus expect that a good fraction of the proton spin comes from the orbital angular momentum of the quarks and gluons.

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Physicists Alice and Bob, ever careful in the woods, are of course aware that there is a certain amount of ambiguity in the division of the proton angular momentum into pieces in this way.

We can also ask for the probability $f_{a/A}(x,\boldsymbol{s}_\perp;\mu^2)$ to find in the proton a parton of flavor a having transverse spin $\mathbf{s}_\perp$. If we polarize the proton so that it has a transverse spin $\boldsymbol{S}_\perp$, then there is a preferred transverse direction and $f_{a/A}(x,\boldsymbol{s}_\perp;\mu^2)$ can have a contribution proportional to $\mathbf{s}_\perp\cdot\boldsymbol{S}_\perp$. This gives us an interesting function, the *transversity* function, which we heard about at this conference.

6.3 Parton transverse momentum

The partons in a proton carry momentum components transverse to the beam direction. Thus there are transverse momentum dependent (TMD) parton distributions $f_{a/A}(x,\boldsymbol{k}_\perp,Q^2)$. Finding themselves deep in the woods, physicists Alice and Bob are aware that there are some subtle issues in the definition of these functions [19]. However, the idea is pretty intuitive. On an intuitive level, the inclusive parton distribution $f_{a/A}(x,Q^2)$ is the integral over $\boldsymbol{k}_\perp$ of $f_{a/A}(x,\boldsymbol{k}_\perp,Q^2)$.

The most direct way to measure these functions is through γ^* or Z production in proton-proton collisions, where the transverse momenta of the two incoming quarks sum to the transverse momentum of the γ^* or Z. We heard about the TMD parton distributions in Working Group 1 *Structure functions and parton densities*, Working Group 2 *Small-x, diffraction and vector mesons*, and Working Group 6 *Spin physics*.

6.4 Transverse momentum and spin

As soon as we bring measured parton transverse momentum into the picture, there is a rich dependence on the spin of partons that is possible. There is quite a variety of spin and $\boldsymbol{k}_\perp$ dependent parton distribution that we can try to determine from experiment. To take just one example, even if we look inside an unpolarized proton, there can be a correlation between the transverse spin $\boldsymbol{s}_\perp$ of a quark and its transverse momentum $\boldsymbol{k}_\perp$:

$$f_{a/A}(x,\boldsymbol{k}_\perp,\boldsymbol{s}_\perp)\propto\varepsilon^{ij}k_\perp^i s_\perp^j\ , \tag{6.3}$$

where ε^{ij} is the antisymmetric tensor in the two transverse directions. This correlation is known as the Boer-Mulders effect and contributes to the angular distribution of final state leptons in $\mathrm{p}+\mathrm{p}$ to γ^* or Z [20].

With a rich menu of correlations among proton spin, parton spin, and parton transverse momentum available, there was lots to talk about in Working Group 6 *Spin physics*.

6.5 Location of the partons in the proton

Where are the partons located in transverse position $\mathbf{b}$? For that we need distributions $f_{a/A}(x,\mathbf{b}^2)$ derived from generalized parton distributions (GPDs). These are something like x dependent form factors. They are studied using *exclusive* final states obtained with a highly virtual probe. Results for these functions appeared in Working Group 1 *Structure functions and parton densities* and in Working Group 6 *Spin physics*.

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6.6 Hadron structure at small x

When we study deeply inelastic scattering or hadron-hadron scattering or hadron-nucleus scattering, the scattering process is controlled by parton distribution functions $f_{a/A}(x,\mu^2)$. The renormalization scale μ^2 specifies the resolving power of our microscope: we can see details down to a transverse size scale $\Delta b \sim 1/\mu$. If we see a gluon at this scale, the gluon has substructure at smaller transverse separations, but we don't see it. Thus the gluon effectively has a transverse size $1/\mu$.

Let us consider $\mu \sim 2$ GeV. Then $1/\mu$ is pretty small, but not enormously small. Now what if we look at very small values of x? For very small x, the number of gluons per unit of $\log x$, namely $x f_{a/A}(x,\mu^2)$, is very large. Then the gluons of size $1/\mu$ can fill the available transverse area of the proton. We are in the *saturation* regime. The behavior of scattering processes can change. The theory needed changes too. One sort of theory for this situation is the description in terms of the *color glass condensate*. These issues were analyzed in Working Group 2 *Small-x, diffraction and vector mesons*.

6.7 Parton correlations

Knowing one-particle-inclusive distributions like $f_{a/A}(x,\boldsymbol{k}_\perp,Q^2)$ leaves a lot of questions unanswered. Does a quark come with its own little cloud of gluons? Is it attracted to another quark to form a di-quark system? How do any effects like this depend on the colors and spins of the other partons?

In part, one can investigate this sort of question by looking for double parton scattering in hadron-hadron collisions. For instance, suppose that high x partons come in clumps, so that there is a large probability for two high x partons to have a small transverse separation. When there is a high $P_\perp$ scattering of a parton from hadron A from a parton in hadron B, the two partons must have had close to the same transverse position before they scattered. That means that the corresponding clumps of partons from hadron A and hadron B had close to the same transverse position. If that is the case, then the probability to have a second hard collision when you have had one is large. In contrast, if the partons in each hadron are spread uniformly over the whole transverse extent of the hadrons, then the probability to have a second hard collision when you have had one is small. That implies that one can investigate the two parton correlation function in transverse position by looking at the cross section for double parton scattering. This topic was explored in Working Group 4 *QCD and hadronic final states*.

I hope that there will be more explorations of how all of the partons correlate with each other inside of hadrons.

6.8 Experimental tools

We, along with Alice and Bob, would like to understand everything. However, we can't do that without suitable experimental apparatuses. The outlook for future experiments was explored in Working Group 7 *Future experiments*. I was pleased to hear informally this week that the U.S. Nuclear Science Advisory Committee has given an electron-ion collider a high enough priority that it seems likely that such a machine will be built. This will make possible a wide variety of investigations along many of the lines that I have outlined above.

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7. Conclusions

The XXIII International Workshop on Deep-Inelastic Scattering encompassed a rich variety of interesting experimental results and theoretical analyses. Since there were summaries from each of the six working groups, I have not attempted to summarize the workshop as a whole. Rather, I have provided a little theoretical background to some of the topics and I have attempted to provide a personal outlook on how the subjects discussed at the workshop fit into the broader endeavor of particle physics. After listening to many talks at the working groups, I find that I share the enthusiasm expressed by the speakers from whom I learned.

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