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On the physical DGLAP evolution of structure functions and its use for detecting saturation effects*

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Physical anomalous dimensions are a formulation of the DGLAP evolution of Deep Inelastic structure functions which is indepedent of factorization scheme and -scale. In this proceedings we provide an outlook on possible applications, in particular in the search of saturation effects. As an original contribution we present a short study of the scale dependence of physical evolved structure functions for large initial scale $Q_0^2 = 30\text{GeV}^2$

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1. Introduction

Analysis of Deep-inelastic scattering (DIS) cross sections is generally performed through global fits of scale-dependent quark and gluon distribution functions $f_i(x,Q^2)$, $i = q,\bar{q},g$. The underlying theoretical framework of such analysis is based on the collinear factorization theorem [1], which organizes the computation of DIS structure functions $F_{2,L}(x,Q^2)$ into the convolution of short-distance Wilson coefficients and long-distance parton distribution functions (PDFs) (see *e.g.* [2] for a recent review). In order to formulate such a theorem it is necessary to introduce a factorization scale μ_f which separates long- and short-distance physics. Independence of physical observables on μ_f allows then to derive renormalization group equations (RGEs) which govern the scale dependence of PDFs, known as the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations. Since there is an infinite number of different ways to realize factorization, one is left with an additional choice of the factorization scheme with the $\overline{\text{MS}}$ prescription the generally adapted one. For observables, such as DIS structure functions, any residual dependence on factorization scheme and - scale μ_f is suppressed by an additional power of α_s, *i.e.*, is formally one order higher in the perturbative expansion but not necessarily numerically small.

As an alternative to this conventional treatment it is also possible to formulate QCD scale evolution equations directly for observables without referring to auxiliary, convention-dependent PDFs, which avoids introduction of an artificial factorization scheme and -scale dependence altogether [3]. The framework is suited best for theoretical analyses of DIS data; in particular one remains in this case with the renormalization scale μ_r as the only theoretical ambiguity. Since theory uncertainties are in this way reduced to a minimum, it is therefore this 'physical' formulation of DGLAP evolution which is most suitable for extractions of α_s from inclusive DIS data (for a first study see *e.g.* [4]). Moreover, physical evolution allows for the most stringent tests of DGLAP evolution itself. This is of particular interest for regions of phase space where a breakdown of collinear factorization is expected, such as the limit $x \to 0$ of DIS structure functions, where large parton densities eventually saturate and require a more complete description including terms usually suppressed by powers of Q^2. A potential application of physical evolution for such studies is illustrated and discussed in Fig. 1. The outline of these proceedings is as follows: in Sec. 2 we provide some details on the derivation and definition of physical evolution kernels while Sec. 3 presents an analysis of the remaining renormalization scale dependence of physical evolution up to next-to-next-to-leading order (NNLO) in α_s for large input scales $Q^2 = 30\text{GeV}^2$. For more details we refer the interested reader to [5].

2. Physical evolution kernel

To define physical evolution kernels we use that the x-space convolutions of coefficient functions and PDFs turn into products in conjugate Mellin space, $a(n) = \int_0^1 dx x^{n-1} a(x)$. Moments of DIS structure functions $F_I(x,Q^2)$ can then be expressed as

$$F_I(n,Q^2) = \sum_k C_{I,k}\left(n, \alpha_s(\mu^2), \frac{Q^2}{\mu^2}, \frac{\mu_r^2}{\mu_f^2}\right) \cdot f_k\left(n, \alpha_s(\mu^2), \frac{\mu_f^2}{Q_0^2}, \frac{\mu_r^2}{\mu_f^2}\right). \tag{2.1}$$

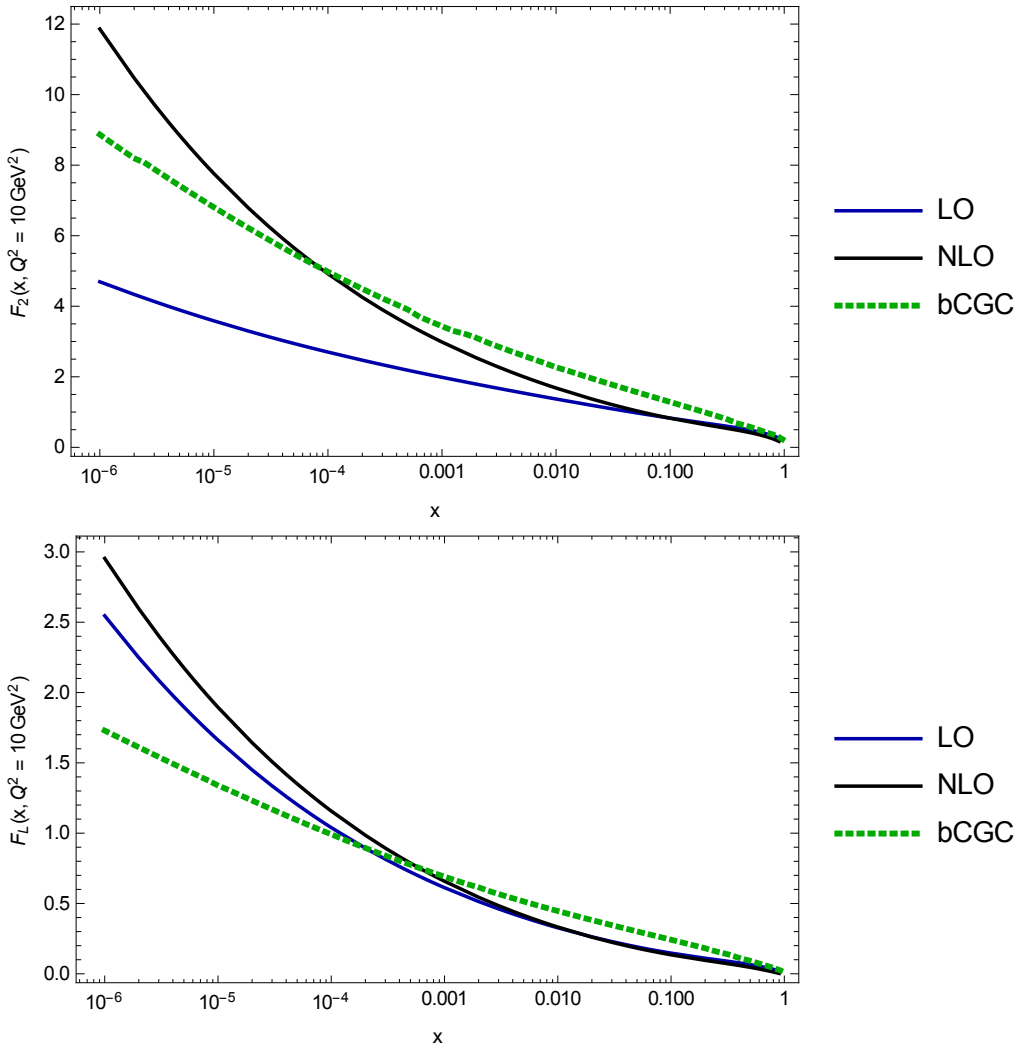

Figure 1: *Gold structure functions F_2 and F_L have been calculated at $Q^2 = 2GeV^2$ from the bCGC-model, using the fit [6] to HERA proton DIS data, combined with a scaling $Q_s^2 \to Q_S^2 A^{1/3}$. The result has been fitted and used as input for physical DGLAP evolution. The plots show a comparison of physical DGLAP evolution at leading (LO) and next-to-leading (NLO) order for the doublet (F_2, F_L) from $Q_0^2 = 2GeV^2 \to Q^2 = 10GeV^2$ and the corresponding bCGC result at $Q^2 = 10GeV^2$.*

The sum runs over all contributing quark flavors and the gluon, each represented by a PDF f_k. The non-perturbative PDFs $f_k(n,\mu^2)$ obey the DGLAP evolution equations

$$\frac{df_k(n,\mu^2)}{d\ln\mu^2} = \sum_l P_{kl}(n,\alpha_s(\mu^2),\frac{Q^2}{\mu^2}) f_l(n,\mu^2), \tag{2.2}$$

while coefficient functions $C_{I,k}$ [7, 8, 9] and splitting kernels P_{kl} [10, 11, 12] can be calculated in perturbative QCD and exhibit the following expansion in α_s

$$P_{kl} = \sum_{m=0} \left(\frac{\alpha_s}{4\pi}\right)^{1+m} P_{kl}^{(m)}(n)\,, \qquad C_{I,k} = \sum_{m=0} \left(\frac{\alpha_s}{4\pi}\right)^{m_0+m} C_{I,k}^{(m)}(n)\,, \tag{2.3}$$

where m_0 depends on the first non-vanishing order in α_s in the expansion for the observable under consideration, *e.g.* $m_0 = 0$ for F_2 and $m_0 = 1$ for F_L. The DGLAP evolution equation are formulated as $n_f - 1$ evolution equations for the different non-singlet quark flavor combinations and a 2×2 matrix valued evolution equation, which evolves the flavor singlet vector (Σ, g); $g(n,\mu^2)$ the gluon distribution and $\Sigma(n,\mu^2) = \sum_f^{n_f} \left[q_f(n,\mu^2) + \bar{q}_f(n,\mu^2)\right]$ the quark flavor singlet. In the following we concentrate ourselves on the flavor singlet sector only; for the physical evolution of the non-singlet sector see *e.g.* [4]. Using that any doublet of flavor singlet observables $F = (F_A, F_B)$ is related to the flavor singlet vector (Σ, g) through a coefficient matrix C

$$C = \begin{pmatrix} C_{Aq} & C_{Ag} \\ C_{Bq} & C_{Bg} \end{pmatrix} \qquad\qquad P = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \quad , \tag{2.4}$$

and introducing further a corresponding 2×2 matrix P for the matrix-valued kernel of the DGLAP evolution in the flavor singlet sector, one finds in a straight forward manner

$$\frac{dF(n,Q^2)}{d\ln Q^2} = \left(4\pi\beta \frac{dC}{d\alpha_s} C^{-1} + C \cdot P \cdot C^{-1}\right) \cdot F. \tag{2.5}$$

where we made in addition use of the RG equation of α_s governed by the QCD beta function

$$\frac{d\alpha_s(\mu)}{d\ln\mu^2} = 4\pi\beta(a_s) = -\alpha_s \sum_m \left(\frac{\alpha_s}{4\pi}\right)^{m+1} \beta_m \,. \tag{2.6}$$

The resulting physical evolution kernels

$$K\left(\alpha_s(\mu_r^2), \frac{Q^2}{\mu_r^2}\right) \equiv \left(\beta \frac{dC}{da_s} C^{-1} + C \cdot P \cdot C^{-1}\right) = \frac{\alpha_s(\mu_r^2)}{4\pi} \sum_{m=0} \left(\frac{\alpha_s(\mu_r^2)}{4\pi}\right)^m K^{(m)}\left(n, \frac{Q^2}{\mu_r^2}\right) \,, \tag{2.7}$$

are independent of factorization scheme and -scale [3] with the renormalization scale μ_r as their only remaining scale ambiguity at finite perturbative order.

3. Renormalization scale dependence

To determine the renormalization scale dependence of physical evolution kernels we use kernels calculated at $\mu_r = Q$, see *e.g.* [5], and recover their full renormalization scale dependence using the same prescription as for conventional DGLAP splitting kernels *i.e.* by Taylor expanding $\alpha_s(Q^2)$ in terms of $\alpha_s(\mu_r^2)$; see *e.g.* [13] for a discussion in the case of splitting kernels. For the following numerical study we fix $n_f = 4$ and use a realistic toy input at $Q^2 = 30\text{GeV}^2$ for quark singlet and gluon distribution [14],

$$\begin{aligned} x\Sigma(x) &= 0.6\, x^{-0.3}(1-x)^{3.5}(1+5x^{0.8}) \\ xg(x) &= 1.6\, x^{-0.3}(1-x)^{4.5}(1-0.6x^{0.3}) \,, \end{aligned} \tag{3.1}$$

from which we calculate structure functions using LO coefficients, independent of the actual studied perturbative order. The strong coupling is fixed to $\alpha_s(Q_0) = 0.2$. We study as examples the flavor singlet sector of the doublets (F_2, F_L) and (F_2, F_S) with F_S the F_2 scaling violations

$$F_S(x,Q^2) \equiv \frac{dF_2(x,Q^2)}{\ln Q^2} \,. \tag{3.2}$$

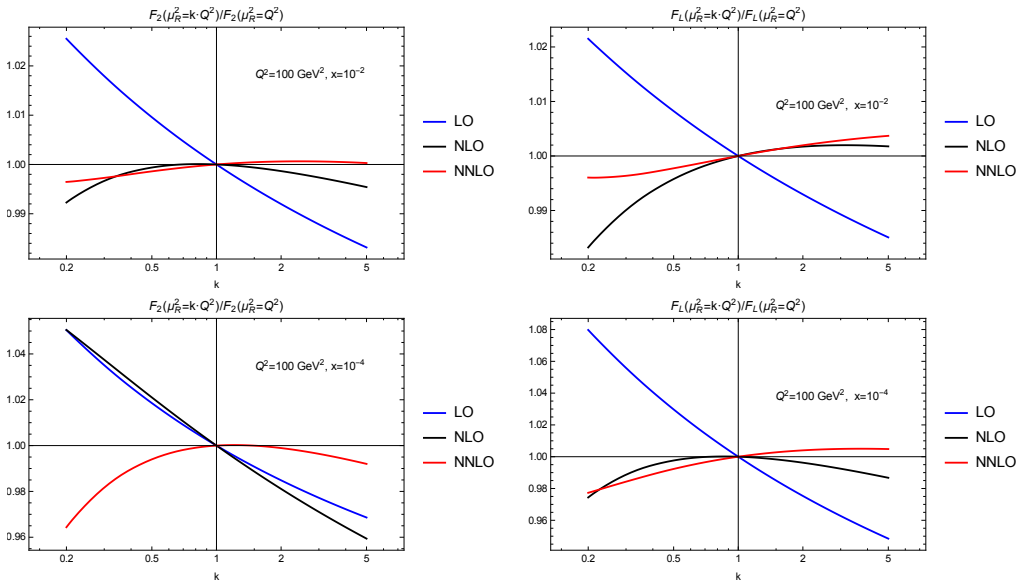

Figure 2: *Renormalization scale dependence of the doublet* (F_2, F_L)

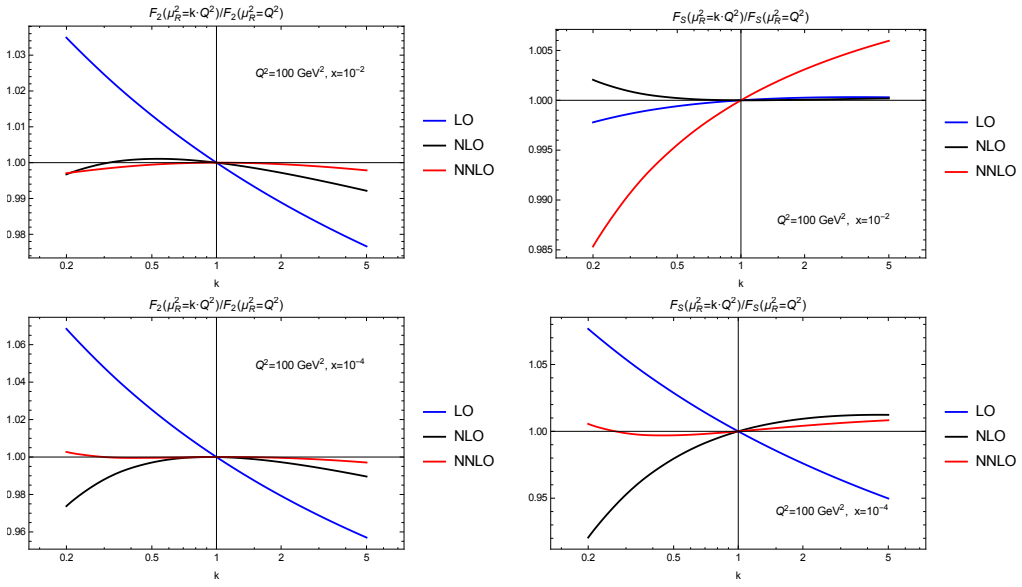

Figure 3: *Renormalization scale dependence of the doublet* (F_2, F_S)

Our result are depicted in Fig. 2 and Fig. 3 and reveal a very small dependence on the chosen renormalization at NNLO, with the doublet (F_2, F_S) slightly less sensitive to the variation of the renormalization scale. Note that the variations have been performed over a very wide range *i.e.* $\mu_{r,0}^2/\text{GeV}^2 \in [6, 150]$ and $\mu_r^2/\text{GeV}^2 \in [20, 500]$ respectively.

In conclusion we find a very mild dependence on the renormalization scale, if the initial scale for DGLAP evolution is rather large. For the case of small initial scales we refer to the discussion in [5].

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