PROCEEDINGS OF SCIENCE

Higher Fock states in CGC

A. V. Grabovsky[*]
Budker Institute of Nuclear Physics and Novosibirsk State University
630090 Novosibirsk, Russia
E-mail: a.v.grabovsky@inp.nsk.su

The NLO low - x evolution equation for the 3 quark Wilson loop operator describing the Fock state of 3 quarks with baryon quantum numbers is discussed.

XXIII International Workshop on Deep-Inelastic Scattering,
27 April - May 1 2015
Dallas, Texas

[*]Speaker.

PoS(DIS2015)074

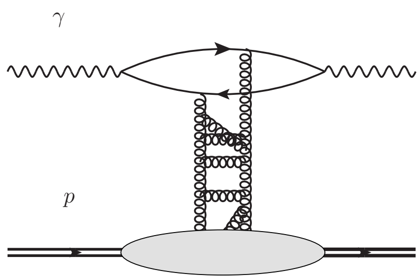

Figure 1: Dipole picture. The Wilson line operator is a color dipole.

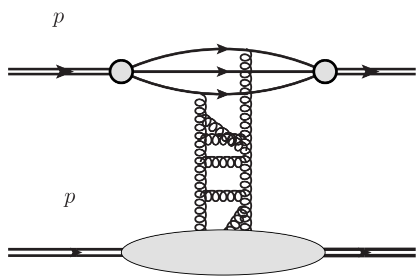

Figure 2: Dipole picture with a proton instead of a photon. The Wilson line operator is a 3QWL.

1. Introduction

This talk is based on [1]. It presents the development of the Wilson line approach to the high energy scattering proposed in [2] to the case of the Fock state of 3 quarks with baryon quantum numbers. This state is known as 3-quark Wilson loop (3QWL)

$$B_{123} \equiv \varepsilon^{i'j'h'} \varepsilon_{ijh} U^i_{1i'} U^j_{2j'} U^h_{3h'} \equiv U_1 \cdot U_2 \cdot U_3, \tag{1.1}$$

where

$$U_i = U(\vec{r}_i, \eta) = Pe^{ig\int_{-\infty}^{+\infty} b_\eta^-(r^+,\vec{r})dr^+}, \quad b_\eta^- = \int \frac{d^4p}{(2\pi)^4} e^{-ipz} b^-(p)\,\theta(e^\eta - p^+). \tag{1.2}$$

and b_η^- is the external shock wave field built from only slow gluons. Here η is the rapidity divide separating the slow gluons in the shock wave and the fast ones in the impact factors; $\vec{r}_{1,2,3}$ and $\vec{r}_{0,4}$ are the transverse coordinates of the quarks and gluons correspondingly.

Such an operator naturally appears in the high energy operator product expansion [2] if one treats the proton as a projectile instead of a photon (see figures 1,2). For the photon the relevant Wilson line operator is the color dipole obeying the Balitsky-Kovchegov equation [2, 3]. The next to leading order (NLO) of this equation was derived in [4], [5]. The leading order (LO) linear evolution equation for 3QWL was studied in the C-odd case and proved equivalent to the C-odd Bartels-Kwiecinski-Praszalowicz (BKP) equation [6] in [7] and its nonlinear evolution equation was derived within Wilson line approach in [8]. The connected contribution to the kernel of the equation was calculated in [9]. In the momentum representation the evolution of this operator was first studied in [10], and the nonlinear equation was worked out in [11]. In the C-odd case the linear NLO evolution equation for the odderon Green function was obtained in [12].

The building blocks for the kernels presented here are the next to leading order (NLO) hierarchy of the evolution equations for Wilson lines obtained in [13] (see also [14]) and the connected contribution from [9].

2. Evolution equations for the 3QWL

The NLO evolution equation for the 3QWL operator reads

$$
\begin{aligned}
\frac{\partial B_{123}}{\partial \eta} &= \frac{\alpha_s(\mu^2)}{8\pi^2}\int d\vec{r}_0 \Big[(B_{100}B_{320}+B_{200}B_{310}-B_{300}B_{210}-6B_{123})\\
&\times\left\{\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}-\frac{3\alpha_s}{4\pi}\beta\left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}}-\frac{1}{\vec{r}_{01}^{\,2}}\right)-\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right)\right]\right\}\\
&-\frac{\alpha_s}{\pi}\ln\frac{\vec{r}_{20}^{\,2}}{\vec{r}_{21}^{\,2}}\ln\frac{\vec{r}_{10}^{\,2}}{\vec{r}_{21}^{\,2}}\left\{\frac{1}{2}\left[\frac{\vec{r}_{13}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{30}^{\,2}}-\frac{\vec{r}_{32}^{\,2}}{\vec{r}_{30}^{\,2}\vec{r}_{20}^{\,2}}\right](B_{100}B_{320}-B_{200}B_{310})\right.\\
&\left.-\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{20}^{\,2}}\left(9B_{123}-\frac{1}{2}\left[2\left(B_{100}B_{320}+B_{200}B_{130}\right)-B_{300}B_{120}\right]\right)\right\}+(1\leftrightarrow 3)+(2\leftrightarrow 3)\Big]\\
&-\frac{\alpha_s^2 n_f}{16\pi^4}\int d\vec{r}_0 d\vec{r}_4\left[\left\{\left(\frac{1}{3}(U_1U_0{}^{\dagger}U_4+U_4U_0{}^{\dagger}U_1)\cdot U_2\cdot U_3-\frac{1}{9}B_{123}tr(U_0{}^{\dagger}U_4)\right.\right.\right.\\
&\left.\left.+\ (U_1U_0{}^{\dagger}U_2)\cdot U_3\cdot U_4+\frac{1}{6}B_{123}-\frac{1}{4}(B_{013}B_{002}+B_{001}B_{023}-B_{012}B_{003})\ +(1\leftrightarrow 2)\right)+(0\leftrightarrow 4)\right\}\\
&\times L_{12}^{q}+(1\leftrightarrow 3)+(2\leftrightarrow 3)\Big]-\frac{\alpha_s^2}{8\pi^4}\int d\vec{r}_0 d\vec{r}_4\ \Big[\{\tilde{L}_{12}\left(U_0U_4{}^{\dagger}U_2\right)\cdot\left(U_1U_0{}^{\dagger}U_4\right)\cdot U_3\\
&+L_{12}\left[\left(U_0U_4{}^{\dagger}U_2\right)\cdot\left(U_1U_0{}^{\dagger}U_4\right)\cdot U_3+tr\left(U_0U_4{}^{\dagger}\right)\left(U_1U_0{}^{\dagger}U_2\right)\cdot U_3\cdot U_4\ -(0\leftrightarrow 4)\right]\\
&+(M_{13}-M_{12}-M_{23}+M_2^{13})\left[\left(U_0U_4{}^{\dagger}U_3\right)\cdot\left(U_2U_0{}^{\dagger}U_1\right)+\left(U_1U_0{}^{\dagger}U_2\right)\cdot\left(U_3U_4{}^{\dagger}U_0\right)\right]\cdot U_4\\
&+\ (\text{all 5 permutations } 1\leftrightarrow 2\leftrightarrow 3)\}+\ (0\leftrightarrow 4)\Big]\,.
\end{aligned}
\tag{2.1}
$$

Here the functions $L_{12},\tilde{L}_{12},M_{12},M_2^{13}$ are defined in (A.2-A.5), L_{12}^q is defined in (A.1); $\vec{r}_{ij}=\vec{r}_i-\vec{r}_j$. The notation $(i\leftrightarrow j)$ stands for the permutation. It means that we have to change $\vec{r}_i\leftrightarrow\vec{r}_j$ and $U_i\leftrightarrow U_j$. As a result performing $(1\leftrightarrow 3)$, we change $B_{100}\to B_{300}$, $B_{320}\to B_{120}$, etc. The $\overline{MS}$ renormalization scale μ^2 is related to scale $\tilde{\mu}^2$ through

$$
\beta\ln\frac{1}{\tilde{\mu}^2}=\left(\frac{11}{3}-\frac{2}{3}\frac{n_f}{3}\right)\ln\left(\frac{\mu^2}{4e^{2\psi(1)}}\right)+\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{3},\quad \beta=\left(\frac{11}{3}-\frac{2}{3}\frac{n_f}{3}\right). \tag{2.2}
$$

One can see that this evolution equation contains nonconformal terms not related to renormalization. As in the dipole case one can cancel them rewriting the evolution equation in terms of composite conformal operators [5] (see also [14]). The evolution equation for the composite 3QWL operator B_{123}^{conf}

$$
\begin{aligned}
B_{123}^{conf} =& B_{123}+\frac{\alpha_s 3}{8\pi^2}\int d\vec{r}_4\left[\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{41}^{\,2}\vec{r}_{42}^{\,2}}\ln\left(\frac{a\vec{r}_{12}^{\,2}}{\vec{r}_{41}^{\,2}\vec{r}_{42}^{\,2}}\right)\right.\\
&\left.\times\left(-B_{123}+\frac{1}{6}(B_{144}B_{324}+B_{244}B_{314}-B_{344}B_{214})\right)+(1\leftrightarrow 3)+(2\leftrightarrow 3)\right]
\end{aligned}
\tag{2.3}
$$

reads

$$
\begin{aligned}
\frac{\partial B_{123}^{conf}}{\partial\eta} &= \frac{\alpha_s\left(\mu^2\right)}{8\pi^2}\int d\vec{r}_0\left[\left((B_{100}B_{320}+B_{200}B_{310}-B_{300}B_{210})-6B_{123}\right)^{conf}\right.\\
\times&\left.\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}-\frac{3\alpha_s}{4\pi}\beta\left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}}-\frac{1}{\vec{r}_{01}^{\,2}}\right)-\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right)\right]\right)+(1\leftrightarrow 3)+(2\leftrightarrow 3)\right]
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\alpha_s^2}{32\pi^3}\int d\vec{r}_0 \left(B_{003}B_{012}\left[\frac{\vec{r}_{32}{}^2}{\vec{r}_{03}{}^2\vec{r}_{02}{}^2}\ln^2\left(\frac{\vec{r}_{32}{}^2\vec{r}_{10}{}^2}{\vec{r}_{13}{}^2\vec{r}_{20}{}^2}\right)-\frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2\vec{r}_{02}{}^2}\ln^2\left(\frac{\vec{r}_{12}{}^2\vec{r}_{30}{}^2}{\vec{r}_{13}{}^2\vec{r}_{20}{}^2}\right)\right]\right.\\
&\left.+\ (\text{all 5 permutations } 1\leftrightarrow 2\leftrightarrow 3)\right)\\
&-\frac{\alpha_s^2 n_f}{16\pi^4}\int d\vec{r}_0 d\vec{r}_4\left[\left\{\left(\frac{1}{3}(U_1U_0{}^\dagger U_4+U_4U_0{}^\dagger U_1)\cdot U_2\cdot U_3-\frac{1}{9}B_{123}tr(U_0{}^\dagger U_4)\right.\right.\right.\\
&\left.+(U_1U_0{}^\dagger U_2)\cdot U_3\cdot U_4+\frac{1}{6}B_{123}-\frac{1}{4}(B_{013}B_{002}+B_{001}B_{023}-B_{012}B_{003})+\ (1\leftrightarrow 2)\right)\\
&\left.+\ (0\leftrightarrow 4)\right\}L_{12}^q+(1\leftrightarrow 3)+(2\leftrightarrow 3)\Big]-\frac{\alpha_s^2}{8\pi^4}\int d\vec{r}_0 d\vec{r}_4\ \left(\left\{\tilde{L}_{12}^C\left(U_0U_4{}^\dagger U_2\right)\cdot\left(U_1U_0{}^\dagger U_4\right)\cdot U_3\right.\right.\\
&+L_{12}^C\left[\left(U_0U_4{}^\dagger U_2\right)\cdot\left(U_1U_0{}^\dagger U_4\right)\cdot U_3+tr\left(U_0U_4{}^\dagger\right)\left(U_1U_0{}^\dagger U_2\right)\cdot U_3\cdot U_4\ -(0\leftrightarrow 4)\right]\\
&+M_{12}^C\left[\left(U_0U_4{}^\dagger U_3\right)\cdot\left(U_2U_0{}^\dagger U_1\right)\cdot U_4+\left(U_1U_0{}^\dagger U_2\right)\cdot\left(U_3U_4{}^\dagger U_0\right)\cdot U_4\right]\\
&\left.\left.+\ (\text{all 5 permutations } 1\leftrightarrow 2\leftrightarrow 3)\right\}+(0\leftrightarrow 4)\right). \qquad (2.4)
\end{aligned}
$$

Here the functions $L_{12}^C, \tilde{L}_{12}^C, M_{12}^C$ are defined in (A.6-A.8).

The equation for composite 3QWL operator B_{123}^{conf} (2.3) linearized in the 3-gluon approximation has the form

$$
\begin{aligned}
&\frac{\partial B_{123}^{conf}}{\partial\eta}\overset{3g}{=}\frac{3\alpha_s\left(\mu^2\right)}{4\pi^2}\int d\vec{r}_0\left[\left(B_{100}^{conf}+B_{320}^{conf}+B_{200}^{conf}+B_{310}^{conf}-B_{300}^{conf}-B_{210}^{conf}-B_{123}^{conf}-6\right)\right.\\
&\times\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}-\frac{3\alpha_s}{4\pi}\beta\left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}}-\frac{1}{\vec{r}_{01}^{\,2}}\right)-\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right)\right]\right)+(1\leftrightarrow 3)+(2\leftrightarrow 3)\Bigg]\\
&-\frac{9\alpha_s^2}{64\pi^3}\int d\vec{r}_0\left(\tilde{F}_{100}B_{100}+\tilde{F}_{230}B_{230}+(1\leftrightarrow 3)+(1\leftrightarrow 2)\right)\\
&+\frac{27\alpha_s^2}{4\pi^2}\zeta(3)(3-\delta_{23}-\delta_{13}-\delta_{21})(B_{123}-6)\\
&-\frac{9\alpha_s^2}{8\pi^4}\int d\vec{r}_0 d\vec{r}_4\left(L_{12}^C+L_{13}^C+L_{23}^C-\frac{n_f}{54}(L_{12}^q+L_{13}^q+L_{23}^q)\right)(B_{044}+B_{004}-12)\\
&-\frac{\alpha_s^2 n_f}{24\pi^4}\int d\vec{r}_0 d\vec{r}_4\ \left\{(2B_{014}-B_{001}-B_{144})(L_{12}^q+L_{13}^q-2L_{32}^q)+(1\leftrightarrow 3)+(1\leftrightarrow 2)\right\}\\
&-\frac{9\alpha_s^2}{64\pi^4}\int d\vec{r}_0 d\vec{r}_4\ \left(\{F_0B_{040}+F_{140}B_{140}+(0\leftrightarrow 4)\}+(\text{all 5 permutations } 1\leftrightarrow 2\leftrightarrow 3)\right). \qquad (2.5)
\end{aligned}
$$

Here $\delta_{ij}=1$, if $\vec{r}_i=\vec{r}_j$ and $\delta_{ij}=0$ otherwise; the functions F_0 and F_{140} are defined in (A.9) and (A.10); $\tilde{F}_{100}$ and $\tilde{F}_{230}$ are defined in (A.11-A.12). This equation goes into the BK equation for the color dipole $B_{122}=2tr(U_1U_2^\dagger)$ in the 3-gluon approximation which reads

$$
\begin{aligned}
&\frac{\partial B_{122}^{conf}}{\partial\eta}\overset{3g}{=}\frac{3\alpha_s\left(\mu^2\right)}{2\pi^2}\int d\vec{r}_0(B_{100}^{conf}+B_{220}^{conf}-B_{122}^{conf}-6)\\
&\times\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}-\frac{3\alpha_s}{4\pi}\beta\left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}}-\frac{1}{\vec{r}_{01}^{\,2}}\right)-\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right)\right]\right)+\frac{27\alpha_s^2}{2\pi^2}\zeta(3)(B_{122}-6)\\
&-\frac{9\alpha_s^2}{4\pi^4}\int d\vec{r}_0 d\vec{r}_4\left(L_{12}^C-\frac{n_f}{54}L_{12}^q\right)(B_{044}+B_{004}-12)-\frac{9\alpha_s^2}{4\pi^4}\int d\vec{r}_0 d\vec{r}_4\ \tilde{L}_{12}^C(B_{044}-B_{040})\\
&-\frac{\alpha_s^2 n_f}{12\pi^4}\int d\vec{r}_0 d\vec{r}_4\ \{(2B_{014}-B_{001}-B_{144})-(2B_{024}-B_{002}-B_{244})\}L_{12}^q. \qquad (2.6)
\end{aligned}
$$

As is clear from the last line, the evolution of the color dipole in the 3-gluon approximation depends on the 3QWL operators which have nondipole structure.

3. Conclusion

In this talk I have presented the results of [1] for the NLO evolution equation for the 3 quark Wilson loop operator $\varepsilon^{i'j'h'}\varepsilon_{ijh}U^{i}_{1i'}U^{j}_{2j'}U^{h}_{3h'}$. As for the NLO BK, this evolution equation (2.1) has nonconformal contributions which can be eliminated if one writes the equation for composite 3QWL operator (2.3). This operator obeys the NLO evolution equation with the quasi-conformal kernel (2.4). The linearized quasi-conformal equation in the 3-gluon approximation is given in (2.5). All the results have correct dipole limits when the coordinates of the two quarks coincide. As a byproduct the latter equation gives the 3-gluon approximation of the BK equation, which contains non-dipole 3QWL operators (2.6).

The 3QWL operator may have many phenomenological applications. First, it is a natural $SU(3)$ model for a baryon Green function in the Regge limit. Also, it is the irreducible operator describing C-odd (odderon) exchange. Moreover, even the NLO evolution equation for the dipole C-odd Green function in the 3-gluon approximation (2.6) in QCD can not be written without the introduction of the 3QWL operator.

The evolution equation for the C-odd part of the 3QWL operator is the generalization of the BKP equation for odderon exchange to the saturation regime. However, it is valid for the colorless object, i.e. for the function $B^{-}_{ijk} = B^{-}(\vec{r}_i, \vec{r}_j, \vec{r}_k)$, which vanishes as $\vec{r}_i = \vec{r}_j = \vec{r}_k$. The linear approximation of the equation for the C-odd part of the 3QWL should be equivalent to the NLO BKP for odderon exchange acting in the space of such functions. One may try to restore the full NLO BKP kernel from our result via the technique similar to the one developed for the 2-point operators in [15].

The result for the evolution of the 3QWL operator was also presented in [16]. However, the equivalence of this result and (2.1) has not been proved yet.

Acknowledgments

I thank the organizers for the invitation to give a talk and for hospitality in Dallas. I acknowledge support of the Dynasty Foundation, Russian Fund for Basic Research grant 13-02-01023 and president scholarship MP–171.2015.2.

A. Building blocks for the kernels

$$L^{q}_{12} = \frac{1}{\vec{r}_{04}{}^{4}}\left\{\frac{\vec{r}_{02}{}^{2}\vec{r}_{14}{}^{2}+\vec{r}_{01}{}^{2}\vec{r}_{24}{}^{2}-\vec{r}_{04}{}^{2}\vec{r}_{12}{}^{2}}{2(\vec{r}_{02}{}^{2}\vec{r}_{14}{}^{2}-\vec{r}_{01}{}^{2}\vec{r}_{24}{}^{2})}\ln\left(\frac{\vec{r}_{02}{}^{2}\vec{r}_{14}{}^{2}}{\vec{r}_{01}{}^{2}\vec{r}_{24}{}^{2}}\right)-1\right\}. \tag{A.1}$$

$$L_{12} = \left[\frac{1}{\vec{r}_{01}{}^{2}\vec{r}_{24}{}^{2}-\vec{r}_{02}{}^{2}\vec{r}_{14}{}^{2}}\left(-\frac{\vec{r}_{12}{}^{4}}{8}\left(\frac{1}{\vec{r}_{01}{}^{2}\vec{r}_{24}{}^{2}}+\frac{1}{\vec{r}_{02}{}^{2}\vec{r}_{14}{}^{2}}\right)+\frac{\vec{r}_{12}{}^{2}}{\vec{r}_{04}{}^{2}}-\frac{\vec{r}_{02}{}^{2}\vec{r}_{14}{}^{2}+\vec{r}_{01}{}^{2}\vec{r}_{24}{}^{2}}{4\vec{r}_{04}{}^{4}}\right)\right.$$
$$\left.+\frac{\vec{r}_{12}{}^{2}}{8\vec{r}_{04}{}^{2}}\left(\frac{1}{\vec{r}_{02}{}^{2}\vec{r}_{14}{}^{2}}-\frac{1}{\vec{r}_{01}{}^{2}\vec{r}_{24}{}^{2}}\right)\right]\ln\left(\frac{\vec{r}_{01}{}^{2}\vec{r}_{24}{}^{2}}{\vec{r}_{14}{}^{2}\vec{r}_{02}{}^{2}}\right)+\frac{1}{2\vec{r}_{04}{}^{4}}. \tag{A.2}$$

$$\tilde{L}_{12} = \frac{\vec{r}_{12}^{\,2}}{8}\left[\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{14}^{\,2}\vec{r}_{24}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}} - \frac{1}{\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}}\right]\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{24}^{\,2}}{\vec{r}_{14}^{\,2}\vec{r}_{02}^{\,2}}\right). \tag{A.3}$$

$$M_{12} = \frac{\vec{r}_{12}^{\,2}}{16}\left[\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{14}^{\,2}\vec{r}_{24}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}} - \frac{1}{\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}}\right]\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}{\vec{r}_{14}^{\,2}\vec{r}_{24}^{\,2}}\right). \tag{A.4}$$

$$M_2^{13} = \frac{1}{4\vec{r}_{01}^{\,2}\vec{r}_{34}^{\,2}}\left(\frac{\vec{r}_{12}^{\,2}\vec{r}_{23}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{24}^{\,2}} - \frac{\vec{r}_{14}^{\,2}\vec{r}_{23}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}} - \frac{\vec{r}_{03}^{\,2}\vec{r}_{12}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}} + \frac{\vec{r}_{13}^{\,2}}{\vec{r}_{04}^{\,2}}\right)\ln\left(\frac{\vec{r}_{02}^{\,2}}{\vec{r}_{24}^{\,2}}\right). \tag{A.5}$$

$$L_{12}^{C} = L_{12} + \frac{\vec{r}_{12}^{\,2}}{4\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}}\ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{14}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}}\right) + \frac{\vec{r}_{12}^{\,2}}{4\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{24}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}}\right). \tag{A.6}$$

$$\tilde{L}_{12}^{C} = \tilde{L}_{12} + \frac{\vec{r}_{12}^{\,2}}{4\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}}\ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{14}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}}\right) - \frac{\vec{r}_{12}^{\,2}}{4\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{24}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}}\right). \tag{A.7}$$

$$\begin{aligned}
M_{12}^{C} &= \frac{\vec{r}_{12}^{\,2}}{16\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{34}^{\,4}}{\vec{r}_{03}^{\,4}\vec{r}_{14}^{\,2}\vec{r}_{24}^{\,2}}\right) + \frac{\vec{r}_{12}^{\,2}}{16\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}}\ln\left(\frac{\vec{r}_{03}^{\,4}\vec{r}_{04}^{\,4}\vec{r}_{12}^{\,4}\vec{r}_{24}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,6}\vec{r}_{14}^{\,2}\vec{r}_{34}^{\,4}}\right)\\
&+ \frac{\vec{r}_{23}^{\,2}}{16\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{34}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,4}\vec{r}_{03}^{\,2}\vec{r}_{24}^{\,6}\vec{r}_{34}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,4}\vec{r}_{14}^{\,4}\vec{r}_{23}^{\,4}}\right) + \frac{\vec{r}_{23}^{\,2}}{16\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}}\ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{14}^{\,4}}{\vec{r}_{01}^{\,4}\vec{r}_{24}^{\,2}\vec{r}_{34}^{\,2}}\right)\\
&+ \frac{\vec{r}_{13}^{\,2}}{16\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}}\ln\left(\frac{\vec{r}_{02}^{\,4}\vec{r}_{14}^{\,2}\vec{r}_{34}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{24}^{\,4}}\right) + \frac{\vec{r}_{13}^{\,2}}{16\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{34}^{\,2}}\ln\left(\frac{\vec{r}_{02}^{\,4}\vec{r}_{14}^{\,2}\vec{r}_{34}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{24}^{\,4}}\right)\\
&+ \frac{\vec{r}_{03}^{\,2}\vec{r}_{12}^{\,2}}{8\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{34}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{24}^{\,4}}{\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}\vec{r}_{34}^{\,2}}\right) + \frac{\vec{r}_{23}^{\,2}\vec{r}_{12}^{\,2}}{8\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{24}^{\,2}\vec{r}_{34}^{\,2}}\ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{12}^{\,2}\vec{r}_{34}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{23}^{\,2}\vec{r}_{24}^{\,2}}\right)\\
&+ \frac{\vec{r}_{14}^{\,2}\vec{r}_{23}^{\,2}}{8\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}\vec{r}_{34}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{23}^{\,2}\vec{r}_{24}^{\,2}}{\vec{r}_{02}^{\,4}\vec{r}_{14}^{\,2}\vec{r}_{34}^{\,2}}\right).
\end{aligned} \tag{A.8}$$

$$\begin{aligned}
F_{0} &= \frac{\vec{r}_{12}^{\,2}}{2\vec{r}_{14}^{\,2}\vec{r}_{24}^{\,2}}\left(\frac{\vec{r}_{24}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{34}^{\,4}}{\vec{r}_{14}^{\,2}\vec{r}_{24}^{\,2}\vec{r}_{03}^{\,4}}\right) - \frac{\vec{r}_{13}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{13}^{\,2}\vec{r}_{24}^{\,2}}{\vec{r}_{03}^{\,2}\vec{r}_{12}^{\,2}\vec{r}_{14}^{\,2}}\right)\right.\\
&\left.+ \frac{2\vec{r}_{34}^{\,2}}{\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{34}^{\,2}}{\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}}\right)\right) - (0 \leftrightarrow 4).
\end{aligned} \tag{A.9}$$

$$\begin{aligned}
F_{140} &= \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}}\ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}\vec{r}_{34}^{\,4}}{\vec{r}_{03}^{\,4}\vec{r}_{14}^{\,2}\vec{r}_{24}^{\,4}}\right)\\
&- \frac{\vec{r}_{01}^{\,2}\vec{r}_{23}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{24}^{\,2}\vec{r}_{34}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}\vec{r}_{23}^{\,2}}\right) - \frac{\vec{r}_{23}^{\,2}\vec{r}_{12}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{14}^{\,2}\vec{r}_{24}^{\,2}}\ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{14}^{\,2}\vec{r}_{23}^{\,2}}{\vec{r}_{03}^{\,2}\vec{r}_{12}^{\,2}\vec{r}_{24}^{\,2}}\right)\\
&+ \frac{\vec{r}_{23}^{\,2}}{\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}}\ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{34}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{23}^{\,2}}\right) + \frac{\vec{r}_{02}^{\,2}\vec{r}_{13}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}}\ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{34}^{\,4}}{\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{13}^{\,2}\vec{r}_{24}^{\,2}}\right).
\end{aligned} \tag{A.10}$$

$$\begin{aligned}
\tilde{F}_{100} &= \left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} - \frac{\vec{r}_{13}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}} - \frac{2\vec{r}_{23}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,2}}\right)\ln^2\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{13}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{23}^{\,2}}\right) + \frac{\vec{r}_{23}^{\,2}}{2\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,2}}\ln^2\left(\frac{\vec{r}_{03}^{\,2}\vec{r}_{12}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{13}^{\,2}}\right)\\
&+ \tilde{S}_{123} I\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}, \frac{\vec{r}_{13}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}}, \frac{\vec{r}_{23}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,2}}\right) + (2 \leftrightarrow 3).
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
\tilde{F}_{230} &= \left(\frac{2\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} - \frac{\vec{r}_{23}^{\,2}}{2\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,2}}\right)\ln^2\left(\frac{\vec{r}_{03}^{\,2}\vec{r}_{12}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{13}^{\,2}}\right) + \left(\frac{\vec{r}_{13}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}} - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}\right)\ln^2\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{13}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{23}^{\,2}}\right)\\
&- \tilde{S}_{123} I\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}, \frac{\vec{r}_{13}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}}, \frac{\vec{r}_{23}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,2}}\right) + (2 \leftrightarrow 3).
\end{aligned} \tag{A.12}$$

$$\tilde{S}_{123} = \left(\frac{\vec{r}_{12}^{\,4}}{\vec{r}_{01}^{\,4}\vec{r}_{02}^{\,4}} + \frac{\vec{r}_{13}^{\,4}}{\vec{r}_{01}^{\,4}\vec{r}_{03}^{\,4}} + \frac{\vec{r}_{23}^{\,4}}{\vec{r}_{02}^{\,4}\vec{r}_{03}^{\,4}} - \frac{2\vec{r}_{13}^{\,2}\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,4}\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,2}} - \frac{2\vec{r}_{23}^{\,2}\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,4}\vec{r}_{03}^{\,2}} - \frac{2\vec{r}_{13}^{\,2}\vec{r}_{23}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,4}}\right). \tag{A.13}$$

$$I(a,b,c) = \int_0^1 \frac{dx}{a(1-x)+bx-cx(1-x)} \ln\left(\frac{a(1-x)+bx}{cx(1-x)}\right). \quad \text{(A.14)}$$

References

[1] I. Balitsky and A. V. Grabovsky, "NLO evolution of 3-quark Wilson loop operator," JHEP **1501**, 009 (2015) [arXiv:1405.0443 [hep-ph]].

[2] I. Balitsky, *Operator expansion for high-energy scattering, Nucl. Phys. B* **463** (1996) 99 [hep-ph/9509348].

[3] Yu.V. Kovchegov, *Small x F(2) structure function of a nucleus including multiple pomeron exchanges, Phys. Rev. D* **60** (1999) 034008, [hep-ph/9901281]; *Unitarization of the BFKL pomeron on a nucleus, Phys. Rev. D* **61** (2000) 074018, [hep-ph/9905214].

[4] I. Balitsky, *Quark contribution to the small-x evolution of color dipole, Phys. Rev. D* **75**, (2007) 014001, [hep-ph/0609105].

[5] I. Balitsky and G.A. Chirilli, *Next-to-leading order evolution of color dipoles, Phys. Rev. D* **77** (2008) 014019 [hep-ph/0710.4330]; *NLO evolution of color dipoles in N=4 SYM, Nucl. Phys. B* **822**, 45 (2009) [arXiv:0903.5326 [hep-ph]].

[6] J. Bartels, *High-Energy Behavior in a Nonabelian Gauge Theory. 2. First Corrections to T(n→m) Beyond the Leading LNS Approximation,* Nucl. Phys. B **175**, 365 (1980); J. Kwiecinski and M. Praszalowicz, *Three Gluon Integral Equation and Odd c Singlet Regge Singularities in QCD,* Phys. Lett. B **94**, 413 (1980).

[7] Y. Hatta, E. Iancu, K. Itakura and L. McLerran, *Odderon in the color glass condensate, Nucl. Phys. A* **760**, 172 (2005) [hep-ph/0501171].

[8] R. E. Gerasimov and A. V. Grabovsky, *Evolution equation for 3-quark Wilson loop operator*, *JHEP* **1304**, 102 (2013) [arXiv:1212.1681 [hep-th]].

[9] A. V. Grabovsky, *Connected contribution to the kernel of the evolution equation for 3-quark Wilson loop operator*, JHEP **1309**, 141 (2013) [arXiv:1307.5414 [hep-ph]].

[10] M. Praszalowicz and A. Rostworowski, *Problems with proton in the QCD dipole picture, Acta Phys. Polon. B* **29** (1998) 745 [hep-ph/9712313].

[11] J. Bartels and L. Motyka, *Baryon scattering at high energies: Wave function, impact factor, and gluon radiation, Eur. Phys. J. C* **55** (2008) 65 [hep-ph/0711.2196].

[12] J. Bartels, V. S. Fadin, L. N. Lipatov and G. P. Vacca, *NLO Corrections to the kernel of the BKP-equations, Nucl. Phys. B* **867**, 827 (2013) [arXiv:1210.0797 [hep-ph]].

[13] I. Balitsky and G. A. Chirilli, *Rapidity evolution of Wilson lines at the next-to-leading order*, arXiv:1309.7644 [hep-ph].

[14] A. Kovner, M. Lublinsky and Y. Mulian, *Complete JIMWLK Evolution at NLO*, arXiv:1310.0378 [hep-ph]; *Conformal symmetry of JIMWLK Evolution at NLO, JHEP* 1404 (2014) 030, [arXiv:1401.0374 [hep-th]].

[15] V. S. Fadin, R. Fiore, A. V. Grabovsky and A. Papa, *Connection between complete and Moebius forms of gauge invariant operators, Nucl. Phys. B* **856** (2012) 111 [arXiv:1109.6634 [hep-th]].

[16] A. Kovner, M. Lublinsky and Y. Mulian, *NLO JIMWLK evolution unabridged,* arXiv:1405.0418 [hep-ph].