The perturbative Pomeron with NLO accuracy:
Jet-Gap-Jet Observables

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We give an overview of the calculation of the forward jet vertex associated to a rapidity gap
(coupling of a hard pomeron to a jet) in the Balitsky-Fadin-Kuraev-Lipatov (BFKL) formalism at
next-to-leading order (NLO). This result allows, together with the NLO non-forward gluon Green
function, to perform NLO studies of jet production in diffractive events (Mueller-Tang dijets, as
a well-known example).

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1. Introduction

Understanding of the phenomenological very successful results of Regge-theory from first principles remains till today one of the big open questions of nuclear and particle physics. Within this framework $t$-channel exchanges of vacuum quantum number are described by a special kind of Regge trajectory, the Pomeron. It is not only responsible for the asymptotic growth of inclusive cross-sections (associated with a high multiplicity final state), but also governs the high energy behavior of diffractive cross-sections (associated with a low multiplicity final state). Within QCD perturbation theory, the Pomeron is obtained from high energy factorization and corresponding resummation of large logarithms in the center of mass energy, as provided by the famous BFKL result [1]. For diffractive events, which require the non-forward BFKL Pomeron at finite momentum transfer $t$, the BFKL Green’s function, which achieves the high energy resummation, has been calculated up to next-to-leading logarithmic (NLL) accuracy in [2]. While the bulk of diffractive cross-sections is of non-perturbative nature, it is possible to identify kinematic configurations with a hard scale which are in principle accessible to a perturbative treatment. A possible observable, original proposed in [3], to test the BFKL Green’s function for finite $t$ is given by forward-backward jets separated by a large empty region in the detector, a so-called rapidity gap. A complete description of such events at NLL requires apart from the Green’s function also the couplings of the Pomeron to the jet at next-to-leading order (NLO). While virtual corrections can be extracted from [4], real corrections have been calculated recently in [5]. This calculation employs Lipatov’s high energy effective action [6], making use of a framework for higher order corrections within this action which has been developed and tested in [7]. In the following we present our final result for the jet vertex for jets with rapidity gap within collinear factorization. For details we refer the interested reader to [5].

2. The NLO Mueller-Tang Jet Vertex

To define an infrared and collinear safe jet cross sections at NLO, it is necessary to convolute the partonic cross section with a jet function $S_J$:

\[
\frac{d\hat{\sigma}}{dJ_1 dJ_2 d^2k} = d\hat{\sigma} \otimes S_{J_1} S_{J_2}, \quad dJ_i = d^{2+\epsilon}k_i dy_i, i = 1, 2. \tag{2.1}
\]

Infrared finiteness imposes general constraints on the jet function [8]. For two final state partons, the jet function $S_{J}^{(3)} (p, q, zx, x)$ must be \{q, z\} $\leftrightarrow$ \{p, 1-z\} symmetric, and must reduce to the one final state parton distribution $S_{J}^{(2)} (p,x) = x \delta \left( x - \frac{k_{j,i} \epsilon}{\sqrt{s}} \right) \delta^{2+\epsilon}(p - k_j)$ in the soft and collinear limits. In particular

\[
S_{J}^{(3)} (p, q, zx, x) \xrightarrow{p=0} S_{J}^{(2)} (k, zx), \quad S_{J}^{(3)} (p, q, zx, x) \xrightarrow{q=0} S_{J}^{(2)} (k, x). \tag{2.2}
\]

Adding to our result the virtual corrections calculated in [4], as well as corresponding UV renormalization of the QCD Lagrangian, and absorbing initial state collinear emissions into a redefinition
of parton distribution functions, we obtain,
\[
\frac{d\sigma_{J,H;H}}{dJ_1 dJ_2 d^3k} = \frac{1}{\pi^2} \int d\ell_1 d\ell_2 d^3\ell \frac{dV\left(\ell_1,\ell_2, k, p_f, y, s_0\right)}{dJ_1} \times G\left(\ell_1,\ell_2, k, \frac{s}{s_0}\right) \frac{dV\left(\ell_1',\ell_2', k, p_f, y, s_0\right)}{dJ_2},
\]
(2.3)
where \(s = x_1 x_2 s_0\), \(x_0 = -t/(M^2_{t,\text{max}} - t)\) and
\[
\frac{dV}{dJ} = \sum_{j=q_\mu,q_\mu} \int_{x_0}^1 dx f_{j/H(x,\mu^2)} \left(\frac{d\bar{\nu}_j}{dJ} + \frac{d\bar{\nu}_j}{dJ} + \frac{d\nu_j}{dJ} + \frac{d\nu_j}{dJ}\right),
\]
\[
\frac{d\bar{\nu}_j}{dJ} = \int d\Gamma(2) \left(\frac{d\bar{\nu}_j}{dJ} + \frac{d\nu_j}{dJ} + \frac{d\nu_j}{dJ} + \frac{d\nu_j}{dJ}\right),
\]
\[
\frac{d\nu_j}{dJ} = \left\{ h_j^{(1)} + h_j^{(1)} \right\} \left(\frac{\alpha_s e}{2\pi} \frac{\bar{\nu}_j}{S_j(2)k,x}\right),
\]
\[
\frac{d\bar{\nu}_j}{dJ} = \left\{ h_j^{(1)} + h_j^{(1)} \right\} \left(\frac{\alpha_s e}{2\pi} \frac{\bar{\nu}_j}{S_j(2)k,x}\right),
\]
(2.4)
with \(\beta_0 = \frac{11}{2} C_a - \frac{1}{2} n_f\), \(P_{ij}^{(0)}(z)\) the leading order DGLAP splitting functions and \(C_{q,q} = C_f, C_\mu = C_a\). The result for \(dV\) can be extracted from [4]. Note that, if the result for the resummed jet cross-section is truncated at next-to-leading order in \(\alpha_s\), our result is independent of the scale \(s_0\) as required. To arrive at a physical representation of this vertex in dimension four we introduce \(\lambda^2 \ll k^2\), to regularize the singular regions in phase space. Using the limits in Eq. (2.2) we can rewrite \(dV_{q,\mu}/dJ\) in terms of \(\lambda\) and, introducing the notations \((i = 1, 2)\)
\[
P_0(z) = C_d \left[\int (1-z) + z(1-z)\right], \quad P_1(z) = C_d \left[\frac{2z}{1-\lambda_z} + z(1-z)\right],
\]
\[
P_{qq}^{(0)}(z) = \frac{1}{2} + \frac{z^2}{1-z}, \quad P_{qq}^{(0)}(z) = \frac{z^2 + (1-z)^2}{2},
\]
\[
P_{gg}^{(0)}(z) = \frac{1}{2} \left(1 - \frac{1}{z}\right)^2, \quad P_{gg}^{(0)}(z) = P_0(z) + P_1(z) + \frac{\beta_0}{2} \delta(1-z),
\]
\[
\alpha_s = \alpha_s(\mu^2), \quad \phi_i = \arccos \left(\frac{t_i}{t_0}\right),
\]
\[
J_1(q,k,I_1) = \int \left[ 2k^2 \left(\frac{1}{\Delta^2} - \frac{1}{q^2} - \frac{2}{\mu^2}\right) - \frac{1}{\Sigma^2} \left(\frac{l_i - k}{\Delta^2} - \frac{l_i^2}{q^2}\right) - \frac{1}{\Delta^2} \left(\frac{l_i - (1-z)k}{\lambda^2} - \frac{l_i - k^2}{\mu^2}\right) \right], \quad i = 1, 2;
\]
\[
J_2(q,k,I_1,1) = \int \left[ 2k^2 \left(\frac{1}{\Delta^2} - \frac{1}{q^2} - \frac{2}{\mu^2}\right) - \frac{1}{\Sigma^2} \left(\frac{l_i - (1-z)k}{\lambda^2} - \frac{l_i - k^2}{\mu^2}\right) \right], \quad i = 1, 2.
\]
(2.5)
we present our expression for those jets with a quark as the initial state, i.e.

\[
\frac{d\hat{\Gamma}^{(1)}(x, k, l_1, l_2; x_f, k_f; M_{X, \text{max}}, s_0)}{dJ} = v^{(0)} \frac{\alpha_s}{2\pi} (Q_1 + Q_2 + Q_3) \tag{2.6}
\]

\[
Q_1 = S_j^{(2)}(k, x)C_j \left[ -\frac{\beta_0}{2} \left\{ \ln \left( \frac{l_1^2}{\mu^2} \right) + \ln \left( \frac{(l_1 - k)^2}{\mu^2} \right) + \{1 \leftrightarrow 2\} \right\} - \frac{20}{3} \right] - 4C_f + C_{\alpha} \left( \frac{3}{2k^2} \left[ l_1^2 \ln \left( \frac{(l_1 - k)^2}{l_1^2} \right) + (l_1 - k)^2 \right] + \ln \left( \frac{(l_1 - k)^2}{k^2} \right) \right)
\]

\[
\ln \left( \frac{l_1^2}{(l_1 - k)^2} \right) \left\{ \ln \left( \frac{l_1^2}{l_1^2} \right) \ln \left( \frac{(l_1 - k)^2}{s_0} \right) - \ln \left( \frac{(l_1 - k)^2}{k^2} \right) \right\}. \tag{2.7}
\]

\[
Q_2 = \int_{s_0}^{1} dz S_j^{(2)}(k, x) \left[ \ln \frac{\lambda^2}{\mu^2} \left( C_j^2 P_{qq}^{(0)}(z) + C_{\alpha}^2 P_{qq}^{(0)}(z) \right) + C_f (1 - z) \left( C_j^2 - \frac{2z}{z} C_{\alpha}^2 \right) + 2C_f (1 + z^2) \left( \frac{\ln(1 - z)}{1 - z} \right) \right], \tag{2.8}
\]

\[
Q_3 = \int_{0}^{1} dz \int \frac{d^2 q}{\pi} \left[ \Theta \left( \hat{M}_{X, \text{max}}^2 - \frac{(p - zk)^2}{z(1 - z)} \right) S_j^{(3)}(p, q, (1 - z)x, x)C_j \right] - P_{qq}^{(0)}(z) \Theta \left( \frac{|q|}{1 - z} - \lambda^2 \right) \frac{k^2}{q^2(p - zk)^2} + \Theta \left( \hat{M}_{X, \text{max}}^2 - \frac{\Delta^2}{z(1 - z)} \right) \right]
\]

\[
S_j^{(3)}(p, q, z, x, x)P_{qq}^{(0)}(z) \left\{ C_f C_{\alpha} [J_1(q, k, l_1) + J_2(q, k, l_2)] + C_{\alpha}^2 J_2(q, k, l_1, l_2) \Theta (p^2 - \lambda^2) \right\}. \tag{2.9}
\]

In a similar way, the equivalent gluon-generated forward jet vertex reads

\[
\frac{d\hat{\Gamma}^{(1)}(x, k, l_1, l_2; x_f, k_f; M_{X, \text{max}}, s_0)}{dJ} = v^{(0)} \frac{\alpha_s}{2\pi} (G_1 + G_2 + G_3) \tag{2.10}
\]
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These expressions are in a form suitable for phenomenological studies. It is important to note that its convolution with the nonforward BFKL Green function with exact treatment of the running
of the QCD coupling is complicated. The use of Monte Carlo integration techniques [9] appears therefore to be preferable since they allow to generate exclusive distributions needed to describe different diffractive data in hadronic collisions. For more inclusive observables, analytic methods might be a valuable alternative where one might – for a complete NLL treatment – follow the procedures suggested in [10, 11].

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References


