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Matching collinear and small x factorization calculations for inclusive hadron production in pA collisions

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We discuss the forward inclusive hadron production in hadron-nucleus collisions which includes the effects of gluon saturation at low values of x. The leading order and next-to-leading order formalism is presented together with the numerical evaluation and the comparison with the experimental data. Finally, we discuss the matching between the small x and the collinear formalism and the role of the exact kinematics.

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1. Introduction

Proton or deuteron-nucleus collisions are excellent processes for investigating nuclear structure. Both RHIC and LHC colliders perform these type of high energy collisions. A signature measurement in these collisions is the single inclusive hadron production, with the hadron measured at very forward rapidity. In this kinematical range, the proton (or deuteron) projectile is probed at large values of x - the longitudinal momentum fraction of the projectile carried by the parton which takes part in the collision. On the other hand, the nucleus target is probed at very small x. Therefore the target parton density is expected to be very large, both due to the low x and large mass number A of the nucleus. In such regime one could expect that the typical saturation scale for the gluon density of the nucleus is rather large $Q_s(x_g) \gg \Lambda_{\rm QCD}$ and perturbative. If the produced hadron has the transverse momentum of the order of the saturation scale $p_T \sim Q_s$ then one can expect that the saturation effects in the nuclear structure are important and therefore the standard collinear formalism might not be sufficient to describe this process.

2. The small x formalism for the single inclusive production in the forward region

The formalism which includes the low x saturation effects in the nuclear density was developed in [1, 2]. For the quark production, one considers an energetic quark which undergoes multiple rescattering in the gluon field of the nucleus. The leading order cross section for the quark production in proton-nucleus collisions can be expressed as

$$\frac{d\sigma_{LO}^{pA\to qX}}{d^2k_\perp dy} = \sum_f xq_f(x) \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp-y_\perp)} \frac{1}{N_c}\langle {\rm Tr}\, U(x_\perp)U^\dagger(y_\perp)\rangle_Y \ , \tag{2.1}$$

where $(k_\perp, y)$ are the transverse momentum and the rapidity of the outgoing quark correspondingly. The longitudinal momentum fractions of the incoming quark and the participating gluon are defined as $x = \frac{k_\perp}{\sqrt{s}}e^y$ and $x_g = \frac{k_\perp}{\sqrt{s}}e^{-y}$ and the variable Y (which is the rapidity interval between the target and the struck quark) is defined as $Y = \ln 1/x_g$. The correlator involves Wilson lines in the fundamental representation and can be related to the dipole unintegrated gluon distribution function

$$\mathscr{F}(k_\perp) \equiv \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp-y_\perp)} \frac{1}{N_c}\langle {\rm Tr}\, U(x_\perp)U^\dagger(y_\perp)\rangle_Y \ . \tag{2.2}$$

The subscript $_Y$ denotes the fact that the correlator can be evaluated at certain rapidity interval taking into account the quantum evolution for which one needs to use an appropriate small x evolution equation like the Balitsky-Kovchegov equation [3, 4].

For the complete calculation one needs to include the gluon channel, this will involve the correlator of two Wilson lines in the adjoint representation. Finally, the outgoing parton needs to be convoluted with the fragmentation function in order to produce the colorless hadron. The final formula for the single inclusive hadron production in this formalism reads then

$$\frac{d\sigma_{LO}^{pA\to hX}}{d^2p_\perp dy} = \int_\tau^1 \frac{dz}{z^2}\left[\sum_f x_p q_f(x_p)\mathscr{F}(k_\perp)D_{h/q}(z) + x_p g(x_p)\widetilde{\mathscr{F}}(k_\perp)D_{h/g}(z) \ ,\right] \tag{2.3}$$

where $\widetilde{\mathscr{F}}(k_\perp)$ is the Fourier transform of the correlator of two Wilson lines in the adjoint representation, $x_p g(x_p)$ is the (integrated) gluon density in the incoming proton projectile and $D_{h/f}$ are the fragmentation functions. Note that, the above formula is of the form of the hybrid factorization which involves both the unintegrated gluon distributions which depend on the transverse momentum of nuclear gluon distribution and collinear quark and gluon distributions on the projectile which are evaluated at large values of x and do not contain the transverse momentum dependence.

At lowest order of this formalism there is formally no scale dependence in the collinear distributions and no rapidity dependence in the unintegrated gluon distribution. However, for the realistic phenomenology this is typically included, as the collinear parton distributions are taken from the standard parametrizations based on the DGLAP evolution equations and extracted from the fits to the experimental data.

3. Forward production beyond lowest order

The higher order calculation was completed in [5]. The calculation involves four different channels: quark to quark, gluon to gluon, quark to gluon and gluon to quark. Figure 1 shows a cartoon of a typical process which involves gluon correction to quark to quark channel. After

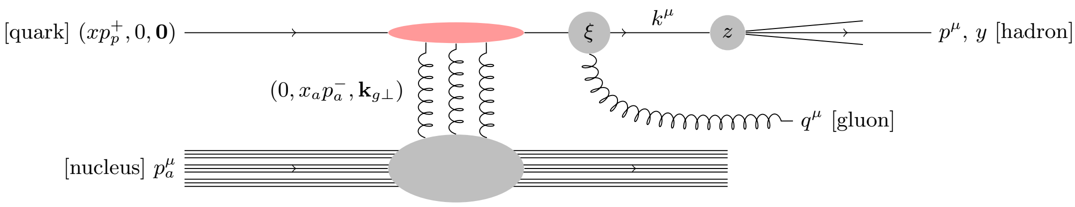

Figure 1: A cartoon of the $2 \to 2$ subprocess of the NLO $q \to qg$ channel interaction, with selected momenta and momentum fractions labeled, as well as the rapidity y of the detected hadron. The momentum fractions z and ξ are defined by $p_\perp = zk_\perp$ and $k^+ = (1-\xi)xp_p^+$, and x is the longitudinal momentum fraction of the incoming quark with respect to the proton projectile.

computing all the real and virtual diagrams, the subtractions are performed according to the appropriate renormalization group equations. There are different type of singularities which undergo the subtractions. The collinear divergences are subtracted into the parton distribution functions corresponding to the initial parton from the projectile and fragmentation functions for the outgoing produced hadron. In addition to those collinear divergencies there are rapidity divergencies which are included into the Balitsky-Kovchegov evolution. The remaining parts are finite and form the hard factor. The expression for the cross section therefore, does factorize into the different parts and can be written in the following form

$$\frac{d^3\sigma^{pA\to hX}}{dy\,d^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\,\xi\, xf_a(x,\mu) D_{h/c}(z,\mu) \int [dx_\perp]\, S^Y_{a,c}([x_\perp])\, \mathscr{H}_{a\to c}(\alpha_s,\xi,[x_\perp],\mu)\,, \tag{3.1}$$

where xf_a is the parton distribution function for parton of type a, D is the fragmentation function, S^Y is the appropriate correlator which is evolved up to the rapidity Y, and $\mathscr{H}$ is the finite hard factor. In addition, μ is the factorization scale, $[x_\perp]$ denotes all the transverse coordinate variables which need to be integrated over.

The next-to-leading calculation (3.1) has been fully implemented in the numerical code called SOLO (saturation physics at One Loop Order) [6]. The unintegrated gluon distribution has been obtained from the solution to the leading order Balitsky-Kovchegov equation with the running coupling. In addition, calculations with phenomenological parametrizations like Golec-Biernat-Wusthoff model and McLerran-Venugopalan model have been used as well.

For the integrated parton distributions, MSTW 2008 NLO [7] parton set has been used. For the fragmentation functions we have used the DSS NLO [8] fragmentation functions. The simulations were performed for the RHIC and LHC experiments. For the RHIC kinematics, hadron pseudo-rapidity was equal to $\eta = 2.2, 3.2$ (BRAHMS) and $\eta = 4$ (STAR). The numerical calculations at NLO accuracy showed positive correction for low values of p_T of the produced hadron. The overall shape was comparable with the leading order calculations, as can be observed in Fig. 2. As expected, the scale dependence at NLO is significantly reduced as compared with the leading order. The calculation showed however significant instability at higher values of p_T. The NLO correction in that region is negative and dominates the overall result, which becomes negative. The point at which the calculation breaks down, depends on the details of the unintegrated gluon distribution. A characteristic feature of that instability is that the point at which the calculation breaks down shifts to higher values of transverse momentum as the rapidity is increased.

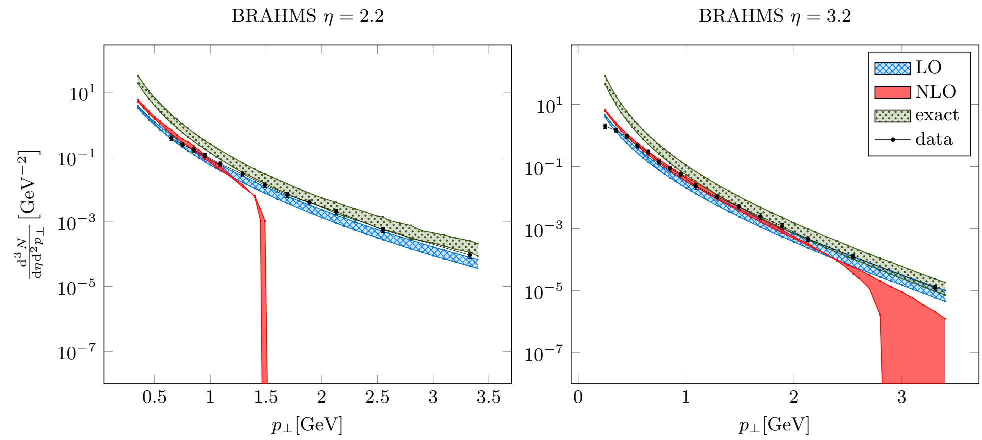

Figure 2: Comparison between the BRAHMS h^- data [10] at pseudorapidities $\eta = 2.2, 3.2$ and the LO and NLO small-x computations, as well as the large $p_\perp$ perturbative results with exact kinematics, at $y = 2.2, 3.2$. The edges of the band were computed with $\mu^2 = 10 GeV^2$ and $\mu^2 = 50 GeV^2$, thus the width of the band indicates the theoretical uncertainty due to the factorization scale. Calculated results use the unintegrated gluon distribution from the running coupling BK evolution.

4. Matching small x calculation to the collinear factorization

This instability at high values of the transverse momentum raises questions about the applicability of the whole small x formalism to the large values of the momentum transfer. It is however expected that in this regime the collinear approximation should work well, and therefore the calculation should match that collinear calculation in this limit. The small x calculation should be justified in the region when the transverse momentum of the gluon is of the order of the saturation scale: $k_T \simeq Q_s$. In this approach, the whole transverse momentum of the outgoing produced hadron comes from the multiple scattering off the target. In the collinear formalism, the transverse momentum comes from the hard scattering only and this framework should be applicable when $k_T \gg Q_s$. In [6] the relation between the two formalisms was thoroughly investigated.

In order to perform this matching a close look at kinematics is necessary, in particular one needs to inspect the kinematical variables which are arguments of the corresponding parton distribution functions. In Fig. 1 the relevant variables are shown for the production of the hadron. In the exact kinematics for the $2 \to 2$ case the longitudinal momentum fraction of the gluon from the target is equal to

$$x_a = \frac{k_\perp}{\sqrt{s}} e^{-y} + \frac{(\mathbf{k_{g\perp}} - \mathbf{k}_\perp)^2}{\sqrt{s} k_\perp} \frac{\xi}{1-\xi} e^{-y} \,. \tag{4.1}$$

In the collinear limit, the transverse momentum of the gluon is zero, $k_{g\perp} = 0$, and this variable becomes

$$x_a(k_{g\perp} = 0) = x' = \frac{k_\perp}{\sqrt{s}} e^{-y} + \frac{k_\perp}{\sqrt{s}} \frac{\xi}{1-\xi} e^{-y} \,. \tag{4.2}$$

On the other hand in the small x limit, this momentum fraction is taken to be equal to

$$x_a = \frac{k_\perp}{\sqrt{s}} e^{-y} \,. \tag{4.3}$$

This choice is dictated by level of the accuracy that one is working in the saturation formalism. To be precise, the approximation (4.3) is consistent with the accuracy of the small x calculation. However, in order to perform the matching to the collinear approach one needs to use the more exact kinematics, and therefore to use x_a as the argument in the unintegrated gluon distribution. In order to compare the small x calculation with the collinear approach, one performs the expansion of the off-shell matrix elements in powers of $1/k_T^2$ and keep the term in the lowest order. This corresponds to the leading twist, and should give the collinear formula. Indeed, taking this limit of the small x expressions (which are taken before any subtractions), the collinear expression for the cross section is obtained provided one can identify the integral over the unintegrated gluon distribution with the collinear gluon density as below

$$\int^{\mu^2} dk_{g\perp}^2 \, k_{g\perp}^2 \mathscr{F}_{x_a}(k_{g\perp}) \simeq \frac{2\pi\alpha_s}{N_c} \, x' G(x', \mu^2) \,, \tag{4.4}$$

where $\mu^2 \simeq Q_s^2$. We stress again the importance of the exact kinematics in the argument of the gluon distribution function in order to match the collinear result. The comparison between the small x calculations at the LO, NLO level and with the collinear approach which uses the gluon obtained from (4.4) is shown in Fig. 2. The calculations were performed for the BRAHMS data [10]. The left plot is for the pseudorapidity value equal $\eta = 2.2$ and the right plot for the pseudorapidity value equal to $\eta = 3.2$. The NLO small x calculation suffers from the instability at large values of the transverse momentum where the negative NLO correction dominates the cross section and the whole result becomes negative. The collinear limit with the small x gluon stays of course positive and nicely matches onto the NLO result. We note that the matching point moves to higher value of p_T as the rapidity is increased, which is consistent with the fact that the matching point should be around the value of the saturation scale Q_s. At lower p_T the NLO result is positive and it nicely describes the data, on the other hand the collinear calculation overshoots the experimental data in that region. This indicates that the resummation of all the twists, as performed in the small x calculation, is necessary to properly describe the experimental data in the low momentum region.

5. Summary

In conclusion, we have demonstrated that the saturation formalism for the forward inclusive hadron production in hadron-nucleus collisions in the large transverse momentum region can be matched to the corresponding collinear factorization result. We have shown that, in order to properly perform this matching, exact kinematics for the partonic subprocess has to be taken into account. We have shown that the matching between NLO calculation and the perturbative calculation occurs in the region of the saturation scale and moves to higher values of the transverse momenta with the increasing rapidity of the measured hadron. The perturbative calculation describes the data at higher p_T quite well, whereas the NLO small x calculation describes the low p_T data, where the perturbative calculation overestimates the data. This indicates the importance of the resummation of the higher twists in the low x, high density region.

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References

[1] A. Dumitru and J. Jalilian-Marian, Phys. Lett. B **547**, 15 (2002) [hep-ph/0111357].

[2] A. Dumitru and J. Jalilian-Marian, Phys. Rev. Lett. **89**, 022301 (2002) [hep-ph/0204028].

[3] I. Balitsky, Nucl. Phys. B **463**, 99 (1996) [hep-ph/9509348].

[4] Y. V. Kovchegov, Phys. Rev. D **60**, 034008 (1999) [hep-ph/9901281].

[5] G. A. Chirilli, B. W. Xiao and F. Yuan, Phys. Rev. D **86**, 054005 (2012) [arXiv:1203.6139 [hep-ph]].

[6] A. M. Stasto, B. W. Xiao, F. Yuan and D. Zaslavsky, Phys. Rev. D **90**, no. 1, 014047 (2014) [arXiv:1405.6311 [hep-ph]].

[7] A. D. Martin, W. J. Stirling, R. S. Thorne and G. Watt, Eur. Phys. J. C **63**, 189 (2009) [arXiv:0901.0002 [hep-ph]].

[8] D. de Florian, R. Sassot and M. Stratmann, Phys. Rev. D **75**, 114010 (2007) [hep-ph/0703242 [HEP-PH]].

[9] A. M. Stasto, B. W. Xiao and D. Zaslavsky, Phys. Rev. Lett. **112**, no. 1, 012302 (2014) [arXiv:1307.4057 [hep-ph]].

[10] I. Arsene *et al.* [BRAHMS Collaboration], Phys. Rev. Lett. **93**, 242303 (2004) [nucl-ex/0403005].