PROCEEDINGS OF SCIENCE

Transverse Momentum Distributions at Small-x

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We review recent theoretical developments connecting two fields: transverse momentum distributions in hadron physics and small-x saturation physics. Both fields use the same language to describe nucleon/nucleus structure in terms of parton distributions. We present the current understanding of the TMDs at small-x, and discuss future perspectives as well.

XXIII International Workshop on Deep-Inelastic Scattering,
27 April - May 1 2015
Dallas, Texas

*This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under contract number DE-AC02-05CH11231.

PoS(DIS2015)096

1. Introduction

There have been strong theoretical arguments and compelling experimental evidence that suggest that the gluon distribution saturates at small Bjorken-x [1, 2, 3, 4]. The emergence of the gluon saturation at small-x changes the landscape of parton distributions inside the nucleon/nucleus. In the dense region, the QCD dynamics will be different as compared to that in the dilute region. For example, the evolution equation has to be modified to account for the high density gluon distribution. In the dilute region, the DGLAP evolution has been employed to understand the scale dependence of parton distributions, whereas the small-x BFKL evolution (and the nonlinear extension: BK-JIMWLK) plays a crucial role. These QCD effects are summarized in Fig. 1, which is usually referred as the phase structure of the cold nuclear matter. The QCD dynamics in the dilute region, aka, the DGLAP evolution, has been systematically studied in the last three decades thanks to the vast experimental data generated from various high energy facilities. On the other hand, the investigation of the small-x dynamics (either BFKL or BK-JIMWLK) had just started very recently, although the theoretical arguments have been put in place in late 70s. A central question is to identify the boundary between the dilute and dense regions, the so-called saturation limit.

It was realized that the semi-inclusive processes, which involve a hard momentum scale Q in addition to the transverse momenta of the observables, have a unique feature to probe the saturation physics. The most important advantage is that they can directly access to the unintegrated gluon distributions (UGDs), which are important ingredients in the saturation physics. They unveil the importance of the multiple interaction effects in the factorization of the hard processes in the small-x calculations. Furthermore, the UGDs are unified with the transverse momentum distributions (TMDs) of gluon in nucleon/nucleus [5]. The TMDs are closely related to the tomography concept of partons which are developed in the last few years in hadron physics community. Therefore, the TMDs at small-x shall provide a unique opportunity to connect the underlying physics associated with wide range of phenomena. In this talk, we will review some of recent progress toward to a unified picture for the TMDs at small-x [4].

2. Gluon Distributions at Small-x

In saturation physics, two different unintegrated gluon distributions (UGDs) have been widely used in the literature. The first gluon distribution, which is known as the Weizsäcker-Williams (WW) gluon distribution, is calculated from the correlator of two classical gluon fields of relativistic hadrons [3, 6]. The WW gluon distribution has a clear physical interpretation as the number density of gluons inside the hadron in light-cone gauge, and can be defined following the conventional gluon distribution [7, 8]

$$xG^{(1)}(x,k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{L}_\xi^\dagger \mathcal{L}_0 F^{+i}(0)|P\rangle \,, \qquad (2.1)$$

where $F^{\mu\nu}$ is the gauge field strength tensor $F_a^{\mu\nu}$

$$\mathcal{L}_\xi = \mathscr{P}\exp\{-ig\int_{\xi^-}^\infty d\zeta^- A^+(\zeta,\xi_\perp)\}\mathscr{P}\exp\{-ig\int_{\xi_\perp}^\infty d\zeta_\perp\cdot A_\perp(\zeta^-=\infty,\zeta_\perp)\}$$

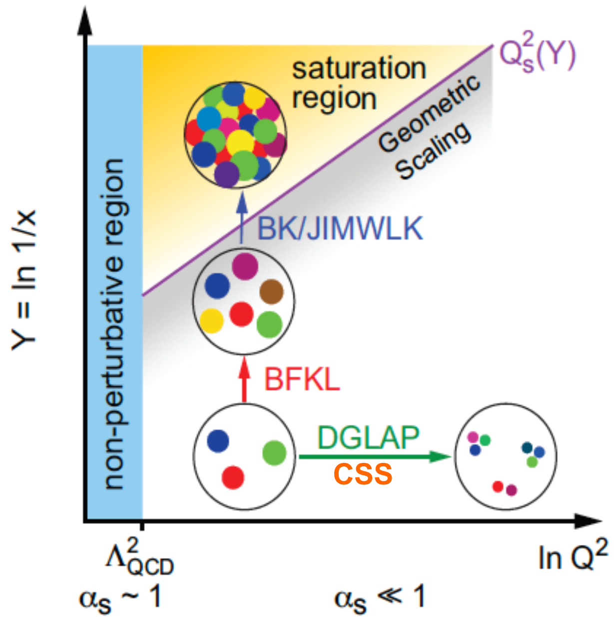

Figure 1: The so-called phase structure of the cold nuclear matter: parton distributions in the dilute and dense region and the associated QCD dynamics controlling the evolutions. Along the momentum scale (Q^2) in the dilute region, we have the well-known DGLAP evolution, whereas the BFKL evolution plays important role at small-x. In the dense region, a non-linear term in BFKL has to be taken into account, which leads to the BK-JIMWLK evolution. We note that the DGLAP and BFKL are dealing with different forms of the parton distributions: the integrated and unintegrated ones, respectively.

is the gauge link in the adjoint representation. This gluon distribution can also be defined in the fundamental representation [9],

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathscr{U}^{[+]\dagger}F^{+i}(0)\mathscr{U}^{[+]}\right]|P\rangle \, , \quad (2.2)$$

where the gauge link $\mathscr{U}_\xi^{[+]} = U^n[0,+\infty;0]\,U^n[+\infty,\xi^-;\xi_\perp]$ with U^n being reduced to the light-like Wilson line in covariant gauge. It is straightforward to see that $\mathscr{U}^{[+]}$ represents the final state interactions according to its future integration path to $+\infty$. By choosing the light-cone gauge with certain boundary condition for the gauge potential ($A_\perp(\zeta^- = \infty) = 0$ for the specific case above), we can drop out the gauge link contribution in Eqs. (2.1) and (2.2) and find that this gluon distribution has the number density interpretation. Then, it can be calculated from the wave functions or the WW field of the nucleus target [3, 6]. Within the CGC framework, this distribution can be written in terms of the correlator of four Wilson lines as,

$$xG^{(1)}(x,k_\perp) = -\frac{2}{\alpha_S}\int \frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}\, e^{-ik_\perp\cdot(v-v')}\left\langle \mathrm{Tr}\left[\partial_i U(v)\right]U^\dagger(v')\left[\partial_i U(v')\right]U^\dagger(v)\right\rangle_{x_g} , \quad (2.3)$$

where the Wilson line $U(x_\perp)$ is defined as $U^n[-\infty,+\infty;x_\perp]$.

The second gluon distribution, the Fourier transform of the dipole cross section, is defined in the fundamental representation

$$xG^{(2)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathscr{U}^{[-]\dagger}F^{+i}(0)\mathscr{U}^{[+]}\right]|P\rangle \ , \quad (2.4)$$

where the gauge link $\mathscr{U}_\xi^{[-]} = U^n[0,-\infty;0]\,U^n[-\infty,\xi^-;\xi_\perp]$ stands for initial state interactions. Here, the gauge link contribution can not be completely eliminated. In other words, there is no number density interpretation for this gluon distribution. This is also because it contains both initial and final state interaction effects. Due to the gauge link in this gluon distribution from $-\infty$ to $+\infty$, naturally this gluon distribution can be related to the color-dipole cross section evaluated from a dipole of size $r_\perp$ scattering on the nucleus target, and has been calculated in the CGC formalism,

$$xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp} \frac{1}{N_c} \left\langle \mathrm{Tr} U(0) U^\dagger(r_\perp)\right\rangle_{x_g} . \quad (2.5)$$

It is important to investigate the above two gluon distributions by measuring the quark-antiquark correlation in DIS process and direct photon jet correlation in pA collisions, since these processes can directly probe these two gluon distributions separately [5]. A comprehensive study on the dihadron correlations in the forward pA collisions has been performed with the full CGC results [5]. An effective k_t factorization has been established for these processes in the back-to-back correlation limit. In particular, it was found that, the differential cross sections can be expressed in terms of various UGDs, which can be related to the above two fundamental UGDs.

3. QCD Resummaiton: BFKL vs Sudakov

An important application of the perturbative quantum chromodynamics (QCD) is the resummation. In high energy hadronic processes involving largely separated scales, resummation is not only necessary to make reliable predictions, but also crucial to extract the fundamental properties of the strong interaction theory. One of the examples is the resummation of the Sudakov-type double logarithms [10, 11]. The double logarithms appear in, for example, the transverse momentum spectrum of a hard process, where each order of perturbative correction is accompanied by a large double logarithmic term of $\ln^2(Q^2/k_\perp^2)$ with Q the large momentum scale and $k_\perp$ the transverse momentum. This resummation is often referred as the transverse momentum dependent (TMD) resummation. Meanwhile, there is also the small-x resummation which is equally important, in particular, in the energy regime of the large hadron collider (LHC). The small-x resummation is governed by the well-known BFKL evolution [12], which will be extended to the so-called BK-JIMWLK evolution [13] to take into account the non-linear term in the evolution due to high gluon density in nucleons/nuclei at high energy.

One would naturally ask the following question: if there is a place that we have to resum the above two large logarithms and can we do that consistently? The answer is yes. In recent studies of Refs. [14, 15, 16], it has been shown that we can perform the above two resummations (Sudakov and BFKL) consistently in physical processes in high energy scattering. The physics behind the above results can be understood as follows. The gluon radiation comes from three different regions, for a particular partonic channel in pA collisions: (1) collinear gluon parallel to the incoming nucleon; (2) collinear gluon parallel to the incoming nucleus; (3) soft gluon radiation. For example, for one-gluon radiation contribution to the Born diagram in pA collisions, we can parameterize the radiated gluon momentum as $q = \alpha_q p_1 + \beta_q p_2 + q_\perp$, where p_1 and p_2 denote the four momenta of incoming partons in the partonic process. The momentum region of (1) corresponds to $\alpha_q \sim \mathscr{O}(1)$, whereas $\beta_q \ll 1$, which contributes to the collinear divergence associated with the parton distribution from the nucleon. For region (2), the dominant collinear gluon radiation parallel to the nucleus requires not only β_q of order 1, but also close to 1, i.e., $1-\beta_q \ll 1$. Effectively, because of $q_\perp \sim q_\perp \ll Q$, this leads to $\alpha_q \to 0$. This corresponds to the rapidity divergence at one-loop order, which can be absorbed into the renormalization of the un-integrated gluon distribution of the nucleus in the small-x limit. Region (3) concerns the Sudakov double logarithms where $\alpha_q \sim \beta_q \ll 1$. The gluon radiation in this kinematic region depends on the overall color flow in the hard partonic processes [14]. The kinematics of the three regions are well separated, and at the leading power of $q_\perp/Q$, they can be factorized into various factors.

We emphasize that the small-x resummation of high order corrections in terms of $\ln(s/Q^2) \sim \ln(1/x_A)$ is carried out by solving the small-x evolution equations, whereas the Sudakov double logarithms in terms of $\ln(Q^2/q_\perp^2)$ is resummed by solving the TMD evolution equation. In a recent paper, Balitsky and Tarasov have shown that both evolution equations can be derived from the operator definition of the gluon distribution. As a consequence, we immediately find that the Sudakov effects will be dominant effects in the dilute region where $Q_s \ll Q$. On the other hand, in the dense region with $Q \sim Q_s$, the small-x effects dominates. Since the small-x and Sudakov resummations are common features in hard processes in pA collisions, we expect they will play important roles in hard scattering processes in pA collisions.

4. Quark TMD at Small-x

The TMD quark distribution is defined as [17]

$$q(x,k_\perp) = \frac{1}{2}\int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^2} e^{-ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\bar{\Psi}(\xi^-,\xi_\perp)\mathscr{L}_\xi \gamma^+ \mathscr{L}_0 \Psi(0)|P\rangle \ , \tag{4.1}$$

where P is the momentum for the hadron, x and $k_\perp$ are longitudinal momentum fraction of the hadron and transverse momentum carried by the quark. At small-x, we can calculate the TMD quark distribution in nucleon/nucleus in the saturation framework. This is because, at this region, the quark distribution is dominated by gluon splitting [18, 19, 20],

$$xq(x,k_\perp) = \frac{N_c}{4\pi^4}\int d^2b d^2q_\perp F(q_\perp,x)\left(1 - \frac{k_\perp\cdot(k_\perp-q_\perp)}{k_\perp^2-(k_\perp-q_\perp)^2}\ln\frac{k_\perp^2}{(k_\perp-q_\perp)^2}\right) \ , \tag{4.2}$$

where $F(q_\perp)$ is the well-known dipole gluon distribution, A number of interesting features of this quark distribution have been discussed in the literature. For example, in the small $k_\perp$ limit, the quark distribution saturates: $xq(x,k_\perp)|_{k_\perp\to 0} \propto N_c/4\pi^4$; in the large $k_\perp$ limit, it has power-law behavior $xq(x,k_\perp)\big|_{k_\perp \gg Q_s} \propto Q_s^2/k_\perp^2$.

More importantly, we can incorporate both small-x QCD dynamics and Sudakov resummation in the above quark distributions. First, we can calculate the TMD quark distribution from the dipole gluon distribution from the above equation, where the dipole gluon distribution obeys the small-x

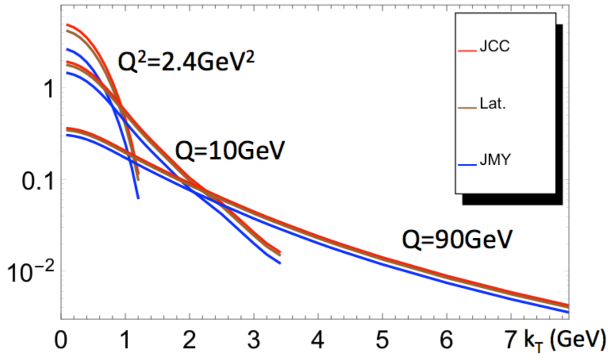

Figure 2: TMD up quark distributions $f_u^{(sub.)}(x=0.1,k_\perp)$ as functions of the transverse momentum (right) at different scale Q for different schemes, from the top to the bottom: JCC, Lat., JMY ($\ln\rho=1$).

BK evolution equation. In the sense, once we have the model parameterization for the dipole gluon distribution at some initial x_0, we can compute not only the dipole gluon distribution at any value of small-x by solving the BK evolution equation, but also the quark distribution. This provides a powerful approach for phenomenology associated with the quark distributions at small-x.

Meanwhile, the Sudakov logarithms can be resummed by solving the TMD evolution equation. This solution help to simplify the differential cross sections in terms of the TMDs at the hard momentum sale, i.e., setting the factorization scale at $\mu_F=Q$. In addition, there is scheme dependence in the TMD definition and factorization, due to the soft factor contribution. The way to subtract the soft factor and regulate the associated light-cone singularity defines the scheme [21, 22, 23]. However, after solving the evolution equations, the scheme dependence can be factorized into perturbative calculable coefficients, which can be compared with different schemes. Thus, we obtain a unified picture for TMD phenomenology [24]. In Fig. 2, as an example, we plot the up quark distributions at $x=0.1$ for different schemes at different scale Q: $\widetilde{f}_u^{(sub.)}(x=0.1,b_\perp)$ as functions of $b_\perp$ (left) and $f_u^{(sub.)}(x=0.1,k_\perp)$ as functions of the transverse momentum $k_\perp$. Here, three chosen schemes have been shown: Collins-2011 scheme (JCC) [21]; Ji-Ma-Yuan scheme (JMY) [22]; Lattice scheme (Lat) [23]. As a general feature, there are broadening effects for TMD distributions at higher scales. These plots also show that the difference between the different schemes becomes less evident at higher scales. Furthermore, the scheme dependence in TMD distributions will be compensated for by the hard factors of each scheme, and the final expressions will be the same when compared with the physical cross sections from experiments. It will be interesting to compare the TMDs at small-x as we discussed above.

5. Conclusion

In this talk, we have reviewed recent theoretical developments on TMDs at small-x and the associated QCD resummation in hard processes in hadronic collisions, It was shown that the two famous resummations in high energy scattering: Sudakov and BFKL can be encoded in the calculations consistently. This has opened a new window to study strong interaction physics in great details. We anticipate more theoretical and phenomenological developments along this direction.

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