The story of WINO Dark Matter

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We calculate the annihilation rate for neutral winos into the state $\gamma + X$, including all leading log(\LL) radiative corrections. We factorize the cross section into a long distance contribution (Sommerfeld enhancement) followed by a short distance annihilation (involving Sudakov double logarithms) using a hybrid SCET effective theory. Using the analysis of the HESS experiment, which looks for gamma ray lines, we place constraints on the wino mass as a function of the wino fraction of the dark matter and the shape of the dark matter profile. We also determine the viability of the wino as a dark matter candidate as a function of the coring of the galactic dark matter halo.

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1. Introduction

All of the astrophysical and cosmological evidence for dark matter has yet to reveal whether it has any non gravitational interactions with the standard model (SM). The WIMP Miracle presents a particularly compelling link between the weak scale and dark matter (see [1]), leading to an $\mathcal{O}(\text{TeV})$-mass particle with electroweak-strength coupling. We consider here the case of the wino, that belongs to a supersymmetric explanation of the weak scale.

A nearly pure wino emerges as the lightest supersymmetric particle (LSP) [2] in theories of anomaly-mediated SUSY-breaking. If we assume that the wino constitutes all of the dark matter and that its relic density was set at freeze-out, then the mass is constrained to the window $M_{\text{Wino}} \equiv (M_\chi) = 2.7-2.9$ TeV [3, 4].

The direct detection cross section for a TeV wino to scatter off nucleons is $\sigma \sim 10^{-47}$ cm$^2$, putting it far below current limits [5, 6]. However, since the wino can annihilate directly to photons, by searching for monochromatic, $\mathcal{O}(\text{TeV})$ photon lines, we can hope to discover it via indirect detection. The authors of [3, 7] used limits from the HESS Cherenkov telescope to argue that the nonobservation of such a photon feature put wino DM in severe tension with experiment. In particular, [7] calculated the annihilation rate to be $\sim 15\times$ larger at $M_\chi = 3$ TeV than the HESS limit.

However, this analysis is subject to astrophysical uncertainties. [3, 7] consider variations of the DM halo profile for the galaxy. To alleviate the tension with HESS, some amount of coring will be necessary to return pure wino dark matter to viability, but the question, which we answer in Section 5, is how much?

The annihilation rate of two heavy WIMPs ($M_\chi \gg M_W$), which are nonrelativistic, is plagued by infrared (IR) divergences which are cut-off by the gauge boson mass, $M_W \sim 100$ GeV. These corrections are of two types: the resummation of these corrections results in a Sommerfeld enhancement to the rate [8]. The second type of IR sensitivity is a Sudakov double-log, $\alpha_W \log(M_\chi/M_W)^2$, which arises due to the non-singlet nature of our external states. This effect is known as “Bloch-Nordsieck Theorem Violation” [9, 10]. We derive a factorization theorem to calculate the leading-log (LL) semi-inclusive wino annihilation rate ($\chi^0\chi^0 \rightarrow \gamma + X$) which is the relevant observable for constraining the wino with HESS since only a single hard photon from annihilation is measured and the resolution of the experiment is too poor to distinguish two-body from $n$-body annihilation (e.g. $\chi^0\chi^0 \rightarrow \gamma + W^+W^-$) [12]. Despite the fact that both the Sommerfeld and Sudakov effects arise from the same hierarchy, $M_\chi \gg M_W$, they can be factorized through a mode decomposition of the relevant fields.

We resum Sudakov logs and present our analysis of the Sommerfeld enhancement for the particular case of wino dark matter. We find that compared to tree level plus Sommerfeld corrected rate, the leading-log radiative corrections lead to a few-percent reduction in the rate. We also present exclusion plots for wino dark matter as a function of the mass, the amount of coring in the dark matter profile, and the wino fraction of the dark matter.

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1The term “divergences” is used despite the fact that the rate is physical.
2. Factorization

To develop a factorization theorem it helps to work in an EFT, which in our case is a hybrid of SCET and NRQCD that power counts in a double expansion in $v$, the relative velocity of the DM particles and $\lambda = M_W/M_\chi$. We will work to leading order in both these parameters, which seems quite reasonable until a discovery is made. The momentum modes we need in our EFT are modes for which, either all components of momenta are small (soft modes) $(\vec{n} \cdot p, n \cdot p, p_\perp) \sim M_\chi (\lambda, \lambda, \lambda)$ or collinear with $p \sim M_\chi (1, \lambda^2, \lambda)$ (SCETII). We choose a lightlike-vector $\vec{n}$ for the direction of the final jet containing the observed photon where $n = (1,0,0,1)$.

We build a factorization theorem at the level of the amplitude squared integrating out all final state particles except the final state jet containing observed photons and accompanying unobserved particles. We thus match the full theory onto a set of operators with six fields, corresponding to the incoming winos and the outgoing, collinear, photon. The minimal operator basis that we can write down is

$$
O_1 = (\bar{\chi} \gamma^5 \chi) |0\rangle \langle 0| (\bar{\chi} \gamma^5 \chi) B^{\mu A}_\perp B^{A\perp}_\mu
$$

$$
O_2 = \frac{1}{2} \left\{ (\bar{\chi} \gamma^5 \chi) |0\rangle \langle 0| (\bar{\chi} \gamma^5 \chi) B^{\mu A}_\perp B^{A\perp}_\mu + (\bar{\chi} \gamma^5 \chi) |0\rangle \langle 0| (\bar{\chi} \gamma^5 \chi) B^{\mu B}_\perp B^{B\perp}_\mu \right\}
$$

$$
O_3 = (\bar{\chi} \gamma^5 \chi D) |0\rangle \langle 0| (\bar{\chi} \gamma^5 \chi D) B^{\mu A}_\perp B^{A\perp}_\mu, \quad O_4 = (\bar{\chi} \gamma^5 \chi C) |0\rangle \langle 0| (\bar{\chi} \gamma^5 \chi B) B^{\mu B}_\perp B^{B\perp}_\mu
$$

(2.1)

where we use the vacuum insertion approximation in the WIMP sector, which is valid up to $O(v^2)$ corrections. Henceforth, we drop the explicit vacuum projector. Implicitly, there is also a projection onto a single-photon state between the $B^{\mu \perp}$ fields i.e.

$$
B^{\mu A}_\perp B^{B\perp}_\mu \equiv \sum_X B^{\mu A}_\perp |\gamma + X\rangle \langle \gamma + X| B^{B\perp}_\mu.
$$

(2.2)

where $X$ contains the accompanying particles in the collinear jet. The only relevant nonrelativistic bilinear is $\bar{\chi} \gamma_5 \chi$. The $B$ field here interpolates for a collinear gauge boson (for details see [13]).

There is method which we call the “method of descent”, developed in [17] which allows us to place the Soft Wilson lines which dresses the operators such that $O_2$ and $O_4$ become

$$
O_2 = \frac{1}{2} \left\{ (\bar{\chi} \gamma^5 \chi) (\bar{\chi} A \gamma^5 \chi_B) + (\bar{\chi} A \gamma^5 \chi_B) (\bar{\chi} \gamma^5 \chi) \right\} B^{\mu A}_\perp B^{B\perp}_\mu S^{\perp}_{vA} S^{\perp}_{vB} S^{\perp}_{nAA} S_{nBB}
$$

$$
O_4 = (\bar{\chi} A \gamma^5 \chi_C) (\bar{\chi} C \gamma^5 \chi_B) B^{\mu A}_\perp B^{B\perp}_\mu S^{\perp}_{vA} S^{\perp}_{vB} S^{\perp}_{nAA} S_{nBB}
$$

(2.3)

The operators $O_1$ and $O_3$ receive no soft corrections. The annihilation spectrum may be written as

$$
\frac{1}{E_\gamma dE_\gamma} \frac{d\sigma}{d\cos\theta} = \frac{1}{4m_\chi^2} \int |0\rangle \langle 0| \left[ \int d\vec{n} \cdot \vec{p} \left\{ C_2(M_\chi, n \cdot p) \langle p_1 p_2 | \left( \bar{\chi} \gamma^5 \chi \right) (\bar{\chi} A \gamma^5 \chi_B) \right\} + \left( \bar{\chi} A \gamma^5 \chi_B \right) (\bar{\chi} C \gamma^5 \chi_B) \langle 0 | p_1 p_2 \right\} + C_4(M_\chi, n \cdot p) \langle p_1 p_2 | (\bar{\chi} A \gamma^5 \chi_C) (\bar{\chi} C \gamma^5 \chi_B) \langle 0 | p_1 p_2 \right\} + C_5(M_\chi, n \cdot p) \right]
$$

(2.4)

$$
+ \left\{ C_1(M_\chi, n \cdot p) \langle p_1 p_2 | (\bar{\chi} \gamma^5 \chi) (\bar{\chi} C \gamma^5 \chi) \langle 0 | p_1 p_2 \right\} + C_5(M_\chi, n \cdot p) \right]
$$

(2.5)

$$
+ \left\{ C_1(M_\chi, n \cdot p) \langle p_1 p_2 | (\bar{\chi} C \gamma^5 \chi_B) \langle 0 | p_1 p_2 \right\} + C_5(M_\chi, n \cdot p) \right]
$$

(2.6)
\[ \times \langle p_1 p_2 | (\mathcal{X}_c \gamma^5 \mathcal{X}_D)(\mathcal{X}_D \gamma^5 \mathcal{X}_c)(0) | p_1 p_2 \rangle \left[ F_{\gamma} \left( \frac{2E_T}{n \cdot p} \right) \right], \]  

where \( O_s^a = S_{A A}^{T} S_{n A}^{B} S_{n A}^{B B} \) and \( F_{Y}^{A B} \) is a fragmentation function defined by

\[ F_{Y}^{A B} \left( \frac{n \cdot k}{n \cdot p} \right) = \int \frac{dx_+}{2\pi} e^{i p x_+} \langle 0 | B_{A}^{\gamma} (x_+) | \gamma + X_n | B_{B}^{\gamma} (0) | 0 \rangle, \]  

and \( F_{Y} = F_{Y}^{A B} \delta_{AB} \). Note that this is an unusual fragmentation function in that we are measuring states which are not gauge singlets. \( C_{1-4} \) are the matching coefficients and \( F_{Y} \) is the canonical fragmentation function giving the probability of an initial photon with momentum \( k \) to yield a photon with momentum fraction \( n \cdot k / n \cdot p \) after splitting.

Thus, we factorized the collinear and soft fields, as the total Hilbert space of the system is a tensor product of the soft and collinear sector.

### 3. Calculating the Anomalous Dimension

We first introduce an operator basis in the collinear and soft sectors

\[ O_s^a = S_{A A}^{T} S_{n A}^{B} S_{n A}^{B B} \]  

\[ O_s^b = \delta_{AB} \delta_{AB} \]  

\[ O_s^c = B_{A}^{\gamma} | \gamma (k_A) + X_A | B_{B}^{\gamma} \]  

\[ O_s^d = B_{B}^{\gamma} | \gamma (k_B) + X_B | B_{A}^{\gamma} \delta_{AB}. \]  

The divergences, that arise from the factorization of the soft sector from the collinear needs a rapidity regulator[16]. This requires a corresponding factorization scale which we call \( \nu \). The operators mix within their respective sectors and we can define anomalous dimension matrices for the scales \( \mu \) and \( \nu \) as follows

\[ \mu \frac{d}{d \mu} \left( \begin{array}{cc} O_s^a & O_s^c \\ O_s^b & O_s^d \end{array} \right) = \left( \begin{array}{cc} \gamma_{s,aa} & \gamma_{s,ab} \\ 0 & 0 \end{array} \right) \left( \begin{array}{cc} O_s^a & O_s^c \\ O_s^b & O_s^d \end{array} \right), \]  

\[ \nu \frac{d}{d \nu} \left( \begin{array}{cc} O_s^a & O_s^c \\ O_s^b & O_s^d \end{array} \right) = \left( \begin{array}{cc} \gamma_{s,aa} & \gamma_{s,ab} \\ 0 & 0 \end{array} \right) \left( \begin{array}{cc} O_s^a & O_s^c \\ O_s^b & O_s^d \end{array} \right). \]  

The anomalous dimensions are given by

\[ \gamma_{s,aa} = \frac{3g^2}{4\pi^2} \log \left( \frac{\nu^2}{4M_W^2} \right), \quad \gamma_{s,ba} = -\frac{3g^2}{4\pi^2} \log \left( \frac{\mu^2}{\nu^2} \right), \]  

\[ \gamma_{s,ab} = -\frac{g^2}{4\pi^2} \log \left( \frac{\nu^2}{4M_W^2} \right), \quad \gamma_{s,bb} = \frac{g^2}{4\pi^2} \log \left( \frac{\nu^2}{\mu^2} \right). \]  

\[ \gamma_{s,aa} = \frac{3g^2}{4\pi^2} \log \left( \frac{\mu^2}{M_W^2} \right), \quad \gamma_{s,ba} = -\frac{3g^2}{4\pi^2} \log \left( \frac{M_W^2}{\mu^2} \right), \]  

\[ \gamma_{s,ab} = -\frac{g^2}{4\pi^2} \log \left( \frac{\mu^2}{M_W^2} \right), \quad \gamma_{s,bb} = \frac{g^2}{4\pi^2} \log \left( \frac{M_W^2}{\mu^2} \right). \]  

At leading double log accuracy we can resum all of the relevant terms by choosing \( \mu = M_W \). We can read off the running of the hard matching coefficients \( C_{1-4} \) of the operators in Eq. 2.1 by imposing that the cross section be RG invariant.
\[ \frac{d}{d\mu} C_{2,4}(\mu) = -(\gamma'_{\mu,aa} + \gamma'_{\mu,ba}) C_{2,4} \]
\[ \mu \frac{d}{d\mu} C_{1,3}(\mu) = -(\gamma'_{\mu,ba} + \gamma'_{\mu,ba}) C_{2,4}. \tag{3.5} \]

Notice that the RHS of Eq. 3.5 is independent of the rapidity scale as it must be. We have \( C_1 = C_4 \), \( C_3 = 0 \) and \( C_2 = -2C_1 \) when matching at the high scale \( M_X \). The cross section can now be obtained by evaluating the effective theory matrix elements at their natural scale \( \mu \sim M_W \). At the low scale, we are working in the broken theory, where the mass eigenstates are the neutralino \( \chi^0 \) and the charginos, \( \chi^\pm \), which are defined as

\[ \chi^0 = \chi^3, \quad \chi^\pm = \frac{1}{\sqrt{2}} (\chi^1 \pm i\chi^2) \tag{3.6} \]

We obtain the final form of the cross section up to corrections in the relative velocity (which is of the order of \( 10^{-3} \)) [11].

\[ \frac{1}{E_\gamma} \frac{d\sigma}{dE_\gamma} = C_1(\mu = E_\gamma) \frac{\delta(E_\gamma - M_X)}{4M_X^2 v} \left[ \frac{2}{3} f_+ |\psi_{00}(0)|^2 + 2f_+ |\psi_+-(0)|^2 \right. \\
\left. + \frac{2}{3} f_- (\psi_{00}\psi_+ - \text{h.c.}) \right] \tag{3.7} \]

where \( f_\pm = 1 \pm \exp[-\frac{3\alpha_c}{8} \log^2 \left( \frac{M_W}{M_X} \right)] \). We define the wavefunctions as

\[ \psi_{00} = \langle 0 | \bar{\chi}^0 \gamma^5 \chi^0 | 0 \rangle, \quad \psi_\pm = \langle 0 | \bar{\chi}^\pm \gamma^5 \chi^\pm | 0 \rangle \tag{3.8} \]

where \( | \chi^0 \chi^0 \rangle = \frac{1}{\sqrt{2}}(|\chi^0(p_1)\chi^0(p_2)\rangle - |\chi^0(p_1)\chi^0(p_2)\rangle) \).

In order to fix the Wilson coefficient \( C_1 \), we match onto the tree level annihilation cross section of a spin singlet chargino state \( \frac{1}{\sqrt{2}}(|\chi^+_1(p_1)\chi^-_0(p_2)\rangle - |\chi^-_1(p_1)\chi^+_0(p_2)\rangle) \).

### 4. Sommerfeld Enhancement

In order to quantify the semi-inclusive rate calculation, we need to determine the wavefunction-at-the-origin factors that enter our final, LL-resummed differential cross section in Eq. 3.7. The wavefunctions themselves are defined in Eq. 3.8 and can be computed in principle in the nonrelativistic effective theory by summing the ladder exchange of electroweak gauge bosons between winos to all orders. Fortunately, this is equivalent to the operationally simpler task of solving the Schrödinger equation for our two, two-body states \( |\chi^0\chi^0\rangle \) and \( |\chi^+\chi^-\rangle \) in the presence of the electroweak potential, details of which can be found in [8, 18, 19].

### 5. Dark Matter Constraints and Conclusion

We can now evaluate the differential cross section for \( \chi^0\chi^0 \rightarrow \gamma + X \), given in Eq. 3.7. We plot this in Fig. 1, where we have digitized the HESS limits given [15]. We note that in contrast to those
The story of WINO Dark Matter

Varun Vaidya

Figure 1: Annihilation cross section to $\gamma + X$. Exclusion taken from [15], assuming an NFW profile.

groups that performed an exclusive two-body calculation, [14, 15], we find the effect of higher order correction to be very modest. This difference is to be expected given the distinct difference in our choice of observables.

The limit from HESS in Fig. 1 shows that the thermal relic wino, $M_\chi \approx 3$ TeV is ruled out by more than an order of magnitude. Unfortunately, the astrophysical uncertainties in the halo profile are sufficient to evade an excess of even this size. Discussions on the ability of different halo models to evade constraints can be found in the earlier papers that found the wino to be in tension with HESS [7, 3]. The exclusion curve we have taken from [15] assumes an NFW profile [20] with a local density, $\rho_{\text{loc}} = 0.4$ GeV/cm$^3$ [21, 22], and $r_s = 20$ kpc [23]. In the discussion that follows, we fix the local density $\rho_{\text{loc}} = 0.4$ GeV/cm$^3$, but we will change the functional form of the distribution along with a possible core radius. It is possible though, that the local density could lie somewhere in the range of 0.2-0.6 GeV/cm$^3$ [23]. One can ask, how much coring is needed to save the wino, given our LL-resummed annihilation rate? For an NFW profile that becomes constant below a certain radius,

$$\rho_{\text{cutoff-NFW}}(r) = \begin{cases} \frac{\rho_0}{(r/r_s)(1+r/r_s)^2}, & r > r_c \\ \frac{\rho_0}{(r_c/r_s)(1+r_c/r_s)^2}, & r \leq r_c \end{cases}$$

in Fig. 3 we plot the value of the core radius, $r_c$, needed to make our semi-inclusive rate calculation consistent with the limit from HESS.

The observation of wino dark matter near the thermal relic mass of 3 TeV would point to the existence of a nontrivial amount of coring in the halo of the galaxy which would require an explanation. However, there is the possibility that the lightest neutralino may not be a pure wino. For example, a thermal relic higgsino is far from constrained, and thus admixtures between these states could certainly be allowed [3]. Sticking with the pure wino, if there were some non-thermal mechanism for its production, then the limit at values other than 3 TeV would be relevant, and $M_\chi$ could be in one of the allowed regions shown in Fig. 1.

Alternatively the wino could make up just a fraction of the dark matter, and thus much of parameter space would remain open, as shown in Fig. 2. The discovery of a wino at future indi-
The story of WINO Dark Matter

Varun Vaidya

The story of WINO Dark Matter

Figure 2: Exclusion plot for an NFW profile with the wino making up only some fraction of the dark matter. Expression for NFW profile with coring given in Eq. 5.1.

Figure 3: The amount of coring required for the wino to become viable with respect to the HESS constraint shown in Fig. 1 for the cutoff-NFW profile (Eq. 5.1). The three curves display the effect of variation in the local dark matter density.

rect detection experiments, such as CTA [24], could give us important windows into further open questions such as the halo distribution, cosmological history of DM production, and the presence of multi-component dark matter.

References


The story of WINO Dark Matter

Varun Vaidya


