PROCEEDINGS OF SCIENCE

NNLL Resummation of Event-Shapes in e^+e^- Annihilation

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Event shape observables are essential tools for studying the behaviour of high energy QCD. Yet in the dijet region, standard perturbation theory is rendered unreliable and the series must be resummed to all orders in the strong coupling. We have recently developed a general method for the resummation of event shapes, in e+e- annihilation, at next-to-next-to-leading logarithmic accuracy. We have implemented the novel method semi-numerically and reproduced four already-known predictions, as well as three new results. We match our findings to fixed-order results, up to NNLO.

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1. Introduction

Obtaining precise predictions for high-energy QCD-events is necessary for both Standard Model tests and beyond the Standard Model searches. However the standard perturbative method, a series expansion in the strong coupling α_s, breaks down in certain regions of phase space. In the dijet region, large logarithms arising from kinematically constrained (soft and collinear) gluon emissions must be resummed in order to restore the calculability of the series.
We calculate the cross-sections of seven event shapes in the regime of $e^+e^- \to$ 2jets. This provides us with a clean and relatively simple environment in which to build an extendable resummation framework.

2. NNLL Resummation: ARES

We work in the regime where $\alpha_s \ln(1/v) \approx 1$, where v is the value of the observable we are resumming. The dominant contributions to $\ln(\Sigma(v))$, with $\Sigma(v)$ the cumulative distribution, are those of the form $\alpha_s^n \ln^{n+1}(1/v)$, for n emissions/virtual corrections. Resumming only contributions of this form gives a leading-logarithmically accurate result (LL). Also including terms of form $\alpha_s^n \ln^n(1/v)$ results in a calculation accurate to next-to-leading logarithmic order (NLL), and so on.

Using the philosophy of CAESAR for NLL resummation [1], we have the parameterisation of a generic observable in the presence of a single soft-collinear gluon emission k in terms of its transverse momentum k_t, and its rapidity η and azimuth ϕ with respect to the emitting hard partons:

$$V(\{\tilde{p}\},k) = d_\ell \left(\frac{k_t^{(\ell)}}{Q}\right)^{a_\ell} e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi^{(\ell)}) \tag{2.1}$$

$\{\tilde{p}\}$ denotes the final-state hard partons after recoil effects, in this case $\{\tilde{p}\} = \{\tilde{p}_1, \tilde{p}_2\}$, where p_1 and p_2 are the high-energy quark-antiquark pair seeding the two final-state jets. Q is the total centre of mass energy of the collision. The subscript ℓ refers to the emitting parton, since this parameterisation holds beyond the 2-leg $e^+e^- \to$ 2jets process.
The observables must also satisfy recursive infrared and collinear (rIRC) safety. In analogy with standard IRC safety, rIRC safety stipulates that the observable's scaling properties should be the same in the presence of any number of extra soft and/or collinear emissions. Additionally, the observables must be continuously global, thereby prohibiting effects of non-global logarithms. [2]
This implies that $a_1 = a_2 = a$.
At NLL, the resummed cumulative distribution for an observable of this form can be written

$$\Sigma(v) = \frac{1}{\sigma}\int_0^v dv' \frac{d\sigma(v')}{dv'} = e^{Lg_1(\lambda)+g_2(\lambda)} \mathscr{F}_{\mathrm{NLL}}(\lambda), \qquad \lambda = \alpha_s(Q)\beta_0 \ln(1/v) \tag{2.2}$$

The exponential function is a Sudakov form factor, containing contributions arising from virtual corrections and unresolved real emissions. The $g_1(\lambda)$ function resums all the leading logarithms, and the $g_2(\lambda)$ next-to-leading logarithms. The $\mathscr{F}_{\mathrm{NLL}}$ function contains NLL contributions from real resolved emissions which are soft and collinear, widely separated in rapidity, and emitted independently from one another. All subsequent gluon-splitting is treated as inclusive.

At NNLL accuracy, the kinematic constraints on a single real emission are relaxed, one by one. A single emission is allowed to roam to the edges of (previously forbidden) phase space; to either the hard-collinear or soft-large angle regions. The non-inclusive gluon splitting of one emission is also allowed. It is important that every other emission satisfies the stronger NLL constraints in order to neglect subleading effects. Each of these cases leads to a correction $\delta\mathscr{F}_i$, which can be considered as amendments to the multiple emissions function $\mathscr{F}_{\rm NLL}$. Each kinematic contribution is evaluated by a Monte Carlo. The Monte Carlo also subtracts the value of the resummation in the limit that the NLL kinematics are restored, to ensure the result is purely an NNLL one. The Monte Carlo code, Automated Resummer of Event Shapes (ARES) will be made publicly available soon. To achieve the NNLL result we must also include the next term in the Sudakov factor; $g_3(\lambda)$. We obtain the expression for g_3 from previously calculated resummations of the thrust [3],[4] and k_t [5] in the literature. All aspects of the cancellation of real and virtual IR divergences are contained in the Sudakov factor.

The resummed cumulative distribution at NNLL is

$$\Sigma(v) = \frac{1}{\sigma}\int_0^v dv' \frac{d\sigma(v')}{dv'} = e^{Lg_1(\lambda)+g_2(\lambda)+\frac{\alpha_s(Q)}{\pi}g_3(\lambda)}\left[\mathscr{F}_{\rm NLL}(\lambda)+\frac{\alpha_s(Q)}{\pi}\delta\mathscr{F}_{\rm NNLL}(\lambda)\right] \tag{2.3}$$

3. Comparison to Fixed Order

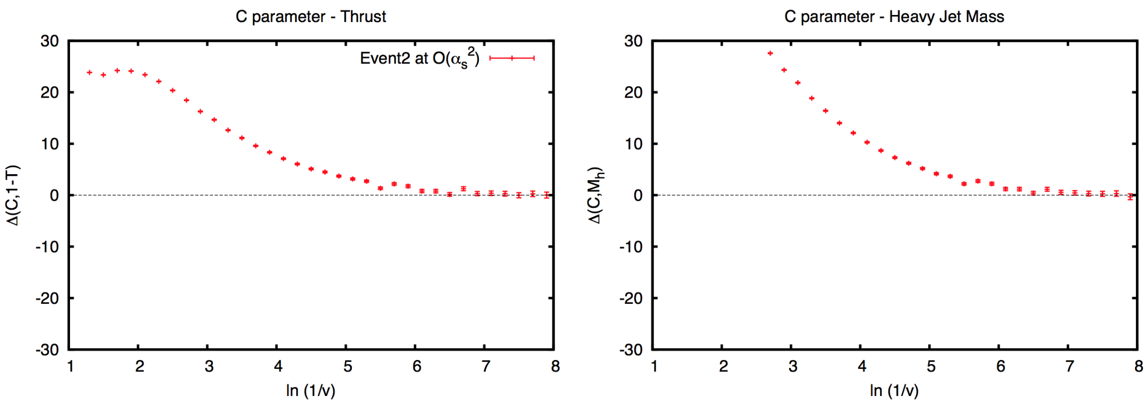

Figure 1: In the region $v \to 0$ the distribution is dominated by the large logs which require resumming, so the difference between the expansion of our results and the full result must go to zero as $\ln(1/v) \to$ large positive values.

There is full agreement to $\mathscr{O}(\alpha_s^3)$ between our results and observables for which an analytic resummation already existed (thrust [3], total and wide broadenings[6], heavy jet mass[7]). To check our new results (oblateness, thrust-major, C-parameter[1]), we compare our resummation to fixed-order by expanding it in powers of α_s. Figure (1) shows the difference between the fixed-order generator `Event2` [8] and the expansion of our resummation to second order in α_s. To

[1] an analytic resummation for the C-parameter appeared whilst the work in [9] was being finalised. [10] We find full agreement between this result and our resummation. We also find full agreement with the numerical resummation of the C-parameter in [11].

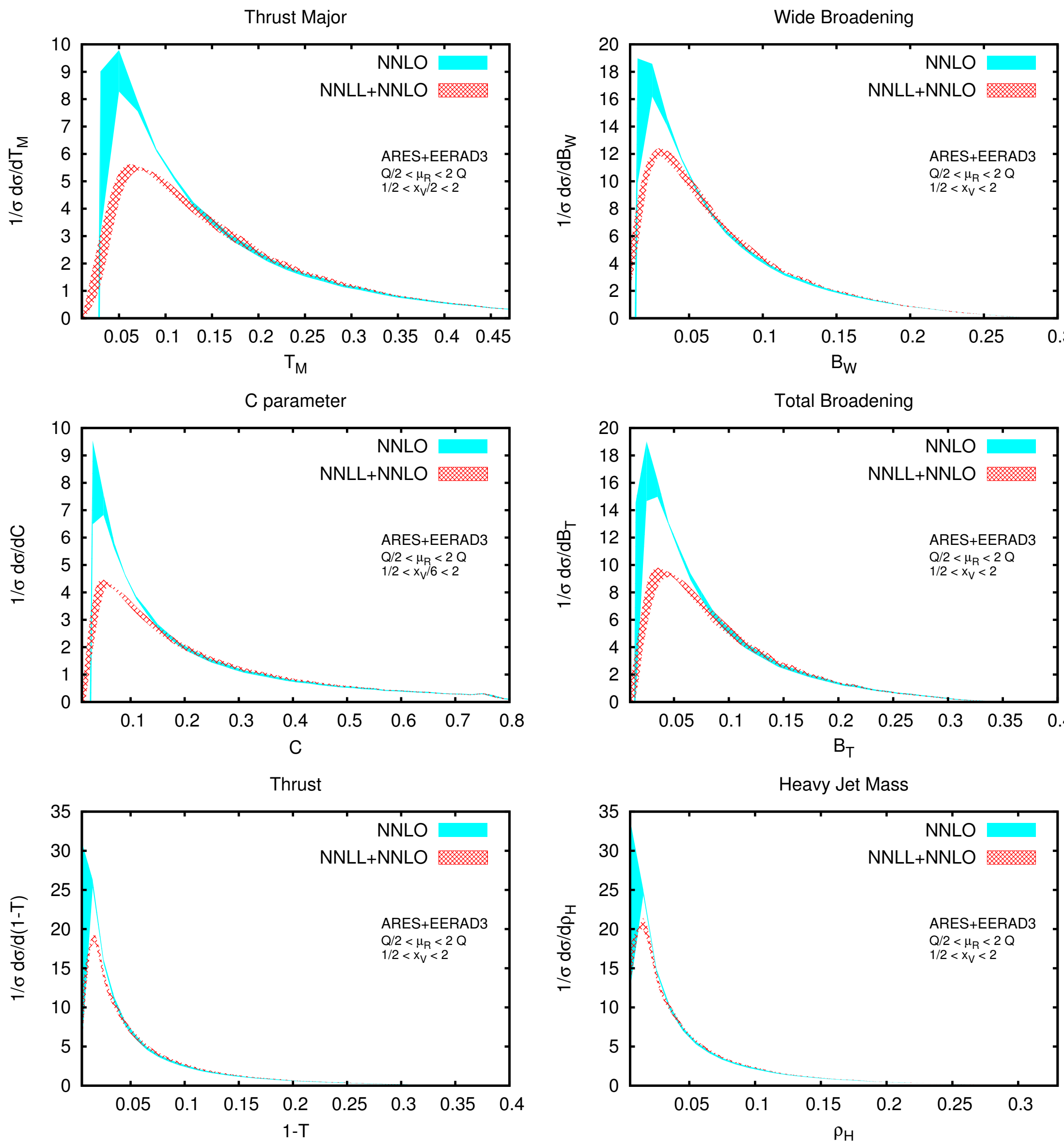

Figure 2: Comparison of the pure fixed order to $\mathcal{O}(\alpha_s^2)$ and the same fixed order plus our resummation, expanded to $\mathcal{O}(\alpha_s^2)$. The convergence of the perturbative prediction is restored in the dijet region.

obtain more stable distributions we plot the *difference* of observables with the same soft-collinear scaling,

$$\Delta(v_1, v_2) = \left(\frac{1}{\sigma_0} \frac{d\sigma^{\text{NLO}}}{d\ln\frac{1}{v_1}} - \frac{1}{\sigma_0} \frac{d\sigma^{\text{NNLL}}|_{\text{expanded}}}{d\ln\frac{1}{v_1}} \right) - \{v_1 \to v_2\}\,. \tag{3.1}$$

As expected, this difference goes to zero in the small-v region where the resummation is necessary to correctly capture the behaviour of the cross-section. This same trend can be seen for all of the observables we have resummed [9]. Figure (2) shows the effect of the resummation on the standard fixed order series in α_s, using the generator EERAD3 [12]. In the small-v region the fixed-order result tends to diverge, and this is tamed by the addition of the resummation. In the large-v region, the combined resummation and fixed-order band smoothly approaches the purely fixed order result, as the resummation becomes unimportant.

We have developed a novel and general framework to calculate the resummed cross-section of observables to NNLL order, utilising the code to resum event-shape variables in $e^+e^- \to 2$jets. We are currently working to extend this framework to include additional observables such as jet rates, and to include further processes such as hadron-hadron collisions.

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