

AdS/QCD predictions for semileptonic and rare B decays to ρ and K^* vector mesons

Mohammad Ahmady*

Department of Physics, Mount Allison University, Sackville, New Brunswick, Canada E4L 1E6

E-mail: mahmady@mta.ca

Sébastien Lord

Département de Mathématiques et Statistique, Université de Moncton, Moncton, N-B. E1A 3E9, Canada

E-mail: es18420@umoncton.ca

Ruben Sandapen

Department of Physics, Mount Allison University, Sackville, New Brunswick, Canada E4L 1E6

E-mail: rsandapen@mta.ca

The light front wavefunction obtained from anti-de Sitter Quantum Chromodynamics is used to calculate the light cone distribution amplitudes for ρ and K^* vector mesons. These distribution amplitudes are then utilized to calculate the $B \rightarrow \rho, K^*$ transition form factors via light cone sum rules. Two-parameter fits of our results for low to intermediate momentum transfer q^2 that includes the QCD lattice data at high q^2 are presented. Consequently, we give predictions for the semileptonic $B \rightarrow \rho \ell \nu$ and dileptonic $B \rightarrow K^* \mu^+ \mu^-$ decays.

XXIII International Workshop on Deep-Inelastic Scattering

27 April - May 1 2015

Dallas, Texas

*Speaker.

1. Introduction

Rare B meson decays have been at the focus of a significant number of theoretical and experimental investigations. The underlying quark processes for these decays are suppressed either due to small Cabbibo-Kobayashi-Mashawa (CKM) quark mixing or lack of flavor changing neutral current (FCNC) at tree level in standard model (SM). A better theoretical understanding of the former decays can help with our knowledge of the least known CKM matrix elements like V_{ub} . On the other hand, since the latter processes occur only through quantum loops, they are sensitive to new unknown particles that are predicted by various new physics (NP) scenarios beyond the SM. These so-called exotic particles can appear as virtual entries in the loops and alter the decay rate and other associated observables from what is expected within the SM. This indirect search for NP in rare B meson decays has been used to constrain various scenarios like supersymmetry and vector-like quark model.

Here, we report a number of predictions on semileptonic and rare dileptonic B decays to light vector mesons based on anti-de Sitter Quantum Chromodynamics (AdS/QCD)[4, 5, 6]. The holographic light-front wavefunctions obtained from AdS/QCD are used to calculate the distribution amplitudes (DAs) of the ρ and K^* vector mesons. These DAs are then inserted in light-cone sum rules (LCSR) formulas for $B \rightarrow \rho, K^*$ transition form factors. Our results contain predictions for a number of observables related to $B \rightarrow \rho \ell \nu$ and $B \rightarrow K^* \mu^+ \mu^-$.

2. Effective hamiltonian

The effective Hamiltonian for $b \rightarrow u \ell \nu, b \rightarrow s \gamma$ and $b \rightarrow s \ell \bar{\ell}$ transitions (Replace s with d for $b \rightarrow d(\gamma, \ell \bar{\ell})$ transition) can be written as:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \left\{ V_{ps}^* V_{pb} \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\dots,10} C_i Q_i \right] + V_{pb} Q_0^p \right\} \quad (2.1)$$

where the operators $Q_i, i = 0..10$ are defined as the following:

$$\begin{aligned} Q_0^p &= (\bar{\nu} \ell)_{V-A} (\bar{p} b)_{V-A} \\ Q_1^p &= (\bar{s} p)_{V-A} (\bar{p} b)_{V-A} & Q_2^p &= (\bar{s}_i p_j)_{V-A} (\bar{p}_j b_i)_{V-A} \\ Q_3 &= (\bar{s} b)_{V-A} \sum_q (\bar{q} q)_{V-A} & Q_4 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A} \\ Q_5 &= (\bar{s} b)_{V-A} \sum_q (\bar{q} q)_{V+A} & Q_6 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A} \\ Q_7 &= \frac{e}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) b_i F_{\mu\nu} & Q_8 &= \frac{g}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) T_{ij}^a b_j G_{\mu\nu}^a \\ Q_9 &= \frac{e^2}{8\pi^2} \bar{s} \gamma^\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu \ell & Q_{10} &= \frac{e^2}{8\pi^2} \bar{s} \gamma^\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu \gamma_5 \ell \end{aligned} \quad (2.2)$$

Q_0^p is relevant to semileptonic decay and $Q_i, i = 1..10$ appear in rare radiative and dileptonic decays. The effective Hamiltonian 2.1 is obtained by integrating out the particles much heavier than the B meson mass, i.e. W and Z bosons as well as the top quark. The Wilson coefficients C_i are evaluated perturbatively at next-to-leading order (NLO) and their numerical values are shown in Table 1.

C_1	C_2	C_3	C_4	C_5	C_6	C_7^{eff}	C_8^{eff}	C_9	C_{10}
-0.148	1.060	0.012	-0.035	0.010	-0.039	-0.307	-0.169	4.238	-4.641

Table 1: Tabulated Wilson coefficients.

The nonperturbative QCD effects appear in the $B \rightarrow V$, $V = \rho, K^*$ transition matrix elements which are parametrized in terms of a number of form factors. Understanding these form factors is crucial in theoretical calculation of the exclusive B meson decays to light mesons.

3. $B \rightarrow \rho, K^*$ transition form factors

From the structure of the operators in 2.2 we observe that the transition matrix element is either of $V - A$ or tensor forms. These two forms can be parametrized in terms of 7 form factors as the following:

$$\begin{aligned}
 \langle V(k, \varepsilon) | \bar{q} \gamma^\mu (1 - \gamma^5) b | B(p) \rangle &= \frac{2iV(q^2)}{m_B + m_{K^*}} \varepsilon^{\mu\nu\rho\sigma} \varepsilon_\nu^* k_\rho p_\sigma - 2m_{K^*} A_0(q^2) \frac{\varepsilon^* \cdot q}{q^2} q^\mu \\
 &- (m_B + m_{K^*}) A_1(q^2) \left(\varepsilon^{\mu*} - \frac{\varepsilon^* \cdot q q^\mu}{q^2} \right) \\
 &+ A_2(q^2) \frac{\varepsilon^* \cdot q}{m_B + m_{K^*}} \left[(p+k)^\mu - \frac{m_B^2 - m_{K^*}^2}{q^2} q^\mu \right] \\
 q_\nu \langle V(k, \varepsilon) | \bar{q} \sigma^{\mu\nu} (1 - \gamma^5) b | B(p) \rangle &= 2T_1(q^2) \varepsilon^{\mu\nu\rho\sigma} \varepsilon_\nu^* p_\rho k_\sigma \\
 &- iT_2(q^2) [(\varepsilon^* \cdot q)(p+k)_\mu - \varepsilon_\mu^* (m_B^2 - m_{K^*}^2)] \\
 &- iT_3(q^2) (\varepsilon^* \cdot q) \left[\frac{q^2}{m_B^2 - m_{K^*}^2} (p+k)_\mu - q_\mu \right] \quad (3.1)
 \end{aligned}$$

We use LCSR[1, 2] to calculate the seven form factors in terms of the DAs of the light vector mesons. LCSR are variations of the traditional QCD Sum Rules whereby non-local matrix elements are expanded in terms of light front DAs. For example, the LCSR for the radiative form factor T_1 is given below:

$$\begin{aligned}
 T_1(q^2) &= \frac{1}{4} \left(\frac{m_b}{f_B m_B^2} \right) \exp\left(\frac{m_B^2}{M^2}\right) \int_\delta^1 \frac{du}{u} \exp\left(-\frac{m_b^2 + p^2 u \bar{u} - q^2 \bar{u}}{uM^2}\right) \left\{ m_b f_V^\perp \phi_\perp(u) + \right. \\
 &\left. f_V m_V \left[\Phi_\parallel(u) + u g_\perp^{(v)}(u) + \frac{g_\perp^{(a)}(u)}{4} + \frac{(m_b^2 + q^2 - p^2 u^2) g_\perp^{(a)}(u)}{4uM^2} \right] \right\} \quad (3.2)
 \end{aligned}$$

In Eqn. 3.2, M is the Borel parameter and δ is associated with the continuum threshold [1]. Using light-cone coordinates, $x^\pm = x^0 \pm x^3$, $x^\perp = x^1, x^2$, the two twist-2 DAs $\phi_{\perp, \parallel}$ along with the two twist-3 DAs $g_\perp^{(v,a)}$ are defined through the following relation[3]:

$$\begin{aligned}
 \langle 0 | \bar{q}(0) \gamma^\mu q(x^-) | V(P, \lambda) \rangle &= f_V M_V \frac{e_\lambda \cdot x}{P^+ x^-} P^\mu \int_0^1 du e^{-iuP^+ x^-} \phi_\parallel(u, \mu) \\
 &+ f_V M_V \left(e_\lambda^\mu - P^\mu \frac{e_\lambda \cdot x}{P^+ x^-} \right) \int_0^1 du e^{-iuP^+ x^-} g_\perp^{(v)}(u, \mu), \quad (3.3)
 \end{aligned}$$

$$\langle 0|\bar{q}(0)[\gamma^\mu, \gamma^\nu]q(x^-)|V(P, \lambda)\rangle = 2f_V^\perp (e_\lambda^\mu P^\nu - e_\lambda^\nu P^\mu) \int_0^1 du e^{-iuP^+x^-} \phi_\perp(u, \mu), \quad (3.4)$$

$$\langle 0|\bar{q}(0)\gamma^\mu\gamma^5s(x^-)|V(P, \lambda)\rangle = -\frac{1}{4}\varepsilon_{\nu\rho\sigma}^\mu e_\lambda^\nu P^\rho x^\sigma \tilde{f}_V M_V \int_0^1 du e^{-iuP^+x^-} g_\perp(a)(u, \mu), \quad (3.5)$$

where

$$\tilde{f}_\rho = f_\rho, \quad (3.6)$$

and

$$\tilde{f}_{K^*} = f_{K^*} - f_{K^*}^\perp \left(\frac{m_s + m_{\bar{q}}}{M_{K^*}} \right). \quad (3.7)$$

m_s and $m_{\bar{q}}$ are the masses of the strange quark and light anti-quark, respectively. As $x^- \rightarrow 0$, in Eqns 3.3 and 3.4 we recover the usual definition for the decay constant f_V and f_V^\perp :

$$\langle 0|\bar{q}(0)\gamma^\mu q(0)|V(P, \varepsilon)\rangle = f_V M_V \varepsilon^\mu, \quad (3.8)$$

$$\langle 0|\bar{q}(0)[\gamma^\mu, \gamma^\nu]q(0)|V(P, \varepsilon)\rangle = 2f_V^\perp (\varepsilon^\mu P^\nu - \varepsilon^\nu P^\mu). \quad (3.9)$$

Also, all the above-defined DAs are normalized, i.e.

$$\int_0^1 du \phi_\rho^{\perp, \parallel}(u, \mu) = \int_0^1 du g_\rho^{\perp(a, v)}(u, \mu) = 1$$

The DAs which are commonly used in the literature are derived from QCD sum rules (SR). In the next section, we put forward alternative DAs which are obtained from AdS/QCD.

4. AdS/QCD predictions for ρ and K^* DAs

The holographic light-front wavefunction for a vector meson ($L=0, S=1$) in AdS/QCD can be written as[7]:

$$\phi_\lambda(z, \zeta) \propto \sqrt{z(1-z)} \exp\left(-\frac{\kappa^2 \zeta^2}{2}\right) \times \exp\left\{-\left[\frac{m_q^2 - z(m_q^2 - m_{\bar{q}}^2)}{2\kappa^2 z(1-z)}\right]\right\} \quad (4.1)$$

with $\kappa = M_V/\sqrt{2}$ and $m_q = m_{\bar{q}}, m_s$ for ρ and K^* , respectively. λ is the polarization of the vector meson. We have introduced the dependence on quark masses following a prescription by Brodsky and de Téramond [8]. This wavefunction for the ρ meson was successfully used to predict the diffractive ρ meson electroproduction at HERA[9].

The relation between the wavefunction 4.1 and DAs for ρ and K^* was derived in [10, 11]. Figures 1 and 2 compare the two twist-2 DAs for these two vector mesons obtained from AdS/QCD and SR.

Consequently, the AdS/QCD predictions for $B \rightarrow \rho, K^*$ form factors can be computed via LCSR. One should note that LCSR results are valid at low to intermediate values of the momentum transfer q^2 . Figure 3 illustrates the AdS/QCD predictions for two $B \rightarrow \rho$ form factors V and T_1 when 3 different values of the quark mass are used. Our results for the all 7 form factors for this transition can be found in [4]. The data points on this figure are from lattice calculation which are

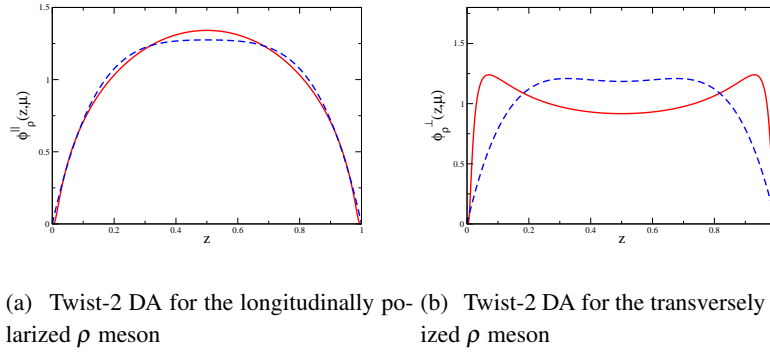


Figure 1: Twist-2 DAs for the ρ meson. Solid: AdS/QCD DAs; Dashed: Sum Rules DAs.

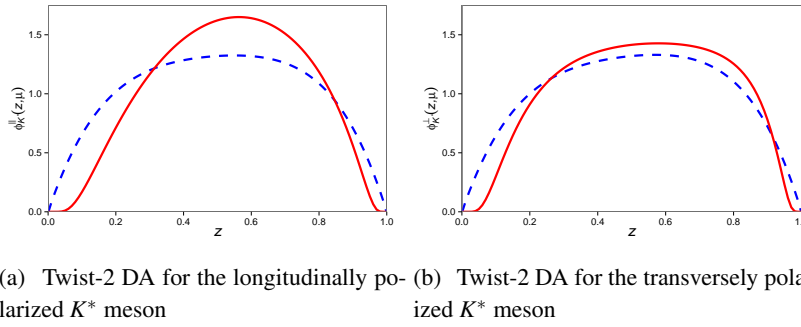


Figure 2: Twist-2 DAs for the K^* meson. Solid: AdS/QCD DAs; Dashed: Sum Rules DAs.

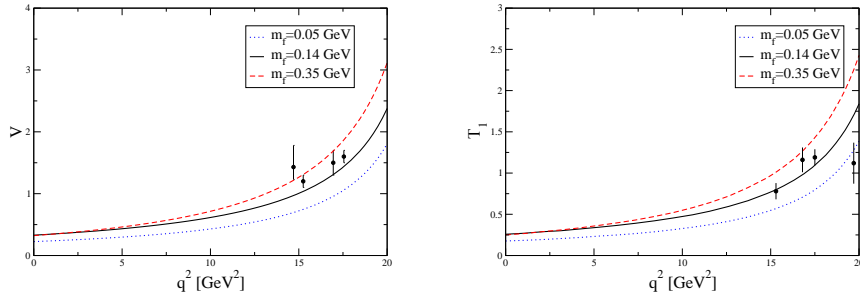


Figure 3: The AdS/QCD prediction for $B \rightarrow \rho$ transition form factors V and T_1 for 3 different quark mass inputs. The available lattice data at high q^2 are shown as well.

available at high q^2 [12]. For our numerical calculations of the semileptonic $B \rightarrow \rho \ell \nu$ decay, we find two-parameter fits for the form factors using AdS/QCD predictions at low to intermediate q^2 and lattice data at high q^2 [4].

Figure 4 shows the AdS/QCD prediction for $B \rightarrow K^*$ form factors V and T_1 as compared with those obtained from SR. The data points at high q^2 are from lattice calculations[13]. Our results for the full set of $B \rightarrow K^*$ transition form factors can be found in [5]. Similar to $B \rightarrow \rho$ case, we use two-parameter fits to AdS/QCD and lattice data combined for the form factors inserted in our

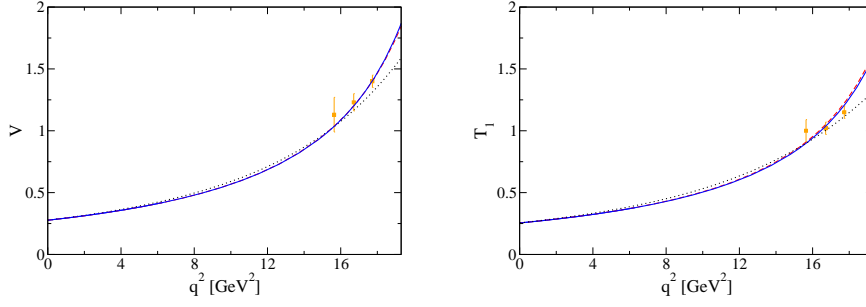


Figure 4: The AdS/QCD prediction for $B \rightarrow K^*$ transition form factors V and T_1 . The solid curve denotes AdS/QCD. The dashed curve denotes the AdS/QCD fit. The dotted curve denotes the fit to AdS/QCD and lattice. The available lattice data at high q^2 are shown as well.

numerical computations.

5. Numerical predictions

The differential decay rate of the semileptonic $B \rightarrow \rho \ell \nu$ is sensitive to V_{ub} . BaBar collaboration has measured the partial branching fractions for this decay channel in three q^2 bins[14]: $\Delta B_{\text{low}} = (0.564 \pm 0.166) \times 10^{-4}$, $\Delta B_{\text{mid}} = (0.912 \pm 0.147) \times 10^{-4}$ and $\Delta B_{\text{high}} = (0.268 \pm 0.062) \times 10^{-4}$ for $0 < q^2 < 8$, $8 < q^2 < 16$ and $16 < q^2 < 20.3 \text{ GeV}^2$ respectively. To eliminate the uncertainty in V_{ub} in comparing our results with the above data, we take the ratios of the partial branching fractions as defined below:

$$R_{\text{low}} = \frac{\Delta B_{\text{low}}}{\Delta B_{\text{mid}}} = 0.618 \pm 0.207, \quad R_{\text{high}} = \frac{\Delta B_{\text{high}}}{\Delta B_{\text{mid}}} = 0.294 \pm 0.083 \quad (5.1)$$

AdS/QCD predictions for these ratios are $R_{\text{low}} = 0.580, 0.424$ and $R_{\text{high}} = 0.427, 0.503$ for $m_q = 0.14 \text{ GeV}$ and 0.35 GeV , respectively. It seems that better agreement with data is achieved at low q^2 .

Our results for the differential decay rate and isospin asymmetry distribution in $B \rightarrow K^* \mu^+ \mu^-$ is shown in Figure 5. Data points for the decay rate are from LHCb[15] and the predictions are given with (solid curve) and without (dashed curve) considering lattice data in two-point fits of the form factors[5]. We also present our prediction for this observable assuming NP contribution to the Wilson coefficient C_9 (dash-dotted curve). The AdS/QCD prediction for isospin asymmetry distribution in $B \rightarrow K^* \mu^+ \mu^-$ in Figure 5 is presented (solid curve) with an uncertainty band due to the renormalization scale dependence. The data points are from Belle and BABAR[16]. For comparison, we also include the SR predictions (dashed curve) for isospin asymmetry distribution. Our prediction for asymmetry at $q^2 = 0$ (relevant to $B \rightarrow K^* \gamma$) is consistent with experimental data.

References

- [1] A. Ali, V. M. Braun, and H. Simma, Z. Phys. C 63, 437 (1994).
- [2] P. Ball and V. M. Braun, Phys. Rev. D 55, 5561 (1997); P. Ball and V. M. Braun, Phys. Rev. D 58, 094016 (1998); P. Ball and R. Zwicky, Phys. Rev. D 71, 014029 (2005); T. Aliev, A. Ozpineci, and M. Savci, Phys. Rev. D 56, 4260 (1997).

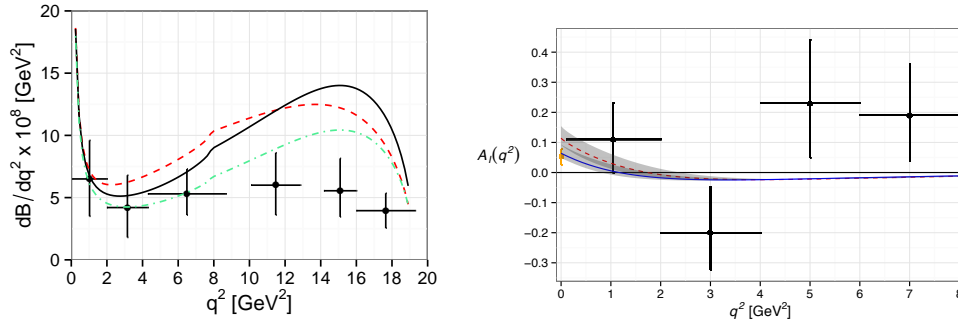


Figure 5: The AdS/QCD prediction for $B \rightarrow K^* \mu^+ \mu^-$ differential decay width and isospin asymmetry distribution. Differential decay rates are predicted with (solid curve) and without (dashed curve) lattice data input in two-parameter fits of the form factors as well as $C_9^{\text{NP}} = -1.5$ (dash-dotted curve). In the Isospin asymmetry graph, AdS/QCD prediction (solid curve) is presented with an uncertainty band and compared with SR expectation (dashed curve).

- [3] P. Ball, V. M. Braun, and A. Lenz, *J. High Energy Phys.* 08, 090 (2007).
- [4] M. Ahmady, R. Campbell, R. Sandapen, and S. Lord, *Phys. Rev. D* 88 074031 (2013).
- [5] M. Ahmady, R. Campbell, S. Lord, and R. Sandapen, *Phys. Rev. D* 89 014042 (2014).
- [6] M. Ahmady, S. Lord, and R. Sandapen, *Phys. Rev. D* 90 074010 (2014).
- [7] G. F. de Téramond and S. J. Brodsky, *Phys. Rev. Lett.* 102, 081601 (2009).
- [8] S. J. Brodsky and G. F. de Téramond, *AIP Conf. Proc.* 1116, 311 (2009).
- [9] J. R. Forshaw and R. Sandapen, *Phys. Rev. Lett.* 109, 081601 (2012).
- [10] M. Ahmady and R. Sandapen, *Phys. Rev. D* 88 054013 (2013).
- [11] M. Ahmady and R. Sandapen, *Phys. Rev. D* 88 014042 (2013).
- [12] D. R. Burford, H. D. Duong, J. M. Flynn, B. J. Gough, N. M. Hazel, J. Nieves, and H. P. Shanahan (UKQCD Collaboration), *Nucl. Phys.* 447 425 (1995); J. M. Flynn, arXiv:hep-lat/9611016; L. Del Debbio, J. M. Flynn, L. Lellouch, and J. Nieves (UKQCD Collaboration), *Phys. Lett. B* 416 392 (1998).
- [13] R. R. Horgan, Z. Liu, S. Meinel, and M. Wingate, *Phys. Rev. D* 89 094501 (2014).
- [14] P. del Amo Sanchez et al. (BABAR Collaboration), *Phys. Rev. D* 83 032007 (2011).
- [15] T. Blake and N. Serra (LHCb Collaboration) Report No. CERN-LHCb-CONF-2012-008; R. Aaij et al. (LHCb Collaboration), *J. High Energy Phys.* 07 133 (2012).
- [16] M. Nakao et al. (BELLE Collaboration), *Phys. Rev. D* 69 112001 (2004); B. Aubert et al. (BABAR Collaboration), *Phys. Rev. Lett.* 103 211802 (2009).