Challenges in the extraction of TMDs from SIDIS data: perturbative vs non-perturbative aspects

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We present our recent results on the study of the Semi-Inclusive Deep Inelastic Scattering (SIDIS) cross section as a function of the transverse momentum, $q_T$. Using the Collins-Soper-Sterman (CSS) formalism, we study the matching between the region where fixed order perturbative QCD can successfully be applied and the region where soft gluon resummation is necessary. We find that the commonly used prescription of matching through the so-called Y-factor cannot be applied in the SIDIS kinematical configurations we examine. We comment on the impact that the non-perturbative component has even at relatively high energies.
1. Introduction

Collinear perturbative QCD computations allow us to predict the behaviour of the cross section of a hadronic process at high resolution scale $Q$, in the large $q_T \gtrsim Q$ region. On the other hand, in the low $q_T$ region one must resum the large (double) logarithmic contributions generated by the emission of soft and collinear gluons.

This can be achieved by applying a soft gluon resummation scheme like, for instance, the Collins-Soper-Sterman (CSS) scheme \cite{1}, which was originally formulated and extensively tested for Drell-Yan (DY) process, $h_1h_2 \rightarrow \ell^+\ell^-X$ \cite{1, 2, 3, 4, 5}. In the case of Semi-Inclusive Deep Inelastic Scattering (SIDIS) process, $\ell N \rightarrow \ell hX$, resummation was studied in Refs. \cite{6, 7, 8}. In the CSS formalism, the resummation is performed in the badly divergent (asymptotic) part of the perturbative cross section $\sigma^{ASY}$, which is separated from the regular part (i.e. less singular than $1/q_T^2$) commonly known as the $Y$-term. In the resummed part, some model-dependence has to be introduced to parametrize the non-perturbative component of the cross section. This model dependence enters in a non-trivial way, since the CSS resummation is done in Fourier space.

A successful resummation scheme should take care of matching the fixed order hadronic cross section, computed in perturbative QCD at large $q_T$, with the resummed cross section, valid at low $q_T \ll Q$, where large logarithms are properly treated. This matching should happen, roughly, at $q_T \sim Q$ where logarithms are small \cite{1}, and is very often realized through the $Y$-term, which should ensure a continuous and smooth matching of the cross section over the entire $q_T$ range.

In this summary we will describe some specific matching procedures, discuss the delicate interplay between the perturbative and non-perturbative parts of the hadronic cross section and give numerical examples, exploring different kinematical configurations of SIDIS experiments.

2. Resummation in Semi-Inclusive Deep Inelastic Scattering

The starting point for the CSS scheme is the separation of the badly divergent part of the pQCD calculation of the cross section, $d\sigma^{ASY}$, from the regular part, the so called $Y$-term. Starting from the Next-to-Leading order SIDIS cross section one has

$$\frac{d\sigma^{NLO}}{dx dy dz dq_T^2} = \frac{d\sigma^{ASY}}{dx dy dz dq_T^2} + Y.$$  

(2.1)

Then, one performs the resummation in the asymptotic term alone. In unpolarized SIDIS processes, $\ell N \rightarrow \ell hX$, the following CSS expression \cite{6, 7} holds

$$\frac{d\sigma^{total}}{dx dy dz dq_T^2} = \pi\sigma_0^{DIS} \int \frac{d^2b_T e^{i\mathbf{q}_T \cdot \mathbf{b}_T}}{(2\pi)^2} W_{SIDIS}(x, z, b_T, Q) + Y_{SIDIS}(x, z, q_T, Q),$$  

(2.2)

where $q_T$ is the virtual photon momentum in the frame where the incident nucleon $N$ and the produced hadron $h$ are head to head, and

$$\sigma_0^{DIS} = \frac{4\pi \alpha_{em}^2}{s x y^2} \left(1 - \frac{y^2}{2}\right).$$  

(2.3)
In the CSS resummation scheme, the term $W^{\text{SIDIS}}(x, z, b_T, Q)$ in Eq. (2.2) resums the soft gluon contributions, large when $q_T \ll Q$:

$$W^{\text{SIDIS}}(x, z, b_T, Q) = \exp[S_{\text{pert}}(b_T, Q)] \sum_j e_j^2 \sum_{i,k} c_{ji}^{\text{in}} \otimes f_i(x, \mu_b^2) c_{kj}^{\text{out}} \otimes D_k(z, \mu_b^2),$$ (2.4)

where $j = q, \bar{q}$ runs over all quark flavors available in the process, $i, k = q, \bar{q}, g$, and

$$S_{\text{pert}}(b_T, Q) = - \int \frac{\text{d} \mu^2}{\mu^2} \left[ A(\alpha_s(\mu)) \ln \left( \frac{Q^2}{\mu^2} \right) + B(\alpha_s(\mu)) \right]$$ (2.5)

is the perturbative Sudakov form factor. In Eq. (2.4), the Wilson coefficients $c_{ji}^{\text{in}}, c_{kj}^{\text{out}}$ are convoluted with the collinear distribution and fragmentation functions, evaluated at the intermediate scale $\mu_b(b_T) = C_1/b_T$. These Wilson coefficients, as well as $A$ and $B$ in Eq. (2.5), are functions that can be expanded in series of $\alpha_s$. For our studies, we use Next-to-Leading Log (NLL) accuracy (for more details, see for instance [1, 9, 4, 7]).

The CSS formalism relies on a Fourier integral (2.2) over $b_T$ which runs from zero to infinity. At very large values of $b_T$, both the Sudakov form factor ($S$) and the collinear functions $f$ and $D$ in (2.4) involve the evaluation of $\alpha_s$ at low scales. In order to avoid this, one must introduce a prescription to "freeze" $b_T$. This can be achieved by making the replacement $\mu_b(b_T) \rightarrow \mu_b(b_*)$ in Eq (2.4), where

$$b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}}.$$ (2.6)

With this replacement, one must also introduce the model-dependent function $S_{NP}$, in order to parametrize the non-perturbative part "avoided" by the $b_*$-prescription. Then, one can write the SIDIS cross section as

$$\frac{d\sigma^{\text{total}}}{dxdydzdq_T^2} = \pi \sigma_0^{\text{DIS}} \int_0^\infty \frac{db_Tq_T}{(2\pi)} J_0(q_Tb_T) W^{\text{SIDIS}}(x, z, b_*, Q) \exp[S_{NP}(x, z, b_T, Q)] + Y(x, z, q_T, Q),$$ (2.7)

where the angular integral in the Fourier transform has been performed. The predictive power of the $b_T$-space resummation formalism is limited by our inability to calculate the non-perturbative distributions at large $b_T$. However, most of these non-perturbative distributions are believed to be universal and can be extracted from experimental data on different processes.

3. Results

Before testing matching prescriptions, we underline the main idea behind Y-term matching. Considering first the expression in Eq. (2.2) the cross section can be written in a short-hand notation as

$$d\sigma^{\text{total}} = W + Y = W + (d\sigma^{\text{NLO}} - d\sigma^{\text{ASY}}).$$ (3.1)
In the region where $q_T \simeq Q$, the resummed cross section $W$ is expected to be very similar to its asymptotic counterpart, $d\sigma^{ASY}$. Therefore, the cross section in Eq. (3.1) should almost exactly match the NLO cross section, $d\sigma^{NLO}$:

$$d\sigma_{total} = W + Y_{q_T \sim Q} d\sigma^{ASY} + Y = d\sigma^{ASY} + d\sigma^{NLO} - d\sigma^{ASY} = d\sigma^{NLO}.$$  

This matching prescription at $q_T \simeq Q$ only works if $W \simeq d\sigma^{ASY}$ over a non-negligible range of $q_T$ values. Of course, in order to write down the cross section, one must include its non-perturbative component. Then one should ask the question of what impact this model-dependent part has in the values. Of course, in order to write down the cross section, one must include its non-perturbative calculation, and ultimately in the matching. Model dependence enters through the function $S_{NP}$, model successfully used in DY processes, namely

$$S_{NP} = \left(-\frac{g_1}{2} - \frac{g_{1f}}{2z^2} - g_2 \ln \left(\frac{Q}{Q_0}\right)\right) b_T^2.$$  

Fig. 1 shows the impact on the calculation of the resummed cross section, when varying the parameters $g_1$ and $g_{1f}$ in Eq. (3.3). Fig. 2 displays the effect of changing the parameter $b_{max}$ in expression (2.6). In these two cases, it is interesting to note that one can observe a mild model-dependence only in the extreme kinematics (left panels of Figures 1 and 2). Of particular interest is the fact that, in the region of current data (HERMES and COMPASS kinematics), at NLL accuracy the model-dependence prevails even at large values of $q_T$.

![Figure 1: Resummed term of the SIDIS cross section including the non-perturbative contribution $S_{NP}$ in the Sudakov factor, calculated at three different values of $g_1$ and $g_{1f}$ and corresponding to the three different SIDIS kinematical configurations: on the left panel $\sqrt{s} = 1$ TeV, $Q^2 = 5000$ GeV$^2$, $x = 0.055$ and $z = 0.325$; on the central panel a HERA-like experiment with $\sqrt{s} = 300$ GeV, $Q^2 = 100$ GeV$^2$, $x = 0.0049$ and $z = 0.325$; on the right panel, a COMPASS-like experiment with $\sqrt{s} = 17$ GeV, $Q^2 = 10$ GeV$^2$, $x = 0.055$ and $z = 0.325$. Here $b_{max} = 1.0$ GeV$^{-1}$.](image)

This model dependence is, quite likely, one of the reasons why $Y$-term matching is not possible as it is shown in Fig. 3, where it can be seen that the NLL resummed cross section, including model dependence (and now labeled as $W^{NLL}$), largely overshoots the asymptotic cross section $d\sigma^{ASY}$ in the region where they are expected to have a similar size. This means that a cancellation like the one shown in Eq. (3.2) cannot happen and in turn, the full NLL cross section $W^{NLL} + Y$ can never be match with the pQCD calculation $d\sigma^{NLO}$. It is interesting to notice that in the kinematics where
data is available, the Y-term actually has a sizeable contribution to the cross section at low $q_T$. In this region, it is usually expected that the resummed part $W^{NLL}$ accounts for most of the cross section.

As an attempt to account for the model-dependence in the matching, we write the SIDIS cross section as

$$d\sigma^{total} = W^{NLL} - W^{FXO} + d\sigma^{NLO}, \quad (3.4)$$

where $W^{FXO}$ is the NLL resummed cross section approximated at first order in $\alpha_s$, and contains the same non-perturbative function $S_{NP}$ as does $W^{NLL}$ (see [10] for a precise definition). In the absence of the non-perturbative function and under some approximations involving the leading power of $\alpha_s$, it can be shown that $W^{FXO} \to d\sigma^{ASY}$ so that, in this limit [11, 12]

$$d\sigma^{total} = W^{NLL} - W^{FXO} + d\sigma^{NLO} \to W^{NLL} - d\sigma^{ASY} + d\sigma^{NLO} = W^{NLL} + Y. \quad (3.5)$$

In this limit this prescription is equivalent to the Y-term matching prescription of Eq. (3.2). It is therefore reasonable to expect to find a region in which $W^{FXO} \simeq W^{NLL}$, allowing to match the SIDIS cross section $d\sigma = W^{NLL} - W^{FXO} + d\sigma^{NLO}$ to the purely perturbative cross section $d\sigma^{NLO}$. Fig. 4 shows the different terms of the cross section in Eq. (3.4), where now $W^{FXO}$ plays the role that $d\sigma^{ASY}$ did in Eq. (3.1). There, one can see that $W^{FXO}$ has a better behaviour, relative to $W^{NLL}$, than $d\sigma^{ASY}$ did (see Fig. 3). For instance, both $W^{FXO}$ and $W^{NLL}$ become negative at very similar values of $q_T$. Furthermore, one can see that in the cases shown in Fig. 4, there is a region where these two quantities have the same size. Unfortunately, this does not happen anywhere close to the region $q_T \simeq Q$, where one would expect to match to $d\sigma^{NLO}$. Therefore, no smooth and continuous matching can be performed.

Finally, we would like to point at the fact that the perturbative Sudakov factor plays a central role in the behaviour of the resummed cross section. In fact, the reason why our latter attempt to perform the matching failed can be likely attributed to the problem of expanding $S_{pert}$ to a definite accuracy in powers of $\alpha_s$ (see [10] for a more detailed discussion of this point). In general, $S_{pert}$ is a very intricate quantity that should never be overlooked. In order to illustrate this, we compare two common approaches to calculate $S_{pert}$. In the first one, we numerically compute $S_{pert}$ from

Figure 2: The resummed cross section $W^{NLL}(q_T)$ corresponding to the three different SIDIS kinematical configurations defined in Fig. 1. Here $b_{max}$ varies from 1.5 GeV$^{-1}$ to 0.5 GeV$^{-1}$, while $g_1$ and $g_{1f}$ are fixed at $g_1 = 0.3$ GeV$^2$, $g_{1f} = 0.1$ GeV$^2$. 

$$\sqrt{s}=300 \text{ GeV}, Q^2=100 \text{ GeV}^2$$
shows both the standard and modified PoS(DIS2015)196

\[ \sqrt{s} = 1 \text{ TeV}, \, Q^2 = 5000 \text{ GeV}^2 \]

\[ \sqrt{s} = 300 \text{ GeV}, \, Q^2 = 100 \text{ GeV}^2 \]

\[ \sqrt{s} = 17 \text{ GeV}, \, Q^2 = 10 \text{ GeV}^2 \]

Figure 3: \( d\sigma^{\text{NLO}}, d\sigma^{\text{ASY}}, W^{\text{NLL}} \) and the sum \( W^{\text{NLL}} + Y \) (see Eq. (3.2)), corresponding to the three different SIDIS kinematical configurations defined in Fig. 1. Here \( b_{\text{max}} = 1.0 \text{ GeV}^{-1}, \, g_1 = 0.3 \text{ GeV}^2, \, g_{1f} = 0.1 \text{ GeV}^2, \, g_2 = 0 \text{ GeV}^2. \)

Figure 4: \( d\sigma^{\text{NLO}}, W^{\text{NLL}} \) and \( W^{\text{FXO}} \) (see Eq. (3.4)), corresponding to three different SIDIS kinematical configurations. Here \( b_{\text{max}} = 1.0 \text{ GeV}^{-1}, \, g_1 = 0.3 \text{ GeV}^2, \, g_{1f} = 0.1 \text{ GeV}^2, \, g_2 = 0 \text{ GeV}^2. \)

Eq. (2.5). In the second one, we use an analytic expression obtained in Ref. [7], for which the replacement

\[ \log(Q^2/\mu_0^2) \rightarrow \log(1 + Q^2/\mu_0^2), \]  

was made in Eq. (2.5). This replacement leads to a modified, better behaved \( S_{\text{pert}} \) as \( b_T \rightarrow 0 \) [13, 14]. Fig. 5 shows both the standard and modified \( S_{\text{pert}} \). For the extreme kinematics, in the left panel, one can see that it is only in the region of large \( b_T \) where significant differences arise. Large \( b_T \) behaviour is commonly associated to non-perturbative physics, which should be accounted for via \( S_{\text{NP}} \), so at these kinematics, both the standard and modified \( S_{\text{pert}} \) seem equally suitable for calculate the cross section. As seen in the right panel of Fig. 5, this is not the case at low energies (compatible to those of COMPASS and HERMES), or even a moderately high energies (central panel). In both cases, there is a sizeable difference even in the low-\( b_T \) regime. Interestingly enough, at kinematics similar to available data by COMPASS and HERMES, the modified \( S_{\text{pert}} \) would have almost no effect in the calculation of the resummed cross section, i.e. \( \exp(-S_{\text{pert}}) \approx 0. \)

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Figure 5: Sudakov factor as given by Eq. (2.5) (solid line), and its modified form given in Eqs. (44)-(47) of Ref. [7] (dashed line), for three different values of $Q^2$.  

References