# Transverse single-spin asymmetries $A_{U T}^{\sin \phi_{S}}$ and $A_{U T}^{\sin \left(2 \phi_{h}-\phi_{S}\right)}$ in SIDIS 

Wenjuan MAO*i<br>Department of Physics, Southeast University, Nanjing 211189, China<br>E-mail: wjmao@seu.edu.cn<br>Zhun LU<br>Department of Physics, Southeast University, Nanjing 211189, China<br>E-mail: zhunduseu.edu.an

We study the single-spin asymmetries with the $\sin \phi_{S}$ and $\sin \left(2 \phi_{h}-\phi_{S}\right)$ angular dependencies of pion production in semi-inclusive deep inelastic scattering off a transversely polarized proton target. We investigate the role of the distributions $f_{T}, h_{T}$ and $h_{T}^{\perp}$ in the $\sin \phi_{S}$ asymmetry, as well as the role of the distributions $f_{T}^{\perp}, h_{T}$ and $h_{T}^{\perp}$ in the $\sin \left(2 \phi_{h}-\phi_{S}\right)$ asymmetry. We calculate the four twist- 3 distributions $f_{T}\left(x, \boldsymbol{k}_{T}^{2}\right), f_{T}^{\perp}\left(x, \boldsymbol{k}_{T}^{2}\right), h_{T}\left(x, \boldsymbol{k}_{T}^{2}\right)$, and $h_{T}^{\perp}\left(x, \boldsymbol{k}_{T}^{2}\right)$ in a spectator-diquark model including vector diquarks. With the model results on these TMD distributions, for the first time we predict the two corresponding asymmetries for $\pi^{+}, \pi^{-}$and $\pi^{0}$ produced off the proton target at the kinematics of HERMES, JLab, and COMPASS.

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## 1. Introduction

In recent years, single-spin asymmetry (SSA) in semi-inclusive deep inelastic scattering (SIDIS) process opened a new window on the understanding of hadron structure (for reviews see Refs. [][], [] []]). Particularly, transverse spin phenomena of the nucleon in scattering processes have attracted growing attentions, as reflected in numerous studies from both theory and experiment,
 (JLab) Hall A Collaboration [ [

As shown in Refs. [【2] [13], besides the leading-twist SSAs, there are two more angular modulations appearing in the the process $l N^{\uparrow} \rightarrow l^{\prime}+h+X$, that is, the $\sin \phi_{S}$ and the $\sin \left(2 \phi_{h}-\phi_{S}\right)$ moments. Most recently, there is also an attempt [[4] to measure these transverse spin asymmetries. However, there are still less systematic studies and calculations on the $\sin \phi_{S}$ and $\sin \left(2 \phi_{h}-\phi_{S}\right)$ asymmetries in literature, especially from the phenomenological view, although these subleadingtwist transverse spin asymmetries can shed light on the transverse spin structure of the nucleon at twist 3. Therefore, the main purpose of this work is to give an attempt to investigate the roles of TMD twist-3 distributions and fragmentation functions(FFs) in the transverse SSAs, and to study the feasibility of experimental measurements on them. In this paper, we will only focus our attention on the contributions from the twist-3 TMDs, and set these FFs to zero in the Wandzura-Wilczek approximation [15] in the practical calculation.

## 2. Model calculation on the twist-3 TMD distributions

Applying the notation from Ref. [■3] , at twist-3 level, the gauge-invariant quark-quark correlator for a transversely polarized nucleon can be decomposed as

$$
\begin{align*}
\left.\Phi\left(x, \boldsymbol{k}_{T}, \boldsymbol{S}_{T}\right)\right|_{\text {twist-3 }} & =\frac{M}{2 P^{+}}\left\{-\varepsilon_{T}^{\rho \sigma} \gamma_{\rho} S_{T \sigma} f_{T}^{\prime}+\frac{\left(k_{T} \cdot S_{T}\right) \varepsilon_{T}^{\rho \sigma} \gamma_{\rho} k_{T \sigma}}{M^{2}} f_{T}^{\perp}\right. \\
- & \left.\frac{k_{T} \cdot S_{T}}{M} \frac{\left[\not h_{+}, \not h_{-}\right] \gamma_{5}}{2} h_{T}+\frac{\left[\phi_{T}, \not k_{T}\right] \gamma_{5}}{2 M} h_{T}^{\perp}+\cdots\right\} \tag{2.1}
\end{align*}
$$

here $\cdots$ denotes the other twist- 3 TMDs that are not relevant in this work. Using the light-cone coordinates, the four involved twist-3 TMDs $f_{T}, f_{T}^{\perp}, h_{T}$, and $h_{T}^{\perp}$ can be obtained from the Dirac traces of $\Phi\left(x, \boldsymbol{k}_{T}, \boldsymbol{S}_{T}\right)$ (see Eqs. (3.23)(3.25)(3.26) of Ref. [때] ]).

Following the way used in Ref. [46] and choosing the form for polarization sum of the axialvector diquark $d_{\mu v}$ adopted in Ref. [[]]], we performed the calculation in a spectator model with an axial-vector diquark, and gave the detailed calculation in Ref. [[8]]. Here we only give the model results of the four TMD distributions contributed by the scalar diquark

$$
\begin{align*}
h_{T}^{s}\left(x, k_{T}^{2}\right) & =\frac{N_{s}^{2}(1-x)^{2}}{16 \pi^{3}} \frac{\left[(1-x)^{2} M^{2}-k_{T}^{2}-M_{s}^{2}\right]}{\left(k_{T}^{2}+L_{s}^{2}\right)^{4}},  \tag{2.2}\\
h_{T}^{\perp s}\left(x, k_{T}^{2}\right) & =\frac{N_{s}^{2}(1-x)^{2}}{16 \pi^{3}} \frac{\left[(1-x)\left(M^{2}+2 m M+x M^{2}\right)-k_{T}^{2}-M_{s}^{2}\right]}{\left(k_{T}^{2}+L_{s}^{2}\right)^{4}} \tag{2.3}
\end{align*}
$$



Figure 1: Model results for $x f^{u}$ (solid line) and $x f^{d}$ (dashed line) as functions of $x$ at $k_{T}=0.3 \mathrm{GeV}$ and of $k_{T}$ at $x=0.3$.

$$
\begin{align*}
f_{T}^{s}\left(x, k_{T}^{2}\right) & =C_{F} \alpha_{s} \frac{N_{s}^{2}(1-x)^{2}}{32 \pi^{3}} \frac{\left(x+\frac{m}{M}\right)\left(L_{s}^{2}-k_{T}^{2}\right)}{L_{s}^{2}\left(L_{s}^{2}+k_{T}^{2}\right)^{3}},  \tag{2.4}\\
f_{T}^{\perp s}\left(x, k_{T}^{2}\right) & =0 \tag{2.5}
\end{align*}
$$

and by the axial-vector diquark component

$$
\begin{align*}
h_{T}^{v}\left(x, k_{T}^{2}\right) & =\frac{N_{v}^{2}(1-x)}{16 \pi^{3}} \frac{\left[(1-x)\left(m^{2}+2 x m M+x M^{2}\right)+k_{T}^{2}-x M_{v}^{2}\right]}{\left(k_{T}^{2}+L_{v}^{2}\right)^{4}}  \tag{2.6}\\
h_{T}^{\perp v}\left(x, k_{T}^{2}\right) & =\frac{N_{v}^{2}(1-x)}{16 \pi^{3}} \frac{\left[(1-x)\left(m^{2}-x M^{2}\right)-k_{T}^{2}+x M_{v}^{2}\right]}{\left(k_{T}^{2}+L_{v}^{2}\right)^{4}},  \tag{2.7}\\
f_{T}^{v}\left(x, k_{T}^{2}\right) & =0,  \tag{2.8}\\
f_{T}^{\perp v}\left(x, k_{T}^{2}\right) & =C_{F} \alpha_{s} \frac{N_{v}^{2}(1-x)^{2} M(m+x M)}{16 \pi^{3}\left(L_{v}^{2}+k_{T}^{2}\right)^{2} k_{T}^{2}}\left[\frac{1}{k_{T}^{2}} \ln \frac{k_{T}^{2}+L_{v}^{2}}{L_{v}^{2}}+\frac{k_{T}^{2}-L_{v}^{2}}{L_{v}^{2}\left(L_{v}^{2}+k_{T}^{2}\right)}\right] . \tag{2.9}
\end{align*}
$$

We find that the above expressions of $h_{T}^{s}$ and $h_{T}^{\perp s}$ are in consistence with the results in Ref. [10]], while the expressions of $f_{T}^{s}$ and $f_{T}^{\perp s}$ have already been presented in Ref. [20]]. Following the way in Ref. [[6]], we construct the distributions for the $u$ and $d$ valence quarks from $f^{s}$ and $f^{v}$ by the relation

$$
\begin{equation*}
f^{u}=c_{s}^{2} f^{s}+c_{a}^{2} f^{a}, \quad f^{d}=c_{a^{\prime}}^{2} f^{a^{\prime}}, \tag{2.10}
\end{equation*}
$$

where $c_{s}, c_{a}$, and $c_{a^{\prime}}$ are the free parameters of the model, and $a$ and $a^{\prime}$ denote the isoscalar and isovector states of the axial-diquark, respectively.

We also show the dependence of the four distributions on the flavors, Bjorken $x$ and the active quark transverse momentum $k_{T}$ in Fig. D. As we can see, in the specified kinematic region ( $x=0.3$ or $k_{T}=0.3 \mathrm{GeV}$ ), the T-even distributions $h_{T}^{u}$ and $h_{T}^{d}$ have similar sizes but opposite signs, while $h_{T}^{\perp u}$ is positive and its size is much larger than that of $h_{T}^{\perp d}$. For the T-odd distributions $f_{T}$ and $f_{T}^{\perp}$, $f_{T}^{d}$ is zero since $f_{T}^{v}$ vanishes in the model we adopt, and there is a node of the distribution $f_{T}^{u}$ in $k_{T}$. The size of $f_{T}$ is much smaller than that of the T-even distributions $h_{T}$ and $h_{T}^{\perp}$. Specially, we
verify that $f_{T}^{u}$ vanishes when it is integrated over the transverse momentum [2]] $\int d^{2} k_{T} f_{T}^{u}\left(x, k_{T}^{2}\right)=$ 0 . This is an expected result from the time-reversal invariance for integrated distributions, and it indicates that the distribution $f_{T}$ will not give any contribution to the transverse SSA in inclusive DIS process [2], [23]. The results for $f_{T}^{\perp}$ show that $f_{T}^{\perp d}$ dominates over $f_{T}^{\perp u}$ in the chosen kinematic regime. This may be explained by the fact that $f_{T}^{\perp s}$ is zero and only $f_{\bar{T}}^{\perp v}$ contributes in our model.

## 3. Predictions on the transverse SSAs in SIDIS

The differential cross section of the process for an unpolarized beam scattering off a transversely polarized hadron can be expressed as [[13]

$$
\begin{align*}
\frac{d \sigma}{d x d y d z d \phi_{S} d \phi_{h} d P_{T}^{2}} & =\frac{\alpha^{2}}{x y Q^{2}} \frac{y^{2}}{2(1-\varepsilon)}\left(1+\frac{\gamma^{2}}{2 x}\right)\left\{F_{\mathrm{UU}}\right. \\
& +\left|S_{T}\right|\left[\sqrt { 2 \varepsilon ( 1 + \varepsilon ) } \left(\sin \phi_{S} F_{\mathrm{UT}}^{\sin \phi_{S}}\right.\right. \\
& \left.\left.\left.+\sin \left(2 \phi_{h}-\phi_{S}\right) F_{\mathrm{UT}}^{\sin \left(2 \phi_{h}-\phi_{S}\right)}\right)\right]+\cdots\right\} . \tag{3.1}
\end{align*}
$$

Based on the tree-level factorization adopted in Ref. [[1]], the structure functions $F_{\mathrm{UU}}, F_{\mathrm{UT}}^{\sin \phi_{s}}$ and $F_{\mathrm{UT}}^{\sin \left(2 \phi_{h}-\phi_{s}\right)}$ can be expressed as the convolutions of twist-2 and twist-3 TMD distributions and FFs:

$$
\begin{align*}
F_{\mathrm{UU}} & =\mathscr{C}\left[f_{1} D_{1}\right],  \tag{3.2}\\
F_{\mathrm{UT}}^{\mathrm{sin} \phi_{S}} & \approx \frac{2 M}{Q} \mathscr{C}\left\{x f_{T} D_{1}+\frac{p_{T} \cdot k_{T}}{2 z M M_{h}}\left(x h_{T} H_{1}^{\perp}-x h_{T}^{\perp} H_{1}^{\perp}\right)\right\},  \tag{3.3}\\
F_{\mathrm{UT}}^{\sin \left(2 \phi_{h}-\phi_{S}\right)} & \approx \frac{2 M}{Q} \mathscr{C}\left\{\frac{2\left(\hat{P_{T}} \cdot k_{T}\right)^{2}-k_{T}^{2}}{2 M^{2}}\left(x f_{T}^{\perp} D_{1}\right)\right. \\
& \left.+\frac{2\left(\hat{P_{T}} \cdot p_{T}\right)\left(\hat{P_{T}} \cdot k_{T}\right)-p_{T} \cdot k_{T}}{2 z M M_{h}} \times\left[x h_{T} H_{1}^{\perp}+x h_{T}^{\perp} H_{1}^{\perp}\right]\right\}, \tag{3.4}
\end{align*}
$$

here $\mathscr{C}$ is the notation

$$
\begin{align*}
\mathscr{C}[w f D] & =x \sum_{q} e_{q}^{2} \int d^{2} \boldsymbol{k}_{T} \int d^{2} \boldsymbol{p}_{T} \delta^{2}\left(z \boldsymbol{k}_{T}-\boldsymbol{P}_{T}+\boldsymbol{p}_{T}\right) \\
& \times w\left(\boldsymbol{k}_{T}, \boldsymbol{p}_{T}\right) f^{q}\left(x, \boldsymbol{k}_{T}^{2}\right) D^{q}\left(z, \boldsymbol{p}_{T}^{2}\right), \tag{3.5}
\end{align*}
$$

which is a transformation from Eq. (4.1) of Ref. [[]3] with our new definitions of $k_{T}, p_{T}$ and $P_{T}$ (see Ref. [[8]]).

With Eqs. (B.2]), (B.3]), and (B.4), therefore the $P_{T}$-dependent transverse SSAs $A_{\mathrm{UT}}^{\sin \phi_{S}}$ and $A_{\mathrm{UT}}^{\sin \left(2 \phi_{h}-\phi_{S}\right)}$ can be defined as

$$
\begin{gather*}
A_{\mathrm{UT}}^{\sin \phi_{S}}\left(P_{T}\right)=\frac{\int d x \int d y \int d z \frac{1}{x y Q^{2}} \frac{y^{2}}{2(1-\varepsilon)}\left(1+\frac{\gamma^{2}}{2 x}\right) \sqrt{2 \varepsilon(1+\varepsilon)} F_{\mathrm{UT}}^{\sin \phi_{S}}}{\int d x \int d y \int d z \frac{1}{x y Q^{2}} \frac{y^{2}}{2(1-\varepsilon)}\left(1+\frac{\gamma^{2}}{2 x}\right) F_{\mathrm{UU}}},  \tag{3.6}\\
A_{\mathrm{UT}}^{\sin \left(2 \phi_{h}-\phi_{S}\right)}\left(P_{T}\right)=\frac{\int d x \int d y \int d z \frac{1}{x y Q^{2}} \frac{y^{2}}{2(1-\varepsilon)}\left(1+\frac{\gamma^{2}}{2 x}\right) \sqrt{2 \varepsilon(1+\varepsilon)} F_{\mathrm{UT}}^{\sin \left(2 \phi_{h}-\phi_{S}\right)}}{\int d x \int d y \int d z \frac{1}{x y Q^{2}} \frac{y^{2}}{2(1-\varepsilon)}\left(1+\frac{\gamma^{2}}{2 x}\right) F_{\mathrm{UU}}} . \tag{3.7}
\end{gather*}
$$



Figure 2: Prediction on the transverse SSA $A_{\mathrm{UT}}^{\sin \phi_{S}}$ (left) and $A_{\mathrm{UT}}^{\sin \left(2 \phi_{h}-\phi_{S}\right)}$ (right) for $\pi^{+}$(upper panel), $\pi^{-}$ (middle panel), and $\pi^{0}$ (lower panel) in SIDIS at HERMES. The dashed, dotted, and dash-dotted curves represent the asymmetries from the $f_{T} D_{1}$ or $f_{T}^{\perp} D_{1}, h_{T} H_{1}^{\perp}$, and $h_{T}^{\perp} H_{1}^{\perp}$ terms, respectively. The solid curves correspond to the total contribution.


Figure 3: Similar to Figure 2, but at JLab 5.5 GeV .

The $x$-dependent and the $z$-dependent asymmetries can be defined in a similar way.
Considering the following kinematical constraints on the intrinsic transverse momenta of the initial quarks [24]

$$
\left\{\begin{array}{l}
\boldsymbol{k}_{T}^{2} \leq(2-x)(1-x) Q^{2}, \quad \text { for } 0<x<1  \tag{3.8}\\
\boldsymbol{k}_{T}^{2} \leq \frac{x(1-x)}{(1-2 x)^{2}} Q^{2}, \quad \text { for } x<0.5
\end{array}\right.
$$

and the kinematical cuts at HERMES [ [ $]$

$$
\begin{align*}
& 0.023<x<0.4, \quad 0.1<y<0.95, \quad 0.2<z<0.7, \\
& W^{2}>10 \mathrm{GeV}^{2}, \quad Q^{2}>1 \mathrm{GeV}^{2} \\
& 0.05<P_{T}<1.2 \mathrm{GeV}, \quad 2 \mathrm{GeV}<E_{h}<15 \mathrm{GeV} \tag{3.9}
\end{align*}
$$



Figure 4: Similar to Figure 2, but at COMPASS.
at JLab 5.5 GeV [25]

$$
\begin{align*}
& 0.1<x<0.6, \quad 0.4<z<0.7, Q^{2}>1 \mathrm{GeV}^{2}, \\
& P_{T}>0.05 \mathrm{GeV}, \quad W^{2}>4 \mathrm{GeV}^{2}, \tag{3.10}
\end{align*}
$$

and at COMPASS [G]

$$
\begin{align*}
& 0.004<x<0.7,0.1<y<0.9, \quad z>0.2, \\
& P_{T}>0.1 \mathrm{GeV}, Q^{2}>1 \mathrm{GeV}^{2}, \\
& W>5 \mathrm{GeV}, \quad E_{h}>1.5 \mathrm{GeV}, \tag{3.11}
\end{align*}
$$

we perform the numerical calculation on the transverse SSAs of charged and neutral pion production in SIDIS, using an unpolarized beam scattered off a transversely polarized proton target. The corresponding results are respectively shown in Figs. [], 回] and 田. As we can see, for the $\sin \phi_{S}$ asymmetry, the $f_{T} D_{1}$ term dominates in $\pi^{+}, \pi^{-}$, and $\pi^{0}$ production, while the $h_{T} H_{1}^{\perp}$ and $h_{T}^{\perp} H_{1}^{\perp}$ terms are almost negligible. Also, the asymmetry from the $f_{T} D_{1}$ term tends to be positive in small $P_{T}$ region, while it turns to be negative in large $P_{T}$ region, due to the $k_{T}$ shape of the distribution $f_{T}$. For the $\sin \left(2 \phi_{h}-\phi_{S}\right)$ asymmetry, in the most cases, the main contribution is from the $f_{T}^{\perp} D_{1}$ term; the effects of the $h_{T} H_{1}^{\perp}$ and $h_{T}^{\perp} H_{1}^{\perp}$ terms might be observed in the asymmetry for $\pi^{-}$production at JLab, according to our numerical calculation.

## 4. Conclusion

According to the results, one can conclude that $\operatorname{sizable} \sin \phi_{S}$ and $\sin \left(2 \phi_{h}-\phi_{S}\right)$ asymmetries may be accessible at the kinematics of HERMES, JLab, and COMPASS, by performing the SIDIS experiments on the transverse polarized proton target or analyzing the available data. The measurements on the $P_{T}$ dependence of the asymmetry $A_{\mathrm{UT}}^{\text {sin } \phi_{S}}$ may be employed to test the transverse momentum dependence of the distribution $f_{T}$, e.g., the existence of a node of $f_{T}$ in $k_{T}$. Moreover,
measuring the $\sin \phi_{S}$ and $\sin \left(2 \phi_{h}-\phi_{S}\right)$ asymmetries for $\pi^{0}$ production, in which the contributions from $h_{T}$ and $h_{T}^{\perp}$ are negligible, are viable to provide clean probes on both the distributions $f_{T}$ and $f_{T}^{\perp}$. Future experiments on these effects can deepen our understanding on the role of twist-3 TMD distributions in transverse spin asymmetries.

## References

[1] V. Barone, A. Drago, and P.G. Ratcliffe, Phys. Rep. 359, 1 (2002).
[2] U. D'Alesio and F. Murgia, Prog. Part. Nucl. Phys. 61, 394 (2008).
[3] V. Barone, F. Bradamante, and A. Martin, Prog. Part. Nucl. Phys. 65, 267 (2010).
[4] HERMES Collab. (A. Airapetian et al.), Phys. Rev. Lett. 94, 012002 (2005).
[5] HERMES Collab. (A. Airapetian et al.), Phys. Rev. Lett. 103, 152002 (2009).
[6] HERMES Collab. (A. Airapetian et al.), Phys. Lett. B 693, 11 (2010).
[7] Jefferson Lab Hall A Collab. (X. Qian et al.), Phys. Rev. Lett. 107, 072003 (2011).
[8] Jefferson Lab Hall A Collab. (Y. X. Zhao et al.), Phys. Rev. C 90, no. 5, 055201 (2014).
[9] COMPASS Collab. (M.G. Alekseev et al.), Phys. Lett. B 692, 240 (2010).
[10] COMPASS Collab. (C. Adolph et al.), Phys. Lett. B 717, 376 (2012).
[11] COMPASS Collab. (C. Adolph et al.), Phys. Lett. B 717, 383 (2012).
[12] M. Diehl and S. Sapeta, Eur. Phys. J. C 41, 515 (2005).
[13] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P.J. Mulders, and M. Schlegel, J. High Energy Phys. 02, 093 (2007).
[14] COMPASS Collab. (B. Parsamyan), Phys. Part. Nucl. 45, 158 (2014).
[15] S. Wandzura and F. Wilczek, Phys. Lett. B 72, 195 (1977).
[16] A. Bacchetta, F. Conti, and M. Radici, Phys. Rev. D 78, 074010 (2008).
[17] S.J. Brodsky, D.S. Hwang, B.-Q. Ma, and I. Schmidt, Nucl. Phys. B593, 311 (2001).
[18] W. Mao, Z. Lu and B. Q. Ma, Phys. Rev. D 90, 014048 (2014).
[19] R. Jakob, P.J. Mulders, and J. Rodrigues, Nucl. Phys. A626, 937 (1997).
[20] Z. Lu and I. Schmidt, Phys. Lett. B 712, 451 (2012).
[21] K. Goeke, A. Metz, and M. Schlegel, Phys. Lett. B 618, 90 (2005).
[22] HERMES Collab. (A. Airapetian et al.), Phys. Lett. B 682, 351 (2010).
[23] A. Metz, D. Pitonyak, A. Schafer, M. Schlegel, W. Vogelsang, and J. Zhou, Phys. Rev. D 86, 094039 (2012).
[24] M. Boglione, S. Melis, and A. Prokudin, Phys. Rev. D 84, 034033 (2011).
[25] H. Avakian, TMD measurements at CLAS, in Proc. 3rd Workshop on the QCD Structure of the Nucleon, (Bilbao, Spain, 2012), Nuovo Cim. C 036, no. 05, 73 (2013).


[^0]:    *Speaker.
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