A study on three gluon correlator of Sivers asymmetry in SIDIS

Ling-Yun Dai*
Center For Exploration of Energy and Matter, Indiana University, Bloomington, IN 47408, USA
Physics Department, Indiana University, Bloomington, IN 47405, USA
E-mail: lingdai@indiana.edu

Zhong-Bo Kang
Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
E-mail: zkang@lanl.gov

Alexei Prokudin
Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA
E-mail: prokudin@jlab.org

Ivan Vitev
Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
E-mail: ivitev@lanl.gov

We study the three-gluon correlation function contribution to the Sivers asymmetry in semi-inclusive deep inelastic scattering. We first establish the matching between the usual twist-3 collinear factorization approach and transverse momentum dependent factorization formalism for the moderate transverse momentum region. We then derive the so-called coefficient functions used in the usual TMD evolution formalism. Finally we perform the next-to-leading order calculation for the transverse-momentum-weighted spin-dependent differential cross section, from which we identify the QCD collinear evolution of the twist-3 Qiu-Sterman function: the off-diagonal contribution from the three-gluon correlation functions.

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1. Introduction

Single transverse-spin asymmetries (SSAs) is a very interesting observable on both experimental and theoretical sides as it helps us understand the novel nucleon structure, in particular the parton's transverse motion [1, 2, 3, 4]. Recently experimental measurements have been performed for both $e+p$ [5, 6, 7, 8] and $p+p$ [9, 10] processes. Theoretically, two mechanisms – transverse momentum dependent (TMD) factorization and twist-3 collinear factorization approach – were developed to describe the SSAs. They are shown to be consistent with each other at the moderate transverse momentum [11, 12]. Here we focus on the SSA generated by Sivers function [13, 14] in semi-inclusive deep inelastic scattering (SIDIS).

At leading order (LO), it is known that Sivers asymmetry is related to the twist-3 quark-gluon correlation function, or Qiu-Sterman function [15]. While at next-to-leading order (NLO), three-gluon correlation functions start to contribute, which could be studied experimentally in open charm production in SIDIS [16, 17]. The purpose of this work is to study the role of three-gluon correlation functions in SIDIS, and their connections to the quark Sivers function. We first calculate the contribution of the three-gluon correlation functions to the transverse spin-dependent differential cross section within the twist-3 collinear factorization formalism, and demonstrate the matching between our result and those based on TMD framework. In the process, we also derive the so-called coefficient functions when the quark Sivers function is expanded in terms of three-gluon correlation functions, which is widely used in the TMD evolution formalism. Finally we study the NLO perturbative QCD corrections to the transverse momentum-weighted spin-dependent cross section. By analyzing the collinear divergence structure, we identify the off-diagonal evolution kernel for the Qiu-Sterman function.

2. Matching between twist-3 collinear factorization and TMD formalism

We focus on SIDIS process, $e(\ell)+p(p,s_\perp)\to e(\ell')+h(p_h)+X$, where the transverse spin dependent differential cross section related to the Sivers effect, the so-called $\sin(\phi_h-\phi_s)$ module, could be written as

$$\frac{d\Delta\sigma}{dx_B dy dz_h d^2p_{h\perp}} = \frac{\alpha_{em}^2 y}{64\pi^3 Q^4 z_h} L^{\mu\nu} W_{\mu\nu}(p,q,p_h). \tag{2.1}$$

Here $L^{\mu\nu}=2(\ell^\mu\ell'^\nu+\ell^\nu\ell'^\mu)-Q^2g^{\mu\nu}$ is the leptonic tensor, and we only consider the metric contribution '$-g^{\mu\nu}$', *i.e.*, the last term. $W^{\mu\nu}$ is the hadronic tensor, including the partonic tensor $w^{\mu\nu}$ and the usual fragmentation function $D_{h/q}(z)$. One can compute the contribution of three gluon correlation function to the spin dependent cross section to the first non-trivial order $\mathscr{O}(\alpha_{\rm em}\alpha_s)$, which has already been studied in [17]. We are interested in its low $p_{h\perp}\ll Q$ limit, which is given by

$$\left.\frac{d\Delta\sigma}{dx_B dy dz_h d^2p_{h\perp}}\right|_{p_{h\perp}\ll Q} = -z_h\sigma_0\left(\varepsilon^{\alpha\beta}s_\perp^\alpha p_{h\perp}^\beta\right)\frac{1}{\left(p_{h\perp}^2\right)^2}\sum_q e_q^2\frac{\alpha_s}{2\pi^2}\int\frac{dz}{z}D_{h/q}(z)\delta(1-\hat{z})$$
$$\times\int\frac{dx}{x^2}P_{q\leftarrow g}(\hat{x})\left(\frac{1}{2}\right)\left[O(x,x)+O(x,0)+N(x,x)-N(x,0)\right], \tag{2.2}$$

where $P_{q\leftarrow g}(\hat{x}) = T_R\left[\hat{x}^2+(1-\hat{x})^2\right]$ is the usual gluon-to-quark splitting kernel, with $T_R = 1/2$ the color factor. On the other hand, the TMD factorization formalism [18, 19, 20] for SIDIS process reads

$$\begin{aligned}\frac{d\Delta\sigma}{dx_B dy dz_h d^2p_{h\perp}} =& \sigma_0 \sum_q e_q^2 \int d^2k_\perp d^2p_\perp d^2\lambda_\perp \delta^2\left(z_h\vec{k}_\perp+\vec{p}_\perp+\vec{\lambda}_\perp-\vec{p}_{h\perp}\right)\\ &\times \frac{\varepsilon^{\alpha\beta}s_\perp^\alpha k_\perp^\beta}{M} f_{1T}^{\perp q}(x_B,k_\perp^2)D_{h/q}(z_h,p_\perp^2)S(\lambda_\perp)H(Q^2),\end{aligned} \tag{2.3}$$

where $f_{1T}^{\perp q}(x_B,k_\perp^2)$ is the quark Sivers function, $D_{h/q}(z_h,p_\perp^2)$ is the TMD fragmentation function, $S(\lambda_\perp)$ and $H(Q^2)$ denotes the soft and hard factors, respectively. To study the connection between Eqs. (2.3) and (2.2), we expand the quark Sivers function $f_{1T}^{\perp q}(x_B,k_\perp^2)$ in terms of the three-gluon correlation functions at $k_\perp \gg \Lambda_{\rm QCD}$,

$$\frac{1}{M} f_{1T}^{\perp q}(x_B,k_\perp^2) = -\frac{\alpha_s}{2\pi^2}\frac{1}{\left(k_\perp^2\right)^2}\int_{x_B}^1 \frac{dx}{x^2} P_{q\leftarrow g}(\hat{x})\left(\frac{1}{2}\right)\left[O(x,x)+O(x,0)+N(x,x)-N(x,0)\right]. \tag{2.4}$$

At the same time all other factors are replaced by their LO results. Finally we find that we arrive at the same result of Eq. (2.2). This demonstrates that collinear twist-3 factorization formalism and TMD factorization formalism for the twist-3 three-gluon correlation functions are consistent at moderate transverse momentum, $\Lambda_{\rm QCD} \ll p_{h\perp} \ll Q$.

3. Coefficient functions in TMD evolution formalism

Coefficient functions are essential ingredients in the usual TMD evolution formalism. We will now derive the coefficient functions associated with the three-gluon correlation functions. Since TMD evolution formalism is often derived in the Fourier transformed coordinate b-space, we thus need to convert the expansion of quark Sivers function in the transverse momentum space to the coordinate b-space. In particular, we need to study the following $k_\perp^\alpha$-weighted quark Sivers function [21, 22]:

$$f_{1T}^{\perp q(\alpha)}(x_B,b) = \frac{1}{M}\int d^2k_\perp e^{-ik_\perp\cdot b} k_\perp^\alpha f_{1T}^{\perp q}(x_B,k_\perp^2). \tag{3.1}$$

In the perturbative region $1/b \gg \Lambda_{\rm QCD}$, one can expand the above quark Sivers function as a product of the coefficient functions $C_{q\leftarrow i}(\hat{x}_1,\hat{x}_2)$ and the corresponding collinear function $f^{(3)}(x_1,x_2)$, *i.e.*, the twist-3 Qiu-Sterman function $T_{q,F}(x_1,x_2)$ as well as the three-gluon correlation functions $O(x_1,x_2)$ and $N(x_1,x_2)$:

$$f_{1T}^{\perp q(\alpha)}(x_B,b) = \left(\frac{ib^\alpha}{2}\right) C_{q\leftarrow i}(\hat{x}_1,\hat{x}_2)\otimes f_i^{(3)}(x_1,x_2). \tag{3.2}$$

where $\otimes$ represents the convolution over the momentum fractions. To study the coefficient function, we redo the calculation of quark Sivers function in $n = 4-2\varepsilon$ dimensions, and perform the Fourier transformation to derive its result in the b-space. Then we need a regularization to subtract the

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divergent part, which is simply the $\mathcal{O}(\alpha_s)$ correction to the Qiu-Sterman function $T_{q,F}(x,x)$ [20, 23]. Finally we have:

$$f_{1T}^{\perp q(\alpha)}(x_B,b) = \left(\frac{ib^\alpha}{2}\right)\int_{x_B}^1 \frac{dx}{x^2}\Big\{C_{q\leftarrow g,1}(\hat{x})\big[O(x,x)+N(x,x)\big]+C_{q\leftarrow g,2}(\hat{x})\big[O(x,0)-N(x,0)\big]\Big\}, \tag{3.3}$$

where

$$\begin{aligned} C_{q\leftarrow g,1}(\hat{x}) &= \frac{\alpha_s}{4\pi}\left[P_{q\leftarrow g}(\hat{x})\ln\left(\frac{c^2}{b^2\mu^2}\right)+\hat{x}(1-\hat{x})\right], \\ C_{q\leftarrow g,2}(\hat{x}) &= \frac{\alpha_s}{4\pi}\left[P_{q\leftarrow g}(\hat{x})\ln\left(\frac{c^2}{b^2\mu^2}\right)-\frac{1}{2}\left(1-6\hat{x}+6\hat{x}^2\right)\right]. \end{aligned} \tag{3.4}$$

We thus have derived the coefficient functions $C_{q\leftarrow g}$ in terms of the three-gluon correlation functions (off-diagonal parts). Such coefficient functions will be exactly *the same* even if one uses the new properly defined TMDs in [20] and/or [24, 25], because there is no contribution from soft factor subtraction at order $\mathcal{O}(\alpha_s)$ [20, 23] for off-diagonal parts.

4. Transverse momentum weighted spin-dependent cross section

We further study the NLO transverse momentum-weighted transverse spin-dependent cross section, which is defined as [26]:

$$\frac{d\langle p_{h\perp}\Delta\sigma\rangle}{dx_B dy dz_h} \equiv \int d^2p_{h\perp}\epsilon^{\alpha\beta}s_\perp^\alpha p_{h\perp}^\beta \frac{d\Delta\sigma}{dx_B dy dz_h d^2p_{h\perp}}. \tag{4.1}$$

The main partonic process is the photon-gluon scattering channel $\gamma^*+g\to q+\bar{q}$. The $p_{h\perp}$-weighted cross section contains collinear divergence, to regularize and isolate such a divergence, we again present all the calculations in $n=4-2\varepsilon$ dimensions. We then perform the usual ε-expansion to isolate the divergent and finite contributions. Collecting these terms, and performing integration by parts to convert all the derivative terms to non-derivative terms, we end up with the following expression:

$$\begin{aligned} \frac{d\langle p_{h\perp}\Delta\sigma\rangle}{dx_B dy dz_h} =& -\frac{z_h\sigma_0}{2}\sum_q e_q^2\int\frac{dz}{z}D_{h/q}(z)\delta(1-\hat{z})\left(-\frac{1}{\hat{\varepsilon}}+\ln\left(\frac{Q^2}{\mu^2}\right)\right) \\ &\times\frac{\alpha_s}{2\pi}\int\frac{dx}{x^2}P_{q\leftarrow g}(\hat{x})\left(\frac{1}{2}\right)[O(x,x)+O(x,0)+N(x,x)-N(x,0)]+\cdots, \end{aligned} \tag{4.2}$$

where $1/\hat{\varepsilon}=1/\varepsilon-\gamma_E+\ln 4\pi$ and "$\cdots$" represents the finite NLO corrections. Comparing Eq. (4.2) with the LO result given by [26]

$$\frac{d\langle p_{h\perp}\Delta\sigma\rangle}{dx_B dy dz_h} = -\frac{z_h\sigma_0}{2}\sum_q e_q^2\int\frac{dx}{x}\frac{dz}{z}T_{q,F}(x,x)D_{h/q}(z)\delta(1-\hat{x})\delta(1-\hat{z}), \tag{4.3}$$

we notice that the divergent part is simply the NLO collinear QCD correction to the LO bare Qiu-Sterman function $T_{q,F}^{(0)}(x_B,x_B)$. It should be absorbed into the definition of the renormalized

$T_{q,F}(x_B, x_B)$. After $\overline{\text{MS}}$ regularization we obtain the evolution equation for the Qiu-Sterman function of the off-diagonal (three gluon correlation function) contribution:

$$\frac{\partial}{\partial \ln \mu_f^2} T_{q,F}(x_B, x_B, \mu_f^2) = \frac{\alpha_s}{2\pi} \int_{x_B}^1 \frac{dx}{x^2} P_{q\leftarrow g}(\hat{x}) \left(\frac{1}{2}\right) \left[O(x,x,\mu_f^2) + O(x,0,\mu_f^2) + N(x,x,\mu_f^2) - N(x,0,\mu_f^2)\right] . \quad (4.4)$$

This result agrees with the earlier findings [27, 28, 29]. After such a subtraction, we finally obtain the NLO corrections (finite parts) of three-gluon correlation function to the $p_{h\perp}$-weighted transverse spin-dependent differential cross section. The complete result can be found in [30]. The result follows the standard form in the usual collinear factorization.

5. Summary

In this contribution we study the contribution of the three-gluon correlation functions to the Sivers asymmetry in SIDIS. We demonstrate at cross section level that twist-3 collinear factorization formalism is consistent with TMD factorization at moderate hadron transverse momentum, $\Lambda_{\text{QCD}} \ll p_{h\perp} \ll Q$. We also derive the $\mathscr{O}(\alpha_s)$ expansion of the quark Sivers function, and identify the off-diagonal coefficient functions used in the usual TMD evolution formalism. We further calculate the NLO perturbative QCD corrections to the transverse-momentum-weighted spin-dependent differential cross section, from which we identify the off-diagonal contribution from the three-gluon correlation functions to the QCD collinear evolution of the twist-3 Qiu-Sterman function.

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