Recent Measurements of the Unitarity Triangle Angles

Markus Röhrken
California Institute of Technology
E-mail: roehrken@caltech.edu

The experimental determination of the angles of the Unitarity Triangle is closely related to the measurement of CP violating effects. Precise knowledge of the angles can put tight constraints on the flavor sector of the Standard Model and on physics beyond. In this proceedings article recent measurements of the angles $\gamma$ and $\beta$ ($\phi_3$ and $\phi_1$, respectively) by the LHCb, BABAR and Belle experiments are reported.
1. Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] defines the couplings of up- to down-type quarks in charged-current $W^\pm$ interactions. The matrix is unitary and can be parametrized by three mixing angles and one $CP$ violating complex phase. The unitarity of the CKM matrix imposes $\sum_i V_{ij}V_{ik}^* = \delta_{jk}$, which gives rise to six vanishing relations. These relations can be interpreted as triangles in the complex plane. Of most importance for the charged and neutral $B$ meson system is the so-called Unitarity Triangle, which arises from

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad (1.1)$$

by normalizing to the best known side given by $V_{cd}V_{cb}^*$. The angles of the Unitarity Triangle are defined by:

$$\alpha = \phi_2 = \arg \left( \frac{V_{ub}V_{tb}^*}{V_{ud}V_{ub}^*} \right) \quad (1.2)$$

$$\beta = \phi_1 = \arg \left( \frac{V_{cb}V_{tb}^*}{V_{cd}V_{cb}^*} \right) \quad (1.3)$$

$$\gamma = \phi_3 = \arg \left( \frac{V_{ub}V_{ub}^*}{V_{cd}V_{cb}^*} \right) \quad (1.4)$$

Precise measurements of the sides and angles of the Unitarity Triangle provide stringent tests of the Standard Model in the flavor sector, and enable constraining physics beyond the Standard Model. Because $CP$ violation arises from the complex phase contributing to the CKM matrix elements, the experimental determination of the angles of the Unitarity Triangle is closely related to the measurements of $CP$ asymmetries.

2. Measurements of $\gamma$

The angle $\gamma$ enters as a relative weak phase in the interference between $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions such as occurring in $B^- \to D^0K^-$ and $B^- \to \bar{D}^0K^-$ decays. If the neutral $D$ meson decays to a final state that is common to $D^0$ and $\bar{D}^0$, then the two interfering amplitudes differ by a factor of $r_B e^{i(\gamma + \delta_B)}$, where $\gamma$ is the relative weak phase, $\delta_B$ is the relative strong phase, and $r_B = |A(B^- \to D^0K^-)/A(B^- \to \bar{D}^0K^-)|$ is the ratio of the interfering amplitudes. Because only tree level amplitudes contribute, the determination of $\gamma$ from $B \to DK$ decays can provide a Standard Model reference. However the current experimental knowledge on $\gamma$ is limited due to the smallness of the involved $B$ and $D$ branching fractions and the smallness of the amplitude ratio $r_B$, which is approximately $0.1$ for charged $B$ meson decays.

Several different methods have been proposed and applied to extract $\gamma$ from $B \to DK$: The GLW method [2, 3] uses $D$ meson decays to a $CP$ eigenstate such as $D \to K^+K^-$, the ADS method [4, 5] uses doubly Cabibbo-suppressed and -favored $D$ meson decays to flavored states such as $D \to K\pi$, the GLS method [6] uses ADS-like singly Cabibbo-suppressed $D$ meson decays such as $D \to K^0\pi^\pm\pi^\mp$, and the GGSZ method [7] uses self-conjugated multi-body $D$ meson decays such as $D \to K^{0}\pi^+\pi^-$. 

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Currently the most sensitive technique to estimate $\gamma$ is the GGSZ method, which enables the extraction of all three observables $\gamma$, $\delta_B$ and $r_B$ by a single measurement. The sensitivity stems from large interferences, which can occur in some regions of the Dalitz plot. However in model-dependent GGSZ analyses a difficulty arises due to the applied $D$ Dalitz model. At measurements at the $B$ factory experiments for example, the $D$ Dalitz model description is obtained from an amplitude analysis of flavor-tagged $D^0$ decays using $c\bar{c}$ data. The uncertainty on $\gamma$ due to the chosen $D$ Dalitz model can be sizeable. In previous $B\bar{B}$ and Belle measurements, a model uncertainty of $3^\circ$ and $9^\circ$ on $\gamma$ has been reported [8, 9]. The model uncertainty contains an irreducible component, and thus limits the overall achievable precision on $\gamma$.

A model-independent approach to extract $\gamma$ using the GGSZ method from $B \to DK$ with multibody $D$ meson decays to self-conjugated states has been pioneered by Belle [10]. In this approach, the $D$ meson Dalitz plot is divided into bins and experimental inputs on the strong phases obtained in quantum-correlated $\psi(3770) \to D\bar{D}$ decays by CLEO are applied [11, 12]. It was demonstrated [11, 14] that this method introduces no bias on the $\gamma$ extraction. The model uncertainty is effectively replaced by the experimental uncertainty on the strong phases, which are expected to be significantly reduced in future measurements at charm-factories such as BESIII, and thus enable precise measurements of $\gamma$ using the GGSZ method.

### 2.1 Model-Independent Measurement of $\gamma$ by LHCb

LHCb reports [15] a measurement of $\gamma$ using the model-independent GGSZ method applied to $B^\pm \to DK^\pm$ decays with $D \to K^0_S \pi^+ \pi^-$ and $D \to K^0_S K^+ K^-$. The measurement is performed on a data sample corresponding to an integrated luminosity of $3 \text{fb}^{-1}$ obtained in $pp$ collisions at centre-of-mass energies of $\sqrt{s} = 7$ and $8 \text{TeV}$. A multivariate technique using boosted decision trees is applied to improve the background suppression of the selection. As control samples $B^\pm \to D\pi^\pm$ and semileptonic neutral $B$ meson decays of the kind $B^0 \to D^{*-}\mu^+\nu_\mu$ with $D^{*-} \to D^0 \pi^-$ and $D \to K^0_S h^+ h^-$ are used, which enable obtaining shape parameters and Dalitz plot efficiency corrections directly from the data. A total yield of $2257 \pm 43 (324 \pm 8) B^\pm \to DK^\pm$ signal events is obtained for $D \to K^0_S \pi^+ \pi^- (D \to K^0_S K^+ K^-)$. For $D \to K^0_S \pi^+ \pi^- (D \to K^0_S K^+ K^-)$ a binning scheme separating the $D$ Dalitz plot into $2 \times 8$ ($2 \times 2$) divisions is used, which has been chosen to optimize the statistical sensitivity on $\gamma$. The cartesian parameters $x_\pm = r_B \cos(\delta_B \pm \gamma)$ and $y_\pm = r_B \sin(\delta_B \pm \gamma)$ are extracted from a simultaneous fit to the bins of the $D$ Dalitz plot in $B^\pm \to DK^\pm$ decays with experimental input on the strong phases provided by CLEO [12]. The results on $x_\pm$ and $y_\pm$ is interpreted in terms of the physics parameters $\gamma$, $\delta_B$ and $r_B$ using a frequentist approach with a Feldman-Cousins ordering. Including statistical and systematic uncertainties, the result of the measurement is:

$$\gamma = (62^{+15}_{-14})^\circ$$
$$r_B = 0.080^{+0.019}_{-0.021}$$
$$\delta_B = (134^{+15}_{-13})^\circ$$ (2.1)

LHCb uses this result to combine it with other GLW, ADS and GLS measurements of $B \to DK$ decays and with time-dependent measurements of $B^0 \to D^+_S K^-$ decays obtained by the same experiment [16]. The result of the LHCb combination is $\gamma = (73^{+9}_{-10})^\circ$ at the 68% C.L.
2.2 Model-Independent Measurement of $\gamma$ by Belle

Belle performed a first measurement of the amplitude ratio $r_S$ of $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$ decays with $D \rightarrow K^0_s \pi^+ \pi^-$ using the model-independent GGSZ approach on a data sample that contains $772 \times 10^6 B\bar{B}$ pairs. The observable $r_S$ depends on the intensities of the $b \rightarrow u$ and $b \rightarrow c$ amplitudes integrated over the $D$ Dalitz plot and is defined as:

$$r_S^2 = \frac{\Gamma(B^0 \rightarrow D^0 K^+ \pi^-)}{\Gamma(B^0 \rightarrow D^0 K^+ \pi^+)} \frac{\int A_{b \rightarrow u}^2(p) dp}{\int A_{b \rightarrow c}^2(p) dp} \tag{2.2}$$

If the neutral $B$ meson decay is saturated by a two-body $DK^{*0}$ decay, then $r_S$ becomes the ratio of the $b \rightarrow u$ and $b \rightarrow c$ amplitudes. Because both amplitudes are color-suppressed in neutral $B$ meson decays, the amplitude ratio $r_S$ is significantly larger than that of the corresponding charged $B$ meson decays. The expected value of $r_S$ is approximately 0.4. Since the statistical precision on $\gamma$ scales with $1/r_S$, the extraction of $\gamma$ from neutral $B$ meson decays can provide better experimental sensitivity.

The measurement reconstructs the excited neutral kaon as $K^{*0}(892) \rightarrow K^+ \pi^-$. The $B$ flavor is identified by the kaon charge. The dominant background arises from $e^+ e^- \rightarrow q\bar{q}$ ($q \in \{u,d,s,c\}$) continuum events. A neural network trained on event shape variables is used to discriminate between continuum and $B\bar{B}$ events. Peaking background originating from $\bar{B}^0 \rightarrow [K^{*0} \pi^+]|_{D^+ [K^0 \pi^-]K^+}$ decays is excluded by a veto. The signal is extracted by a three-dimensional fit to the Dalitz plot and is defined as:

The $D \rightarrow K^0_s \pi^+ \pi^-$ Dalitz plot is divided into $2 \times 8$ bins. With experimental input on the strong phases provided by CLEO [12], the parameters $x_\pm$ and $y_\pm$ are extracted from the $B^0 \rightarrow D^0 K^{*0}$ signal yields in each bin. A frequentist approach with a Feldman-Cousins ordering is applied to determine the physics parameters $\gamma$, $\delta_B$ and $r_B$ from the measured values of $x_\pm$ and $y_\pm$. Due to the low statistics, the measurement is not sensitive to $\gamma$ and $\delta_B$. An upper limit of $r_S < 0.87$ at 68% C.L. including statistical and systematic uncertainties is obtained.

3. Measurements of $\beta$

The angle $\beta$ is experimentally accessible by time-dependent $CP$ violation measurements of neutral $B$ meson decays to $CP$ eigenstates. If neutral $B$ mesons decay to a $CP$ eigenstate $f_{CP}$, then an interference between the decay without $B^0$-$\bar{B}^0$ mixing and the decay following $B^0$-$\bar{B}^0$ mixing emerges. The interference is characterized by the parameter $\lambda = \frac{q\bar{q}}{p\bar{p}}$, where $q/p$ is a phase factor due to $B^0$-$\bar{B}^0$ mixing and $\lambda/|A|$ is the decay amplitude ratio. The $CP$ asymmetry in decays to $CP$ eigenstates is defined as:

$$A(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) - \Gamma(B^0(t) \rightarrow f_{CP})}{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) + \Gamma(B^0(t) \rightarrow f_{CP})} = A \sin(\Delta m t) - \epsilon \cos(\Delta m t) \tag{3.1}$$

The $CP$ asymmetry is time-dependent, and the parameters $A$ and $\epsilon$ measure mixing-induced and direct $CP$ violation, respectively.

A theoretically clean way to access $\beta$ is provided by $b \rightarrow c\bar{c}d$ transitions such as $B^0 \rightarrow J/\psi K_S^{*0}$ decays and by $b \rightarrow c\bar{u}d$ transitions such as $\bar{B}^0 \rightarrow D_{CP}^{(*)} H^0$ decays [17, 18]. To a good approximation,
these decays are only mediated by amplitudes containing real CKM elements. In this case, $B^0$-$\bar{B}^0$ mixing introduces the weak phase $\beta$ such that $\mathcal{A} = -\eta_{CP} \sin(2\beta)$ and $\mathcal{C} = 0$, where $\eta_{CP}$ is the CP eigenvalue of the final state $J_{CP}$. Therefore $\beta$ can be precisely determined from the time-dependent CP asymmetry of $B^0 \rightarrow J/\psi K_S^0$ and $\bar{B}^0 \rightarrow D_{CP}^{(*)} h^0$ decays.

### 3.1 Measurements of $\beta$ in $B^0 \rightarrow J/\psi K_S^0$ by LHCb

LHCb reports [19] a time-dependent CP violation measurement of $\sin(2\beta)$ in $B^0 \rightarrow J/\psi K_S^0$ decays using a data sample corresponding to an integrated luminosity of 3 fb$^{-1}$ obtained in $pp$ collisions at centre-of-mass energies of $\sqrt{s} = 7$ and 8 TeV. $B^0 \rightarrow J/\psi K_S^0$ decays are reconstructed in the $J/\psi \rightarrow \mu^+ \mu^-$ and $K_S^0 \rightarrow \pi^+ \pi^-$ final states. The production flavor of $B$ meson candidates is estimated using flavor tagging algorithms applied to decay products from the $b$ hadron produced in association with the signal $B$ meson (opposite-side tagging) and applied to pions produced in the fragmentation of the $b$ quark of the signal $B$ meson (same-side tagging). The effective tagging efficiency is $(3.02 \pm 0.02)\%$. A total signal yield of 41560 ± 270 flavor-tagged signal events is obtained. The result of the time-dependent measurement is:

$$
\mathcal{A} = +0.731 \pm 0.035 \text{(stat.)} \pm 0.020 \text{(syst.)}
$$

$$
\mathcal{C} = -0.038 \pm 0.032 \text{(stat.)} \pm 0.005 \text{(syst.)}
$$

(3.2)

The result is in good agreement with the world average of $\sin(2\beta) = 0.69 \pm 0.02$ measured in $b \rightarrow c\bar{c}s$ transitions at the $B$ factory experiments [20], and the uncertainty on $\mathcal{A}$ is comparable to that achieved by $B\bar{B}$ and Belle on their final data sets.

### 3.2 Measurements of $\beta$ in $\bar{B}^0 \rightarrow D_{CP}^{(*)} h^0$ by $B\bar{B}$ and Belle Combined

$B\bar{B}$ and Belle report [21] a time-dependent CP violation measurement of $B^0 \rightarrow D_{CP}^{(*)} h^0$ decays that makes simultaneous use of the combined $1240 \times 10^6$ $B\bar{B}$ pairs collected by both experiments on the $\Upsilon(4S)$ resonance. The light neutral hadron $h^0$ is reconstructed as a $\pi^0$, $\eta$ or $\omega$ meson, and the neutral $D$ meson is reconstructed in the two-body $CP$ eigenstates $K^+ K^-$, $K_S^0 \pi^0$ and $K_S^0 \omega$. Excited $D$ mesons are reconstructed in the decay mode $D^{*0} \rightarrow D^0 \pi^0$. The dominant background arises from $e^+ e^- \rightarrow q\bar{q}$ ($q \in \{u,d,s,c\}$) continuum events. This background is suppressed by selection requirements applied to a neural network trained on event shape variables. The similar performance of the $B\bar{B}$ and Belle detectors enable to apply almost the same selection requirements to both data samples. The signal is extracted from fits to the $M_{h^0}$ distributions. A total yield of $508 \pm 31$ ($757 \pm 44$) $\bar{B}^0 \rightarrow D_{CP}^{(*)} h^0$ signal events is obtained for $B\bar{B}$ and Belle (Belle). The joint $B\bar{B}$ +Belle time-dependent measurement is realized by combining both experiments on the likelihood level. The CP violation parameters are estimated by a simultaneous fit to the reconstructed flavor-tagged proper decay time distributions of both experiments. The result of the measurement is:

$$
-\eta_{CP} \mathcal{A} = +0.66 \pm 0.10 \text{(stat.)} \pm 0.06 \text{(syst.)}
$$

$$
\mathcal{C} = -0.02 \pm 0.07 \text{(stat.)} \pm 0.03 \text{(syst.)}
$$

(3.3)

The measurement provides the first observation of CP violation in $\bar{B}^0 \rightarrow D_{CP}^{(*)} h^0$ decays by excluding the no-mixing induced CP violation hypothesis at 5.4 standard deviations. The result is in very good agreement with the world average of $\sin(2\beta) = 0.69 \pm 0.02$ measured in $b \rightarrow c\bar{c}s$ transitions [20].

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4. Summary

LHCb achieves a precision of 10° on the weak phase $\gamma$. Furthermore LHCb reaches the precision of the $B$ factory experiments in time-dependent $CP$ violation measurements and confirms the measurements of $\sin(2\beta)$ in $b \to c\bar{s}s$ transitions. The $B$ factory experiments continue to produce important results such as a first measurement of the amplitude ratio $r_S$ in $B^0 \to D^0 K^{*0}$ decays using $D \to K^0_S \pi^+ \pi^-$ by Belle, and the first observation of $CP$ violation in $\bar{B}^0 \to D^{(*)}_s h^0$ decays by combining BaBar and Belle data. Further improvements on the precision of the Unitarity Triangle angles are expected from Run 2 of the LHC and the high luminosity $B$ factory experiment Belle II.

References