PROCEEDINGS OF SCIENCE

Recent Measurements of the Unitarity Triangle Angles

Markus Röhrken[*†]
California Institute of Technology
E-mail: `roehrken@caltech.edu`

The experimental determination of the angles of the Unitarity Triangle is closely related to the measurement of *CP* violating effects. Precise knowledge of the angles can put tight constraints on the flavor sector of the Standard Model and on physics beyond. In this proceedings article recent measurements of the angles γ and β (ϕ_3 and ϕ_1, respectively) by the LHCb, *BABAR* and Belle experiments are reported.

Flavor Physics & CP Violation 2015,
May 25-29, 2015
Nagoya, Japan

---

[*] Speaker.

[†] On behalf of the *BABAR* Collaboration.

1. Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] defines the couplings of up- to down-type quarks in charged-current $W^{\pm}$ interactions. The matrix is unitary and can be parametrized by three mixing angles and one *CP* violating complex phase. The unitarity of the CKM matrix imposes $\sum_i V_{ij} V_{ik}^* = \delta_{jk}$, which gives rise to six vanishing relations. These relations can be interpreted as triangles in the complex plane. Of most importance for the charged and neutral B meson system is the so-called Unitarity Triangle, which arises from

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \tag{1.1}$$

by normalizing to the best known side given by $V_{cd} V_{cb}^*$. The angles of the Unitarity Triangle are defined by:

$$\alpha = \phi_2 = \arg\left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right) \tag{1.2}$$

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right) \tag{1.3}$$

$$\gamma = \phi_3 = \arg\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right) \tag{1.4}$$

Precise measurements of the sides and angles of the Unitarity Triangle provide stringent tests of the Standard Model in the flavor sector, and enable constraining physics beyond the Standard Model. Because *CP* violation arises from the complex phase contributing to the CKM matrix elements, the experimental determination of the angles of the Unitarity Triangle is closely related to the measurements of *CP* asymmetries.

2. Measurements of γ

The angle γ enters as a relative weak phase in the interference between $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions such as ocurring in $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$ decays. If the neutral D meson decays to a final state that is common to D^0 and $\bar{D}^0$, then the two interfering amplitudes differ by a factor of $r_B \mathrm{e}^{\mathrm{i}(\pm\gamma+\delta_\mathrm{B})}$, where γ is the relative weak phase, δ_B is the relative strong phase, and $r_B = |A(B^- \to \bar{D}^0 K^-)/A(B^- \to D^0 K^-)|$ is the ratio of the interfering amplitudes. Because only tree level amplitudes contribute, the determination of γ from $B \to DK$ decays can provide a Standard Model reference. However the current experimental knowledge on γ is limited due to the smallness of the involved B and D branching fractions and the smallness of the amplitude ratio r_B, which is approximately 0.1 for charged B meson decays.

Several different methods have been proposed and applied to extract γ from $B \to DK$: The GLW method [2, 3] uses D meson decays to a *CP* eigenstate such as $D \to K^+K^-$, the ADS method [4, 5] uses doubly Cabibbo-suppressed and -favored D meson decays to flavored states such as $D \to K\pi$, the GLS method [6] uses ADS-like singly Cabibbo-suppressed D meson decays such as $D \to K_S^0 K\pi$, and the GGSZ method [7] uses self-conjugated multi-body D meson decays such as $D \to K_S^0 \pi^+\pi^-$.

Currently the most sensitive technique to estimate γ is the GGSZ method, which enables the extraction of all three observables γ, δ_B and r_B by a single measurement. The sensitivity stems from large interferences, which can occur in some regions of the Dalitz plot. However in model-dependent GGSZ analyses a difficulty arises due to the applied D Dalitz model. At measurements at the B factory experiments for example, the D Dalitz model description is obtained from an amplitude analysis of flavor-tagged D^0 decays using $c\bar{c}$ data. The uncertainty on γ due to the chosen D Dalitz model can be sizeable. In previous *BaBar* and Belle measurements, a model uncertainty of 3° and 9° on γ has been reported [8, 9]. The model uncertainty contains an irreducible component, and thus limits the overall achievable precision on γ.

A model-independent approach to extract γ using the GGSZ method from $B \to DK$ with multi-body D meson decays to self-conjugated states has been pioneered by Belle [10]. In this approach, the D meson Dalitz plot is divided into bins and experimental inputs on the strong phases obtained in quantum-correlated $\psi(3770) \to D\bar{D}$ decays by CLEO are applied [11, 12]. It was demonstrated [13, 14] that this method introduces no bias on the γ extraction. The model uncertainty is effectively replaced by the experimental uncertainty on the strong phases, which are expected to be significantly reduced in future measurements at charm-factories such as BESIII, and thus enable precise measurements of γ using the GGSZ method.

2.1 Model-Independent Measurement of γ by LHCb

LHCb reports [15] a measurement of γ using the model-independent GGSZ method applied to $B^\pm \to DK^\pm$ decays with $D \to K^0_S\pi^+\pi^-$ and $D \to K^0_S K^+K^-$. The measurement is performed on a data sample corresponding to an integrated luminosity of $3\,\mathrm{fb}^{-1}$ obtained in pp collisions at centre-of-mass energies of $\sqrt{s} = 7$ and $8\,\mathrm{TeV}$. A multivariate technique using boosted decision trees is applied to improve the background suppression of the selection. As control samples $B^\pm \to D\pi^\pm$ and semileptonic neutral B meson decays of the kind $B^0 \to D^{*-}\mu^+\nu_\mu$ with $D^{*-} \to \bar{D}^0\pi^-$ and $D \to K^0_S h^+h^-$ are used, which enable obtaining shape parameters and Dalitz plot efficiency corrections directly from the data. A total yield of 2257 ± 43 (324 ± 8) $B^\pm \to DK^\pm$ signal events is obtained for $D \to K^0_S\pi^+\pi^-$ ($D \to K^0_S K^+K^-$). For $D \to K^0_S\pi^+\pi^-$ ($D \to K^0_S K^+K^-$) a binning scheme separating the D Dalitz plot into 2×8 (2×2) divisions is used, which has been chosen to optimize the statistical sensitivity on γ. The cartesian parameters $x_\pm = r_B \cos(\delta_B \pm \gamma)$ and $y_\pm = r_B \sin(\delta_B \pm \gamma)$ are extracted from a simultaneous fit to the bins of the D Dalitz plot in $B^\pm \to DK^\pm$ decays with experimental input on the strong phases provided by CLEO [12]. The results on $x_\pm$ and $y_\pm$ is interpreted in terms of the physics parameters γ, δ_B and r_B using a frequentist approach with a Feldman-Cousins ordering. Including statistical and systematic uncertainties, the result of the measurement is:

$$\begin{aligned} \gamma &= (62^{+15}_{-14})^\circ \\ r_B &= 0.080^{+0.019}_{-0.021} \\ \delta_B &= (134^{+14}_{-15})^\circ \end{aligned} \tag{2.1}$$

LHCb uses this result to combine it with other GLW, ADS and GLS measurements of $B \to DK$ decays and with time-dependent measurements of $B^0_s \to D^\mp_s K^\pm$ decays obtained by the same experiment [16]. The result of the LHCb combination is $\gamma = (73^{+9}_{-10})^\circ$ at the 68% C.L..

2.2 Model-Independent Measurement of γ by Belle

Belle performed a first measurement of the amplitude ratio r_S of $B^0 \to D^0 K^{*0}$ and $B^0 \to \bar{D}^0 K^{*0}$ decays with $D \to K_S^0 \pi^+ \pi^-$ using the model-independent GGSZ approach on a data sample that contains 772×10^6 $B\bar{B}$ pairs. The observable r_S depends on the intensities of the $b \to u$ and $b \to c$ amplitudes integrated over the D Dalitz plot and is defined as:

$$r_S^2 = \frac{\Gamma(B^0 \to D^0 K^+ \pi^-)}{\Gamma(B^0 \to \bar{D}^0 K^+ \pi^-)} = \frac{\int A^2_{b\to u}(p)dp}{\int A^2_{b\to c}(p)dp} \tag{2.2}$$

If the neutral B meson decay is saturated by a two-body DK^{*0} decay, then r_S becomes the ratio of the $b \to u$ and $b \to c$ amplitudes. Because both amplitudes are color-suppressed in neutral B meson decays, the amplitude ratio r_S is significantly larger than that of the corresponding charged B meson decays. The expected value of r_S is approximately 0.4. Since the statistical precision on γ scales with $1/r_S$, the extraction of γ from neutral B meson decays can provide better experimental sensitivity.

The measurement reconstructs the excited neutral kaon as $K^{*0}(892) \to K^+ \pi^-$. The B flavor is identified by the kaon charge. The dominant background arises from $e^+e^- \to q\bar{q}$ ($q \in \{u,d,s,c\}$) continuum events. A neural network trained on event shape variables is used to discriminate between continuum and $B\bar{B}$ events. Peaking background originating from $\bar{B}^0 \to [\bar{K}^{*0}\pi^+]_{D^+}[K^0\pi^-]_{K^{*-}}$ decays is excluded by a veto. The signal is extracted by a three-dimensional fit to the $M_{\rm bc}$, ΔE and neural network distributions. A total yield of 44^{+13}_{-12} $B^0 \to D^0 K^{*0}$ signal events is obtained. The statistical significance of the signal is 2.8σ.

The $D \to K_S^0 \pi^+ \pi^-$ Dalitz plot is divided into 2×8 bins. With experimental input on the strong phases provided by CLEO [12], the parameters $x_\pm$ and $y_\pm$ are extracted from the $B^0 \to D^0 K^{*0}$ signal yields in each bin. A frequentist approach with a Feldman-Cousins ordering is applied to determine the physics parameters γ, δ_B and r_B from the measured values of $x_\pm$ and $y_\pm$. Due to the low statistics, the measurement is not sensitive to γ and δ_B. An upper limit of $r_S < 0.87$ at 68% C.L. including statistical and systematic uncertainties is obtained.

3. Measurements of β

The angle β is experimentally accessible by time-dependent *CP* violation measurements of neutral B meson decays to *CP* eigenstates. If neutral B mesons decay to a *CP* eigenstate f_{CP}, then an interference between the decay without B^0-$\bar{B}^0$ mixing and the decay following B^0-$\bar{B}^0$ mixing emerges. The interference is characterized by the parameter $\lambda = \frac{q}{p}\frac{\bar{A}}{A}$, where q/p is a phase factor due to B^0-$\bar{B}^0$ mixing and $\bar{A}/A$ is the decay amplitude ratio. The *CP* asymmetry in decays to *CP* eigenstates is defined as:

$$\mathscr{A}(t) = \frac{\Gamma(\bar{B}^0(t) \to f_{CP}) - \Gamma(B^0(t) \to f_{CP})}{\Gamma(\bar{B}^0(t) \to f_{CP}) + \Gamma(B^0(t) \to f_{CP})} = \mathscr{S}\sin(\Delta m t) - \mathscr{C}\cos(\Delta m t) \tag{3.1}$$

The *CP* asymmetry is time-dependent, and the parameters $\mathscr{S}$ and $\mathscr{C}$ measure mixing-induced and direct *CP* violation, respectively.

A theoretically clean way to access β is provided by $b \to c\bar{c}s$ transitions such as $B^0 \to J/\psi K_S^0$ decays and by $b \to c\bar{u}d$ transitions such as $\bar{B}^0 \to D^{(*)}_{CP} h^0$ decays [17, 18]. To a good approximation, these decays are only mediated by amplitudes containing real CKM elements. In this case, B^0-$\bar{B}^0$ mixing introduces the weak phase β such that $\mathscr{S} = -\eta_{f_{CP}} \sin(2\beta)$ and $\mathscr{C} = 0$, where $\eta_{f_{CP}}$ is the *CP* eigenvalue of the final state f_{CP}. Therefore β can be precisely determined from the time-dependent *CP* asymmetry of $B^0 \to J/\psi K^0_S$ and $\bar{B}^0 \to D^{(*)}_{CP} h^0$ decays.

3.1 Measurements of β in $\mathbf{B^0 \to J/\psi K^0_S}$ by LHCb

LHCb reports [19] a time-dependent *CP* violation measurement of $\sin(2\beta)$ in $B^0 \to J/\psi K^0_S$ decays using a data sample corresponding to an integrated luminosity of $3\,\mathrm{fb}^{-1}$ obtained in pp collisions at centre-of-mass energies of $\sqrt{s} = 7$ and $8\,\mathrm{TeV}$. $B^0 \to J/\psi K^0_S$ decays are reconstructed in the $J/\psi \to \mu^+\mu^-$ and $K^0_S \to \pi^+\pi^-$ final states. The production flavor of B meson candidates is estimated using flavor tagging algorithms applied to decay products from the b hadron produced in association with the signal B meson (opposite-side tagging) and applied to pions produced in the fragmentation of the b quark of the signal B meson (same-side tagging). The effective tagging efficiency is $(3.02 \pm 0.02)\%$. A total signal yield of 41560 ± 270 flavor-tagged signal events is obtained. The result of the time-dependent measurement is:

$$\begin{aligned} \mathscr{S} &= +0.731 \pm 0.035\,(\text{stat.}) \pm 0.020\,(\text{syst.}) \\ \mathscr{C} &= -0.038 \pm 0.032\,(\text{stat.}) \pm 0.005\,(\text{syst.}) \end{aligned} \tag{3.2}$$

The result is in good agreement with the world average of $\sin(2\beta) = 0.69 \pm 0.02$ measured in $b \to c\bar{c}s$ transitions at the B factory experiments [20], and the uncertainty on $\mathscr{S}$ is comparable to that achieved by *BABAR* and Belle on their final data sets.

3.2 Measurements of β in $\mathbf{\bar{B}^0 \to D^{(*)}_{CP} h^0}$ by *BABAR* and Belle Combined

BABAR and Belle report [21] a time-dependent *CP* violation measurement of $\bar{B}^0 \to D^{(*)}_{CP} h^0$ decays that makes simultaneous use of the combined 1240×10^6 $B\bar{B}$ pairs collected by both experiments on the $\Upsilon(4S)$ resonance. The light neutral hadron h^0 is reconstructed as a π^0, η or ω meson, and the neutral D meson is reconstructed in the two-body *CP* eigenstates K^+K^-, $K^0_S\pi^0$ and $K^0_S\omega$. Excited D mesons are reconstructed in the decay mode $D^{*0} \to D^0\pi^0$. The dominant background arises from $e^+e^- \to q\bar{q}$ $(q \in \{u,d,s,c\})$ continuum events. This background is suppressed by selection requirements applied to a neural network trained on event shape variables. The similar performance of the *BABAR* and Belle detectors enable to apply almost the same selection requirements to both data samples. The signal is extracted from fits to the M_{bc} distributions. A total yield of 508 ± 31 (757 ± 44) $\bar{B}^0 \to D^{(*)}_{CP} h^0$ signal events is obtained for *BABAR* (Belle). The joint *BABAR* +Belle time-dependent measurement is realized by combining both experiments on the likelihood level. The *CP* violation parameters are estimated by a simultaneous fit to the reconstructed flavor-tagged proper decay time distributions of both experiments. The result of the measurement is:

$$\begin{aligned} -\eta_{f_{CP}}\mathscr{S} &= +0.66 \pm 0.10\,(\text{stat.}) \pm 0.06\,(\text{syst.}) \\ \mathscr{C} &= -0.02 \pm 0.07\,(\text{stat.}) \pm 0.03\,(\text{syst.}) \end{aligned} \tag{3.3}$$

The measurement provides the first observation of *CP* violation in $\bar{B}^0 \to D^{(*)}_{CP} h^0$ decays by excluding the no-mixing induced *CP* violation hypothesis at 5.4 standard deviations. The result is in very good agreement with the world average of $\sin(2\beta) = 0.69 \pm 0.02$ measured in $b \to c\bar{c}s$ transitions [20].

4. Summary

LHCb achieves a precision of 10° on the weak phase γ. Furthermore LHCb reaches the precision of the B factory experiments in time-dependent CP violation measurements and confirms the measurements of $\sin(2\beta)$ in $b \to c\bar{c}s$ transitions. The B factory experiments continue to produce important results such as a first measurement of the amplitude ratio r_S in $B^0 \to D^0 K^{*0}$ decays using $D \to K^0_S \pi^+\pi^-$ by Belle, and the first observation of CP violation in $\bar{B}^0 \to D^{(*)}_{CP} h^0$ decays by combining *BABAR* and Belle data. Further improvements on the precision of the Unitarity Triangle angles are expected from Run 2 of the LHC and the high luminosity B factory experiment Belle II.

References

[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963);
M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] M. Gronau, D. London, Phys. Lett. B **254**, 483 (1991).

[3] M. Gronau, D. Wyler, Phys. Lett. B **265**, 172 (1991).

[4] D. Atwood, I. Dunietz, A. Soni, Phys. Rev. Lett. **78**, 3257 (1997).

[5] D. Atwood, I. Dunietz, A. Soni, Phys. Rev. D **63**, 036005 (2001).

[6] Y. Grossman, Z. Ligeti, A. Soffer, Phys. Rev. D **67**, 071301 (2003).

[7] A. Giri, Y. Grossman, A. Soffer, J. Zupan, Phys. Rev. D **68**, 054018 (2003).

[8] P. Sanchez *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **105**, 121801 (2010).

[9] A. Poluektov *et al.* (Belle Collaboration), Phys. Rev. D **81**, 112002 (2010).

[10] H. Aihara *et al.* (Belle Collaboration), Phys. Rev. D **85**, 112014 (2012).

[11] R.A. Briere *et al.* (CLEO Collaboration), Phys. Rev. D **80**, 032002 (2009).

[12] J. Libby *et al.* (CLEO Collaboration), Phys. Rev. D **82**, 112006 (2010).

[13] A. Bondar, A. Poluektov, Eur. Phys. J. C **47**, 347 (2006).

[14] A. Bondar, A. Poluektov, Eur. Phys. J. C **55**, 51 (2008).

[15] R. Aaij *et al.* (LHCb Collaboration), JHEP **10**, 097 (2014).

[16] LHCb Collaboration, LHCB-CONF-2014-004 (2014).

[17] A.B. Carter, A.I. Sanda, Phys. Rev. D **23**, 1567 (1981).

[18] I.I. Bigi, A.I. Sanda, Nucl. Phys. B **193**, 85 (1981).

[19] R. Aaij *et al.* (LHCb Collaboration), arXiv:1503.07089 (2015).

[20] Y. Amhis *et al.* (Heavy Flavor Averaging Group), arXiv:1412.7515 (2014).

[21] A. Abdesselam *et al.* (*BABAR* and Belle Collaborations), arXiv:1505.04147 (2015).