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Theoretical status of semileptonic and rare heavy meson decays

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Some advances of the theoretical status are reviewed for B-meson decays mediated at the parton level by $b \to (u, c) + \ell\bar{\nu}_\ell$ ("semileptonic") and $b \to (d, s) + (\gamma, \bar{\ell}\ell)$ ("rare"). The latest lattice predictions provide nowadays decay constants and form factors with unprecedented precision, reducing considerably the uncertainties in purely leptonic decays $B \to \ell\bar{\nu}_\ell$ and $B_q \to \bar{\ell}\ell$ as well as exclusive $B \to (\pi, D^{(*)}) + \ell\bar{\nu}_\ell$ decays at high dilepton invariant mass. The latter provide more accurate determinations of $|V_{ub}|$ from exclusive final states. Further, the calculation of higher order short-distance corrections to $B \to X_c\ell\bar{\nu}_\ell$, $B_q \to \bar{\ell}\ell$, $B \to X_s\gamma$ and $B \to X_s\bar{\ell}\ell$ have lead to more accurate standard-model predictions as well as an updated determination of $|V_{cb}|$ from inclusive final states.

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1. Introduction

The importance and the role of so-called "semileptonic" and "rare" B-meson decays derive from the special mechanism of flavor changes as realised in the framework of the standard model (SM). The "semileptonic" B-meson decays, which are mediated at the parton level by $b \to (u,c) + \ell\bar{\nu}_\ell$ ($\ell = e, \mu, \tau$), proceed via the exchange of a charged heavy W-boson at tree-level and will be referred to as "tree" decays throughout. They allow to extract the moduli of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix, $|V_{ub}|$ and $|V_{cb}|$. On the other hand, "rare" B-meson decays, which are mediated by flavor-changing neutral currents (FCNC) $b \to (d,s) + (\gamma, \bar{\ell}\ell, \bar{\nu}\nu, \bar{q}q)$, can arise only at the 1-loop level and will be referred to as "FCNC" decays. They provide tests of the CKM picture at the loop level and offer an enhanced sensitivity to nonstandard effects due to the loop suppression, which implies in the SM for B-decays an additional GIM-enhancement [1] caused by the large value of the top-quark mass.

The decays of B-mesons can be described by an effective theory in complete analogy to Fermi's theory of the β-decay due to the large hierarchy among the external momenta of the order of the bottom-quark mass, $m_b \sim 4$ GeV, and the mass of the W-boson, $m_W \sim 80$ GeV, which mediates these tree and FCNC decays. The according effective Lagrangian involves effective dimension five and six operators O_i, and short-distance couplings $C_i(\mu)$

$$\mathscr{L}_{\text{eff}} \propto G_F V_{\text{CKM}} \sum_i C_i(\mu)\, O_i + \text{h.c.}, \tag{1.1}$$

where the latter are renormalized at the scale $\mu \sim m_b$. For convenience some power of Fermi's constant, G_F, and some combination of the involved CKM elements, V_{CKM}, are factored out of the C_i. In the SM only one $O \propto [\bar{q}\gamma_\mu(1-\gamma_5)b][\bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell]$ ($q = u, c$) arises for tree decays, whereas $|\Delta B| = 1$ FCNC decays are described by several types of operators: 4-quark operators of current-current and QCD- and QED-penguin type, dipole operators and semi-leptonic operators. In the SM, the according matching conditions at the scale $\mu \sim m_W$ [2] and their renormalization group evolution down to $\mu \sim m_b$ [3] are known nowadays at the next-to-next-to leading order (NNLO). This effective theory is the starting point for the evaluation of hadronic matrix elements for the various tree and FCNC B-meson decays.

Here, the main focus lies on some advances of the status of the theoretical methods for the calculation of hadronic matrix elements of tree and FCNC B-meson decays in the SM. Decays into purely leptonic final states are discussed in section 2, decays into exclusive final states are covered in section 3 and into inclusive final states $B \to X_c\ell\bar{\nu}_\ell$ as well as $B \to X_s + (\gamma, \bar{\ell}\ell)$ in section 4.

2. Leptonic decays $B^- \to \ell^-\bar{\nu}_\ell$ and $B_{d,s} \to \bar{\ell}\ell$

The purely leptonic tree and FCNC decays, $B^- \to \ell^-\bar{\nu}_\ell$ and $B_{d,s} \to \bar{\ell}\ell$, receive in the SM an additional helicity suppression, which leads to rare branching ratios scaling with the lepton mass m_ℓ. In consequence, the largest rates arise for tauonic ($\ell = \tau$) final states. Thanks to the helicity suppression, (pseudo-) scalar effective 4-Fermi couplings can be tested beyond the SM.

At leading order (LO) in QED, the only hadronic input in both decay modes is the B-meson decay constant, f_{B_q}, defined as $ip_\mu f_{B_q} \equiv \langle 0|\bar{q}\gamma_\mu\gamma_5 b|B(p)\rangle$. Nowadays it is determined in lattice QCD

(LQCD) calculations and thorough averages of various results are provided by the flavor lattice averaging group (FLAG) [4]. The current accuracy of the averages have reached remarkable 2%.

The theory uncertainty of the branching ratio of the tree decay is dominated by the one of f_{B_u}

$$\mathscr{B}(B^- \to \ell^- \bar{\nu}_\ell) = \frac{m_{B_u} m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_{B_u}^2}\right)^2 G_F^2 \, |V_{ub}|^2 f_{B_u}^2 \, \tau_B \tag{2.1}$$

at LO in QED, being the only theory uncertainty affecting the determination of $|V_{ub}|$ from a measurement. Depending on the exact value of $|V_{ub}|$ and f_{B_u}, the SM predictions are about $\mathscr{B}(B^- \to \ell^- \bar{\nu}_\ell) \approx (8 \times 10^{-12}, 3 \times 10^{-7}, 8 \times 10^{-5})$ for $\ell = e, \mu, \tau$, respectively, with uncertainties of 4% due to 2% accuracy of f_{B_u}. The current experimental upper bounds at 90% CL are $\mathscr{B}(B^- \to \ell^- \bar{\nu}_\ell) < (9.8 \times 10^{-7}, 1.0 \times 10^{-6})$ [5, 6] for $\ell = e, \mu$ and measurements for $\ell = \tau$ from Babar: $(17.9 \pm 4.8) \times 10^{-5}$ $(3.8\,\sigma)$ [6] and Belle: $(9.1 \pm 2.2) \times 10^{-5}$ $(4.6\,\sigma)$ [7]. An accuracy of about 4% on $|V_{ub}|$ for 50 ab^{-1} [9] can be expected from the Belle II experiment for $\ell = \tau$.

The CP-averaged time-integrated branching ratio [10] of the leptonic FCNC decay $B_q \to \bar{\ell}\ell$,

$$\overline{\mathscr{B}}(B_q \to \bar{\ell}\ell) = \frac{m_{B_q} m_\ell^2}{8\pi} \sqrt{1 - \frac{4 m_\ell^2}{m_{B_q}^2}} \left(\frac{G_F m_W}{\pi}\right)^4 |V_{tb} V_{tq}^*|^2 \, |C_{10}(\mu, x_t)|^2 \, \frac{f_{B_q}^2}{\Gamma_H^q}, \tag{2.2}$$

with $q = d, s$, depends on the CKM combination $|V_{tb} V_{tq}^*|^2$ and the short-distance coefficient C_{10}, where $x_t = (m_t/m_W)^2$ is the ratio of the top-quark and the W-boson masses. Here C_{10} incorporates corrections from all scales above m_b including NNLO in QCD [11] and NLO in electroweak interactions [12]. The most recent SM predictions of the various decays [13]

$$\begin{aligned}
\overline{\mathscr{B}}(B_s \to \bar{e}e) &= (8.54 \pm 0.55) \times 10^{-14}, & \overline{\mathscr{B}}(B_d \to \bar{e}e) &= (2.48 \pm 0.21) \times 10^{-15}, \\
\overline{\mathscr{B}}(B_s \to \bar{\mu}\mu) &= (3.65 \pm 0.23) \times 10^{-9}, & \overline{\mathscr{B}}(B_d \to \bar{\mu}\mu) &= (1.06 \pm 0.09) \times 10^{-10}, \\
\overline{\mathscr{B}}(B_s \to \bar{\tau}\tau) &= (7.73 \pm 0.49) \times 10^{-7}, & \overline{\mathscr{B}}(B_d \to \bar{\tau}\tau) &= (2.22 \pm 0.19) \times 10^{-8},
\end{aligned} \tag{2.3}$$

have uncertainties of 7% for $q = s$ and 9% for $q = d$, which are due to f_{B_q} and CKM elements. The latter are determined using the inclusive determination of $|V_{cb}|$ discussed in more detail in section 4. Concerning scales below $\mu \sim m_b$, the above result constitutes the LO in QED, however universal final-state radiation [14] is accounted for on the experimental side [15] and initial-state radiation can be safely neglected within the experimental signal windows [16]. For $\ell = \mu$, the experimental averages of CMS and LHCb of their full LHC Run I data sets [15] are $\overline{\mathscr{B}}(B_s \to \bar{\mu}\mu) = (2.8^{+0.7}_{-0.6}) \times 10^{-9}$ and $\overline{\mathscr{B}}(B_d \to \bar{\mu}\mu) = (3.9^{+1.6}_{-1.4}) \times 10^{-10}$ with a statistical significance of $6.2\,\sigma$ and $3.2\,\sigma$, respectively. Only upper bounds exist for $\ell = e, \tau$.

3. Exclusive decays $B \to M + \ell\bar{\nu}_\ell$ and $B \to M + (\gamma, \bar{\ell}\ell)$

The exclusive decays with at least 3-body final states, or even 4-body final states in the case of subsequent decays $M \to P_1 P_2$, are phenomenologically interesting since they do not suffer from helicity suppression as the purely leptonic decays. Moreover, they provide in principle several observables in angular distributions, which give access to different 4-Fermi effective interactions.

The exclusive tree decays $B \to M + \ell\bar{\nu}_\ell$ are theoretically very simple, since they do not involve contributions due to electro-magnetic currents. The hadronic transition is described by the

FF	method	q^2-region	uncertainty	Ref.	CKM
		$B \to \pi$			$\lvert V_{ub}\rvert \times 10^3$
f_+	LCSR	$q^2 < 10\,\mathrm{GeV}^2$	$\approx 7\,\%$	[18]	$3.32^{+0.26}_{-0.22}$
$f_{+,0}$	LQCD	$19\,\mathrm{GeV}^2 < q^2$	$8-14\,\%$	[19]	3.61 ± 0.32
$f_{+,0}$	LQCD	$20\,\mathrm{GeV}^2 < q^2$	$\approx 4\,\%$	[20]	3.72 ± 0.16
		$B \to \rho, \omega$			$\lvert V_{ub}\rvert \times 10^3$
V, A_i, T_j	LCSR	$q^2 < 14\,\mathrm{GeV}^2$	≈ 10 & $14\,\%$	[21]	6 & 9 % th. err.
		$B \to D$			$\lvert V_{cb}\rvert \times 10^3$
$f_{+,0}$	LCSR	$q^2 < 6\,\mathrm{GeV}^2$	$\approx 27\,\%$	[22]	–
$f_{+,0}$	LQCD	$8.5\,\mathrm{GeV}^2 < q^2$	$\approx 1.5\,\%$	[23]	39.6 ± 1.7
$f_{+,0}$	LQCD	$9.5\,\mathrm{GeV}^2 < q^2$	$\approx 5\,\%$	[24]	40.2 ± 2.1
		$B \to D^*$			$\lvert V_{cb}\rvert \times 10^3$
V, A_i	LCSR	$q^2 < 6\,\mathrm{GeV}^2$	$\approx 27\,\%$	[22]	–
$\mathscr{F}(1)$	LQCD	$q^2 = q^2_{\max}$	$1.4\,\%$	[25]	39.04 ± 0.75

Table 1: Compilation of some recent results on $B \to M$ form factors (FF) from LCSR and LQCD. The approximate q^2-region of the validity and the uncertainty of the FF's are given as well as the values of $|V_{ub}|$ and $|V_{cb}|$, as extracted from data of $B \to M\ell\bar{\nu}_\ell$ decays in the according references.

according heavy-to-light $B \to M$ form factors (FF) for experimentally easy accessible final states $M = \pi, D^{(*)}$, whereas off-resonance $M \to P_1 P_2$ decays are in the phase of exploration, see for example $B \to (\pi\pi) + \ell\bar{\nu}_\ell$ [17]. At low $(\ell\bar{\nu}_\ell)$-invariant masses, q^2, these FF's can be calculated with the help of light-cone sum rules (LCSR) and at high q^2 with the help of LQCD. A list of contemporary results is given in table 1, which shows the progress of LQCD in recent times, where FF uncertainties are now below 5 % for $B \to \pi, D$. Thus, accurate branching-ratio measurements restricted to the high-q^2 region allow for rather precise determinations of $|V_{ub}|$ and $|V_{cb}|$ in exclusive decays, as for example at the 4 % level for $|V_{ub}|$ from $B \to \pi\ell\bar{\nu}_\ell$ [20]. Precise LQCD predictions over the whole high-q^2 region would be also welcome for $B \to D^*$, where currently only results at zero recoil $q^2 = q^2_{\max}$ are available [25], which require an extrapolation of experimental data based on a parameterization of FFs [26] in order to determine $|V_{cb}|$. The according value is currently about $3\,\sigma$ below the inclusive determination discussed in section 4.

Babar [27], Belle [28, 29] and LHCb [30, 31] provide also measurements of the ratios $R(D^{(*)}) \equiv \mathscr{B}(B \to D^{(*)}\tau\bar{\nu}_\tau)/\mathscr{B}(B \to D^{(*)}\ell\bar{\nu}_\ell)$ of tauonic final states over light lepton final states $\ell = e, \mu$. In the SM a substantial cancellation of uncertainties due to FFs takes place in the ratio, which are integrated over the whole available phase space accessible in each decay. Latest LQCD results yield $R(D) = 0.299 \pm 0.011$ [23] and $R(D) = 0.300 \pm 0.008$ [24], whereas one has to resort to the heavy quark limit for $R(D^*) = 0.252 \pm 0.003$ [32]. The experimental results from Babar, Belle and LHCb are compatible with each other and systematically above the SM prediction by $3\,\sigma$, less than $2\,\sigma$ and around $2\,\sigma$, respectively.

The most interesting FCNC decays with exclusive final states are $B \to K^{(*)}\bar{\ell}\ell$ due to large rates, but also $B_s \to \phi\bar{\ell}\ell$ and $B \to \pi\bar{\ell}\ell$ are studied. Dedicated factorization methods based on $1/m_b$ expansions have to be applied at low q^2 [33] and high q^2 [34] due to the variety of operators that contribute. The recent progress on LQCD predictions of FF's [35, 36, 20] and the suppression of

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subleading contributions allow rather precise predictions at high q^2, if duality violating effects are minimised by sufficiently large bins in q^2. In this case, the dominating uncertainties are due to FF's of about $(12-20)$ % for $B \to K^{(*)}\bar{\ell}\ell$ branching ratios. A variety of not yet measured ratios of angular observables in $B \to K^*\bar{\ell}\ell$ have been proposed, with strongly reduced uncertainties due to FF's and that allow to test duality violation and to discriminate among nonstandard effects [37].

In view of a preference of new physics effects in the Wilson coefficient C_9 by global fits[1] [38, 39], special attention was devoted to the study of subleading contributions in $1/m_b$ at low q^2. These unknown contributions are either due to 1) the use of FF-relations, 2) the $b \to s + (\gamma, \bar{\ell}\ell)$ amplitude itself, and 3) the resonance contributions due to $\bar{c}c$ and light quarks $\bar{q}q$ ($q = u, d, s$). They are usually parameterized [41, 42, 39, 21] and the choice of the size of the parameters is based on dimensional arguments [42, 39], in the case of FF-relation breaking contribution also on the spread from different models of FF calculations [41] and for resonance contributions on the explicit study for $B \to (\pi, K, K^*) + \bar{\ell}\ell$ [43]. Further, the choice of the q^2-dependence of the central values of FF's follows either the heavy quark mass limit [41] or the LCSR result. Another important difference throughout the literature concerns the combination of theory uncertainties, ranging from adding in quadrature uncertainties of groups of parameters, to the determination of $1\,\sigma$ CL intervals assuming prior distributions for parameters, to scans over the entire parameter space determining the maximal possible spread in each observable. In consequence, quite different results are obtained, especially for the so-called "optimized" observable P_5' [44], where the central values and uncertainties of [41] deviate from other groups [38, 39, 40, 42, 21], which predict a larger discrepancy of the SM with the LHCb measurements [45]. In this respect, the interested reader has to take care of substantial fine print due to the nonuniform treatment of subleading contributions and sources of uncertainties.

4. Inclusive decays

The tree and FCNC B-meson decays with inclusive final states are complementary probes of the CKM-picture of the SM and nonstandard contributions to the ones with exclusive final states. The channels that received most consideration are $B \to X_{u,c}\,\ell\bar{\nu}_\ell$ and $B \to X_{d,s} + (\gamma, \bar{\ell}\ell)$. The sum over the inclusive hadronic final state X permits to relate $\sum_X \langle B|\mathscr{L}_{\rm eff}^\dagger|X\rangle\langle X|\mathscr{L}_{\rm eff}|B\rangle \simeq \mathrm{Im}\langle B|\mathrm{T}\{\mathscr{L}_{\rm eff}^\dagger \mathscr{L}_{\rm eff}\}|B\rangle$ to the imaginary part of B-meson forward-scattering amplitude, which depends only on the long-distance dynamics of the initial B-meson. The latter can be treated in a local OPE, the so-called heavy quark expansion (HQE), thanks to the hierarchy of the typical energy release $\sim m_b$ and the hadronic scale $\Lambda_{\rm QCD}$ [46]. The matrix elements of the dimension three operators correspond to the free-quark decay, whereas subleading contributions arise only at dimension five with matrix elements of the local operators $\mu_\pi^2 \propto \langle B|\bar{b}_v(\vec{D})^2 b_v|B\rangle$ and $\mu_G^2(\mu) \propto \langle B|\bar{b}_v\sigma_{\mu\nu}G^{\mu\nu}b_v|B\rangle$.

Phase space cuts, required by experiments to suppress backgrounds, lead to less inclusive measurements, which invalidate in principle the applicability of theory predictions based on HQE. They introduce additional scales into the theoretical description that require different factorization techniques[2], which lead to non-local nonperturbative objects, so-called shape functions, which are

[1] It must be noted that a model comparison via Bayes factors favors the SM over new physics extensions due to the lower dimensional parameter space [40].

[2] This so-called endpoint region can be treated within soft-collinear effective theory (SCET) for $B \to X_u\ell\bar{\nu}_\ell$ [47], $B \to X_s\gamma$ [48] and $B \to X_s\bar{\ell}\ell$ [49].

universal at leading order for several processes. Otherwise, experiments extrapolate beyond cuts using models, as for example for $B \to X_{d,s} + (\gamma, \bar{\ell}\ell)$, introducing some model dependence into measurements. In the following the focus will be on theory predictions based on the local OPE.

The most recent advances for $B \to X_c \ell \bar{\nu}_\ell$ are reviewed in [50] and include nowadays various contributions in the double expansion in $\Lambda_{\rm QCD}/m_b$ and $a_s = \alpha_s/(4\pi)$ of quantities of interest

$$\begin{aligned} M_i = M_i^{(0)} + a_s M_i^{(1)} + a_s^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + a_s M_i^{(\pi,0)}\right)\frac{\mu_\pi^2}{m_b^2} + \left(M_i^{(G,0)} + a_s M_i^{(G,0)}\right)\frac{\mu_G^2}{m_b^2} \\ + M_i^{(D)}\frac{\rho_D^3}{m_b^3} + M_i^{(LS)}\frac{\rho_{LS}^3}{m_b^3} + \mathscr{O}\left(a_s^3, \frac{a_s^2}{m_b^2}, \frac{a_s}{m_b^3}, \frac{1}{m_b^4}, \frac{1}{m_b^3 m_c^2}\right), \end{aligned} \tag{4.1}$$

like branching ratios as well as moments in lepton energy and hadronic masses. The $M_i^{(j)}$ depend in general on quark masses m_b, m_c, the renormalization scale μ as well as the choice of renormalisation scheme, and the cut on the lepton energy $E_{\rm cut}$. The whole approach allows to determine $|V_{cb}| = (42.21 \pm 0.78) \times 10^{-3}$ with an accuracy of 2 % from experimental and theoretical uncertainties [51]. The fit utilises data of total and partial widths as well as moments from various experiments and some external input on m_c and μ_G.

The inclusive decays $B \to X_{d,s}\gamma$ provide currently the most sensitive probe of the Wilson coefficient of the dipole operators $O_7 \propto m_b[\bar{q}\sigma_{\mu\nu}P_R b]F^{\mu\nu}$, $(q = d, s)$, and it's chirality-flipped counter part via their branching ratios $\mathscr{B}_{q\gamma} \equiv \mathscr{B}(B \to X_q\gamma) \propto (|C_7|^2 + |C_7'|^2)$. The rate consists of contributions that could be improved systematically in the past, but also a currently irreducible uncertainty of 5 % due to subleading power corrections [52]. A new update of NNLO short-distance corrections to $B \to X_{d,s}\gamma$ [53] yields SM predictions

$$\begin{aligned} \mathscr{B}_{s\gamma}|_{\rm SM} &= (3.36 \pm 0.23) \times 10^{-4}, & \mathscr{B}_{s\gamma}|_{\rm Exp} &= (3.43 \pm 0.22) \times 10^{-4} \\ \mathscr{B}_{d\gamma}|_{\rm SM} &= (1.73^{+0.12}_{-0.22}) \times 10^{-5}, & \mathscr{B}_{d\gamma}|_{\rm Exp} &= (1.41 \pm 0.57) \times 10^{-5} \end{aligned} \tag{4.2}$$

in good agreement with experimental world averages for photon energies $E_\gamma > 1.6$ GeV. The upwards shift of $\mathscr{B}_{s\gamma}|_{\rm SM}$ by around 6.4 % originates mainly from fixing the $m_c = 0$ boundary and including the complete NNLO BLM correction to 3- and 4-body final state channels. The uncertainty budget is due to nonperturbative (5 %), lacking higher order (3 %) and m_c-interpolation of the NNLO (3 %) corrections, as well as of parametric origin (2 %).

Another important inclusive FCNC decay is $B \to X_s\bar{\ell}\ell$, which provides at LO in QED an angular distribution in the dilepton-invariant mass q^2 and the angle θ_ℓ

$$\frac{8}{3}\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} = \left(1 + \cos^2\theta_\ell\right)H_T(q^2) + 2\left(1 - \cos^2\theta_\ell\right)H_L(q^2) + 2\cos\theta_\ell H_A(q^2). \tag{4.3}$$

The decay rate and the normalized lepton forward-backward asymmetry are related to the angular observables as $\Gamma = (H_L + H_T)$ and $A_{\rm FB} = 3/4\, H_A/\Gamma$, respectively. The angular observables have different dependences on the most important Wilson coefficients $C_{7,9,10}$. Recently, the NLO QED corrections where finalized for the fully-differential decay width [54] at low and high q^2. The simple $\cos\theta_\ell$-dependence of (4.3) becomes modified at NLO QED and two additional angular observables $H_{3,4}$ can be measured in principle. Combining NNLO QCD and NLO QED results, the theory uncertainties of $\mathscr{B}$ and $H_{L,T}$ are about $(6-9)\,\%$ in $q^2 \in [1, 3.5], [3.5, 6], [1, 6]$ GeV^2, whereas at high q^2 uncertainties of $\mathscr{B}$ are about 30%. The uncertainty of H_A ranges between $(5-70)\,\%$ depending strongly on q^2-binning around its zero-crossing. The latter can be predicted with an accuracy of $\lesssim 4\,\%$. The QED corrections lead to pronounced differences for $\ell = e$ and $\ell = \mu$, which are smallest for the rates at low q^2 with 4%, but larger for the angular observables $H_{L,T,A,3,4}$.

5. Conclusions

The recent advances in lattice calculations of B-meson decay constants and form factors in combination with the completion of several higher-order short-distance corrections have advanced the theoretical status of tree and FCNC B-meson decays.

Purely leptonic decays are nowadays quite precisely known thanks to recent developments in lattice predictions of decay constants and in the case of FCNC decays $B_{d,s} \to \bar{\ell}\ell$ in addition also NLO-electroweak and NNLO-QCD corrections have been completed. The precision on rates are nowadays at the 4% level for $B \to \tau\bar{\nu}_\tau$ (without CKM uncertainty) and about $(7-9)\,\%$ for $B_{d,s} \to \bar{\ell}\ell$ (including CKM uncertainty). Although numerically important NLO QED corrections are taken into account, isospin and virtual low-energy QED effects have to be considered in order to gain further in accuracy.

Similar comments apply to $b \to (u,c) + \ell\bar{\nu}_\ell$ decays into exclusive final states at high dilepton momenta, again due to the progress in lattice predictions of the relevant $B \to (\pi, D)$ form factors. Most recent determinations of $|V_{ub}|$ from $B \to \pi\ell\bar{\nu}_\ell$ reach the 4% accuracy. On the other hand, the structure of FCNC decays $B \to K^{(*)}\bar{\ell}\ell$ is more involved due to long-distance contributions beyond form factors. The very distinct factorization approaches at low and high dilepton invariant mass imply unknown subleading corrections with substantial size. At high dilepton invariant mass form factor uncertainties are small thanks to lattice and subleading corrections are suppressed, however in addition, duality violation might be sizeable.

Inclusive decays are treated with complementary theoretical methods and provide important cross checks, on the other hand they are experimentally more challenging. The inclusive tree decay $B \to X_c\ell\bar{\nu}_\ell$ can be treated within a local OPE and the latest determination of $|V_{cb}|$ reaches uncertainties at level of 2%. The inclusive FCNC decay $B \to X_s\gamma$ is nowadays known with 7% accuracy and its measurement provides strong constraints on models beyond the SM. The theoretical status of $B \to X_s\bar{\ell}\ell$ has reached a similar accuracy, including also NLO QED corrections and is awaiting improved measurements from Belle II for $\ell = e, \mu$.

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