

Fermi-Boltzmann statistics of neutrinos and relativistic effective degrees of freedom in the early universe

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We investigate the effects of the presence of non-pure fermionic neutrinos on the relativistic effective degrees of freedom in the early universe. The statistics of neutrinos is transformed continuously from Fermi-Dirac to Maxwell-Boltzmann statistics. We show that the relativistic degrees of freedom decreases with the deviation from pure Fermi-Dirac statistics of neutrinos if there are constant and large lepton asymmetries.

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1. Introduction

Neutrinos have spin 1/2 and it is natural that we understand the neutrinos obey purely Fermi-Dirac statistics on the analogy. However, the possibility of the non-pure fermionic neutrinos is not excluded [1, 2]. Although the possibility of the pure bosonic neutrinos is excluded by double-beta decay [3], it does not mean that the neutrinos are pure fermions. Moreover, properties of neutrinos are so different from other spin one half particles, e.g., tiny masses, large mixings, electrically neutral. Thus, the Pauli exclusion principle may be violated for neutrinos and they may possess mixed statistics.

In this paper, we investigate the effect of the presence of non-pure fermionic neutrinos on the relativistic effective degrees of freedom at temperature $T = 0.5 - 500$ GeV (this paper is based on Ref.[4]). The assumption of violation of the Pauli principle makes problems but we put aside discussion of these problems and we restrict ourselves to the phenomenological approach. The large lepton asymmetries at MeV scale are almost excluded by the standard BBN cosmology [5]. However, the large lepton asymmetries at GeV scale are compatible with current observations [6]. We perform the similar analysis as Ref.[6] but we introduce the Fermi-Bose parameter [7] to relax the statistics of neutrinos. We show that the relativistic degrees of freedom decreases with deviation from pure Fermi-Dirac statistics of neutrinos if there are constant and large lepton asymmetries at GeV scale universe.

2. Statistics of neutrinos and relativistic effective degrees of freedom

The distribution function of particle species i is given by

$$f_i = \frac{g_i}{e^{(E-\mu_i)/T} + \kappa_i}, \quad (2.1)$$

where g_i , E , μ_i and T denote number of internal degrees of freedom, energy, chemical potential and temperature of particle i , respectively. The parameter κ_i (we call it the Fermi-Bose parameter [7]) describes the continuous transition either from the Fermi-Dirac ($\kappa_i = 1$) to the Bose-Einstein ($\kappa_i = -1$) via the Maxwell-Boltzmann ($\kappa_i = 0$) distributions. We assume that the spin one half particles i obey "Fermi-Boltzmann" distribution with $0 \leq \kappa_i \leq 1$.

The number density n_i , energy density ρ_i , pressure P_i and entropy density s_i of particle species i are obtained as follows [8]:

$$n_i = \frac{1}{2\pi^2} \int_{m_i}^{\infty} E(E^2 - m_i^2)^{1/2} f_i dE, \quad \rho_i = \frac{1}{2\pi^2} \int_{m_i}^{\infty} E^2(E^2 - m_i^2)^{1/2} f_i dE, \quad (2.2)$$

$$P_i = \frac{1}{6\pi^2} \int_{m_i}^{\infty} (E^2 - m_i^2)^{3/2} f_i dE, \quad s_i = \frac{\rho_i + P_i - \mu_i n_i}{T}, \quad (2.3)$$

According to Stuke, et.al.[6], we define the general relativistic effective degrees of freedom $g_* = g_*(m_i, \kappa_i, \mu_i)$ for any m_i , κ_i and μ_i as follows

$$g_* = \frac{30}{\pi^2 T^4} \rho, \quad (2.4)$$

where $\rho = \sum_i \rho_i(m_i, \kappa_i, \mu_i)$.

To estimate the total energy density ρ , the chemical potentials of all particle species μ_i should be determined [6]. The beta-decay of down quark d via weak interactions provides $\mu_u + \mu_f = \mu_d + \mu_{\nu_f}$ where $f = e, \mu, \tau$. We assume that $\mu_u = \mu_c = \mu_t$ and $\mu_d = \mu_s = \mu_b$ for quarks. All gauge bosons and Higgs boson have vanishing chemical potentials. The five independent chemical potentials $\mu_{\nu_e}, \mu_{\nu_\mu}, \mu_{\nu_\tau}, \mu_u, \mu_d$ are uniquely determined by the following five conservation laws:

$$sq = - \sum_{i=e,\mu,\tau} n_i + \frac{2}{3} \sum_{i=u,c,t} n_i - \frac{1}{3} \sum_{i=d,s,b} n_i, \quad sb = \frac{1}{3} \sum_{i=\text{quarks}} n_i, \quad sl_f = n_f + n_{\nu_f}, \quad (2.5)$$

where $s = \sum_i s_i$ and q denote total entropy density and electric charge of the universe, respectively. Here, baryon number b and lepton flavour number ℓ_f are defined as

$$b = \frac{n_b - n_{\bar{b}}}{s}, \quad \ell_f = \frac{n_f - n_{\bar{f}} + n_{\nu_f} - n_{\bar{\nu}_f}}{s}, \quad (2.6)$$

where n_b and $n_{\bar{b}}$ denote the number densities of baryons and anti-baryons, respectively. From charge neutrality of the universe, we assume $q = 0$. The baryon number is fixed to $b = 9 \times 10^{-11}$ [9]. The neutrino oscillations may ensure the equilibration of the lepton flavour numbers in the early universe [10], we assume that all lepton flavour numbers are the same $\ell = \ell_e = \ell_\mu = \ell_\tau$. Since the measurements of the lepton flavour numbers in the early universe are not established, we keep ℓ as free parameters. We assume that all neutrino masses are same $m_\nu = 1$ eV [11].

The Fermi-Bose parameters κ_i of standard model particles i are assigned as follows: $0 \leq \kappa_\nu \leq 1$ for neutrinos ν_f (we assume that $\kappa_\nu = \kappa_{\nu_e} = \kappa_{\nu_\mu} = \kappa_{\nu_\tau}$), $\kappa_\ell = \kappa_q = +1$ for charged leptons ℓ and quarks q , $\kappa_{\gamma,W,Z,g} = \kappa_H = -1$ for gauge bosons γ, W, Z, g and Higgs boson H . In this case

$$g_* = g_*^{m_i=\mu_i=0} + \Delta g_*(\kappa_\nu = 1, \mu_i \neq 0) + \Delta g_*(\kappa_\nu \neq 1, \mu_i \neq 0), \quad (2.7)$$

where $g_*^{m_i=\mu_i=0} = \sum_{i=\text{bosons}} g_i + (7/8) \sum_{i=\text{fermions}} g_i$ denotes well known relativistic effective degrees of freedom [8] and $\Delta g_*(\kappa_\nu = 1, \mu_i \neq 0)$ denotes the deviation from $g_*^{m_i=\mu_i=0}$ due to the non-zero lepton asymmetries which has been reported [6]. The 3rd term in R.H.S in Eq.(2.7), $\Delta g_*(\kappa_\nu \neq 1, \mu_i \neq 0)$ is the new contribution on the relativistic effective degrees of freedom g_* .

3. Numerical estimation

Figure 1 shows the dependence of the relativistic effective degrees of freedom g_* on temperature T . The left figure shows the numerical solution for $\ell = 0, \ell = 0.01$ and $\ell = 0.03$ cases. There are three curves correspond to $\kappa_\nu = 1, 0.5, 0$ for $\ell = 0.03$. For $\ell = 0.01$ and $\ell = 0$, these three curves almost overlap with each other. The right figure shows the $\ell = 0$ case in more detail. The energy density of pure bosons ($\kappa_i = -1$) is larger than the energy density of pure fermions ($\kappa_i = 1$) by the factor $8/7$ [8]. Thus one may expect, if neutrinos have non-pure Fermi-Dirac statistics ($\kappa_\nu \neq 1$), their larger energy density would correspond to an increase of the effective degrees of freedom g_* in Eq.(2.4). Indeed, if there is no lepton asymmetries (right figure), relativistic effective degrees of freedom for pure Maxwell-Boltzmann neutrinos case is larger than it for pure Fermi-Dirac neutrinos case; $g_*^{\ell=0, \kappa_\nu=0} > g_*^{\ell=0, \kappa_\nu=1}$. However, if there is large lepton asymmetries, such as $\ell = 0.03$ (left figure), the relativistic effective degrees of freedom for pure Maxwell-Boltzmann neutrinos case is smaller than it for pure Fermi-Dirac neutrinos case; $g_*^{\ell=0.03, \kappa_\nu=0} < g_*^{\ell=0.03, \kappa_\nu=1}$.

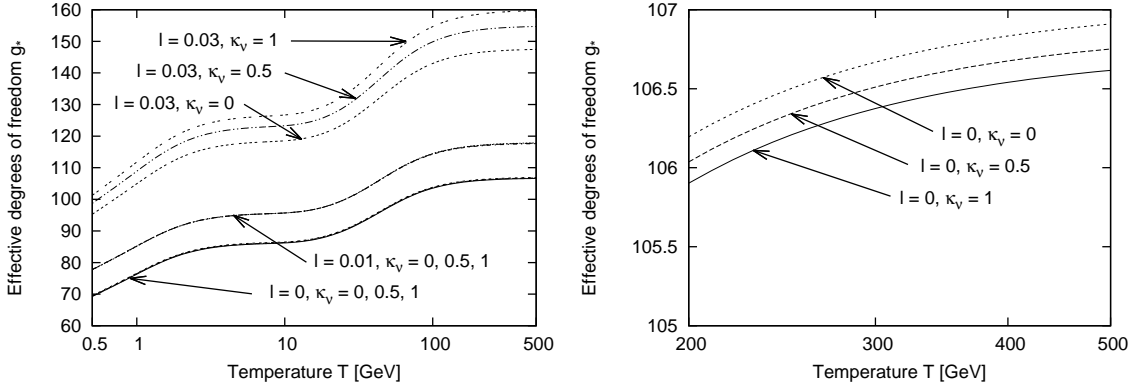


Figure 1: Dependence of the relativistic effective degrees of freedom g_* on temperature T . The left figure shows the numerical solution for $\ell = 0$, $\ell = 0.01$ and $\ell = 0.03$ cases. The right figure shows the $\ell = 0$ case in more detail.

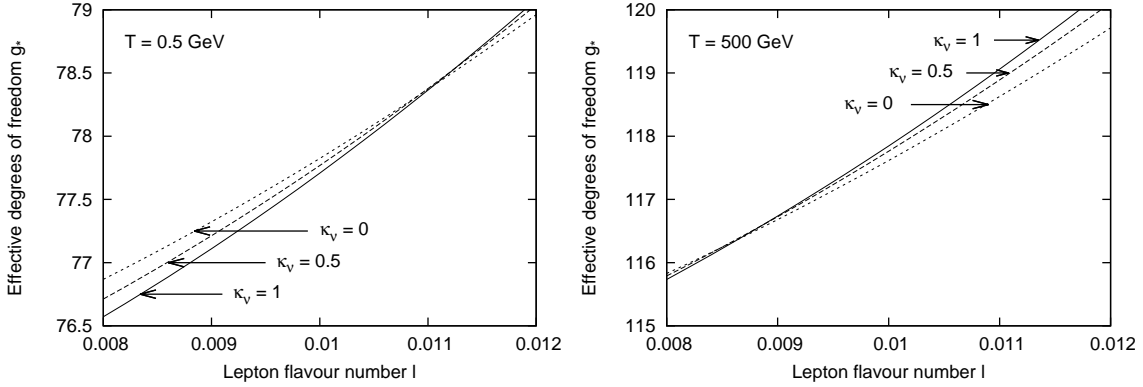


Figure 2: The dependence of the relativistic effective degrees of freedom g_* on lepton flavour number ℓ for $T = 0.5$ GeV (left) and for $T = 500$ GeV (right).

Figure 2 shows the dependence of the relativistic effective degrees of freedom g_* on lepton flavour number ℓ . The left figure shows the result for $T = 0.5$ GeV while the right figure shows the same but for $T = 500$ GeV. In the case of $T = 0.5$ GeV, $g_*^{\kappa_\nu=1}$ and $g_*^{\kappa_\nu=0}$ are equivalent at $\ell^{eq} \sim 0.0112$. In the case of $T = 500$ GeV, the equivalent lepton flavour number is $\ell^{eq} \sim 0.0087$. In the range of $0.5 \leq T[\text{GeV}] \leq 500$, the equivalent lepton flavour number to be $0.0087 \lesssim \ell^{eq} \lesssim 0.0112$ and ℓ^{eq} decreases with the increasing temperature T .

Recall that the relativistic effective degrees of freedom g_* increases with the energy density ρ and this energy density increases with distribution function $f_i = g_i/[e^{(E-\mu_i)/T} + \kappa_i]$. If there is no lepton asymmetry ($\ell = 0$) then the chemical potential of neutrinos vanish $\mu_\nu = 0$ and the small κ_ν induces large f_ν as well as large g_* . Moreover, if there are non-zero lepton asymmetry $\ell \neq 0$, the small κ_ν still induces large g_* as long as the chemical potential μ_ν is constant. In other words, if the chemical potential μ_ν is independent of κ_ν and is constant, the relation of $g_*^{\mu_\nu=const, \kappa_\nu=0} > g_*^{\mu_\nu=const, \kappa_\nu=1}$ is obtained. However, if the Fermi-Bose parameter κ_ν change continuously from

$\kappa_\nu = 1$ to $\kappa_\nu \neq 1$, even if the lepton flavour number is constant, the chemical potential μ_ν is no longer constant with κ_ν . The small κ_ν induces large f_ν while the small μ_ν induces small f_ν . In this case the relation of $g_*^{\mu_\nu=const, \kappa_\nu=0} > g_*^{\mu_\nu=const, \kappa_\nu=1}$ is no longer guaranteed and $g_*^{\mu_\nu \neq const, \kappa_\nu=0} < g_*^{\mu_\nu \neq const, \kappa_\nu=1}$ may be allowed. If the effect of μ_ν on f_ν is dominant, the energy density will be reduced. This is the reason why we encounter $g_*^{\ell=0.03, \kappa_\nu=0} < g_*^{\ell=0.03, \kappa_\nu=1}$ in the Figure 1.

4. Conclusions

We have investigated the effect of the presence of non-pure fermionic neutrinos on the relativistic effective degrees of freedom at temperature 0.5 – 500 GeV. It turned out that the relativistic degrees of freedom decreases with deviation from pure Fermi-Dirac statistics of neutrinos if there are constant and large lepton asymmetries at GeV scale universe.

The relativistic effective degrees of freedom increases with the energy density and this energy density increases with distribution function of particle species i ; $f_i = g_i/[e^{(E-\mu_i)/T} + \kappa_i]$. If the Fermi-Bose parameter κ_ν change continuously from $\kappa_\nu = 1$ to $\kappa_\nu \neq 1$, even if the lepton flavour number is constant, the chemical potential μ_ν is no longer constant with κ_ν . The small κ_ν induces large f_ν while the small μ_ν induces small f_ν . If the effect of μ_ν on f_ν is dominant, the energy density and relativistic effective degrees of freedom are reduced.

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