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Kaon Decay into Three Photons Revisited

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We take another look at the rare kaon decay into three photons. Specifically, after imposing the requirements of gauge invariance and Bose symmetry, we derive a general form of the decay amplitude, including both parity-conserving and parity-violating contributions. Subsequently, we adopt a chiral-Lagrangian approach in conjunction with dimensional analysis arguments to estimate the branching ratios of $K_{\rm L,S} \to 3\gamma$ in the standard model, obtaining values as large as $\mathscr{B}(K_{\rm L} \to 3\gamma) \simeq 7\times10^{-17}$ and $\mathscr{B}(K_{\rm S} \to 3\gamma) \simeq 1\times10^{-19}$, which exceed those found previously by a few orders of magnitude. Measurements of $\mathscr{B}(K_{\rm L,S} \to 3\gamma)$ substantially bigger than these numbers would likely hint at the presence of new physics beyond the standard model.

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The kaon decays into three photons, $K_{\rm L}\to 3\gamma$ and $K_{\rm S}\to 3\gamma$, can happen in the absence of *CP* violation. Based on the experimental branching ratio $\mathscr{B}(K_{\rm L}\to 2\gamma)\simeq 5.5\times 10^{-4}$ [1], one might then naively expect that $\mathscr{B}(K_{\rm L}\to 3\gamma)\sim \alpha_{\rm em}\mathscr{B}(K_{\rm L}\to 2\gamma)\sim 4\times 10^{-6}$. However, this is already way higher than the existing measured limit $\mathscr{B}(K_{\rm L}\to 3\gamma)<7.4\times 10^{-8}$ [1, 2]. As for its $K_{\rm S}$ counterpart, there is currently no empirical information available about it, but its rate is likely to be more suppressed than expected as well.

The considerable smallness of the $K\to 3\gamma$ rate turns out to stem from the conditions imposed on the decay amplitude by gauge invariance and Bose symmetry [3]. Gauge invariance compels the total angular momentum J of any two photons in the 3γ final-state to be nonzero, while Bose statistics disallows the $\gamma\gamma$ pair having $J=1$. Since each pair of the photons has $J\geq 2$, the amplitude suffers from a sizable number of angular-momentum suppression factors.

The $K_{\rm L,S}\to 3\gamma$ rates were first estimated over 2 decades ago in [3], using a simple model in which $K\to 3\gamma$ proceeds from $K\to\pi^0\pi^0\gamma$ with $\pi^0\pi^0$ immediately converting into $\gamma\gamma$. This led to $\mathscr{B}(K_{\rm L}\to 3\gamma)\sim 3\times 10^{-19}$ and $\mathscr{B}(K_{\rm S}\to 3\gamma)\sim 5\times 10^{-22}$ [3]. As this rough determination relied on only 1 diagram, possibly other contributions exist that can enhance the rates. Here we present the results of a more recent study [4] revisiting these decays and attaining much higher numbers.

The $K\to 3\gamma$ amplitude generally consists of two terms describing the parity conserving (PC) and parity violating (PV) components of the transition, namely

$$\mathscr{M}(K\to 3\gamma)=\mathscr{M}^K_{\rm PC}+\mathscr{M}^K_{\rm PV}\,,\qquad \mathscr{M}^K_{\rm PC}=\varepsilon^*_{1\alpha}\varepsilon^*_{2\eta}\varepsilon^*_{3\mu}M^{\alpha\eta\mu}_{\rm PC}\,,\quad \mathscr{M}^K_{\rm PV}=\varepsilon^*_{1\alpha}\varepsilon^*_{2\eta}\varepsilon^*_{3\mu}M^{\alpha\eta\mu}_{\rm PV}\,,\tag{1}$$

where $\varepsilon_{1,2,3}$ are the photon polarization vectors. Each of $\mathscr{M}^K_{\rm PV,PC}$ has to respect gauge invariance and be symmetric under interchange of any two of the photons. As discussed in detail in [4], imposing these requirements with on-shell photons, after some algebra we arrive at[1]

$$\begin{aligned}M^{\alpha\eta\mu}_{\rm PV} &= \big[g^{\alpha\eta}\big(k_1^\mu y-k_2^\mu x\big)+g^{\eta\mu}\big(k_2^\alpha x-k_3^\alpha z\big)+g^{\alpha\mu}\big(k_3^\eta z-k_1^\eta y\big)+k_3^\alpha k_1^\eta k_2^\mu-k_2^\alpha k_3^\eta k_1^\mu\big]G(x,y,z)\\ &\quad+\big(g^{\alpha\eta}z-k_2^\alpha k_1^\eta\big)\big(k_1^\mu y-k_2^\mu x\big)F(x,y,z)+\big(g^{\eta\mu}y-k_3^\eta k_2^\mu\big)\big(k_2^\alpha x-k_3^\alpha z\big)F(z,x,y)\\ &\quad+\big(g^{\alpha\mu}x-k_3^\alpha k_1^\mu\big)\big(k_3^\eta z-k_1^\eta y\big)F(y,z,x)\,,\end{aligned}\tag{2}$$

$$\begin{aligned}M^{\alpha\eta\mu}_{\rm PC} &= \big(g^{\alpha\eta}\varepsilon^{\mu\rho\sigma\tau}+g^{\rho\sigma}\varepsilon^{\alpha\eta\mu\tau}+g^{\eta\rho}\varepsilon^{\alpha\mu\sigma\tau}-g^{\alpha\sigma}\varepsilon^{\eta\mu\rho\tau}+g^{\eta\mu}\varepsilon^{\alpha\rho\sigma\tau}+g^{\sigma\tau}\varepsilon^{\alpha\eta\mu\rho}+g^{\mu\sigma}\varepsilon^{\alpha\eta\rho\tau}\\ &\quad-g^{\eta\tau}\varepsilon^{\alpha\mu\rho\sigma}+g^{\alpha\mu}\varepsilon^{\eta\rho\sigma\tau}+g^{\rho\tau}\varepsilon^{\alpha\eta\mu\sigma}+g^{\mu\rho}\varepsilon^{\alpha\eta\sigma\tau}-g^{\alpha\tau}\varepsilon^{\eta\mu\rho\sigma}\big)k_{1\rho}k_{2\sigma}k_{3\tau}\mathscr{G}(x,y,z)/3\\ &\quad+\big[\big(g^{\alpha\eta}z-k_2^\alpha k_1^\eta\big)\varepsilon^{\mu\rho\sigma\tau}\mathscr{F}(x,y,z)+\big(g^{\eta\mu}y-k_3^\eta k_2^\mu\big)\varepsilon^{\alpha\rho\sigma\tau}\mathscr{F}(z,x,y)\\ &\qquad+\big(g^{\alpha\mu}x-k_3^\alpha k_1^\mu\big)\varepsilon^{\eta\rho\sigma\tau}\mathscr{F}(y,z,x)\big]k_{1\rho}k_{2\sigma}k_{3\tau}\\ &\quad+\big[\big(k_2^\mu k_1^\tau-k_1^\mu k_2^\tau\big)\varepsilon^{\alpha\eta\rho\sigma}\mathscr{H}(x,y,z)+\big(k_3^\alpha k_2^\rho-k_2^\alpha k_3^\rho\big)\varepsilon^{\eta\mu\sigma\tau}\mathscr{H}(z,x,y)\\ &\qquad+\big(k_1^\eta k_3^\sigma-k_3^\eta k_1^\sigma\big)\varepsilon^{\alpha\mu\rho\tau}\mathscr{H}(y,z,x)\big]k_{1\rho}k_{2\sigma}k_{3\tau}\,,\end{aligned}\tag{3}$$

where $k_{1,2,3}$ are the photon momenta, $x=k_1\cdot k_3$, $y=k_2\cdot k_3$, $z=k_1\cdot k_2$, and the functions F, G, $\mathscr{F}$, $\mathscr{G}$, and $\mathscr{H}$ must be free of kinematic singularities and satisfy the relations

$$\begin{aligned}&F(u,v,w)=-F(v,u,w)\,,\qquad G(u,v,w)=-G(v,u,w)=-G(w,v,u)=-G(u,w,v)\,,\\ &\mathscr{F}(u,v,w)=-\mathscr{F}(v,u,w)\,,\qquad \mathscr{H}(u,v,w)=-\mathscr{H}(v,u,w)\,,\\ &\mathscr{G}(u,v,w)=-\mathscr{G}(v,u,w)=-\mathscr{G}(w,v,u)=-\mathscr{G}(u,w,v)\,.\end{aligned}\tag{4}$$

with u,v,w each being any one of the invariants $k_i\cdot k_j$.

[1] We derived $M_{\rm PC}$ in (3) with the aid of Schouten's identity, more examples of which can be found in [5].

In the sum of $|\mathscr{M}^K_{\rm PV}+\mathscr{M}^K_{\rm PC}|^2$ over the photon polarizations, the interference between $\mathscr{M}^K_{\rm PV,PC}$ vanishes. The corresponding decay rate is given by

$$\Gamma(K\to 3\gamma) \;=\; \frac{1}{256\,\pi^3\,m_K^3}\,\frac{1}{3!}\int ds_{12}\,ds_{23}\,\sum_{\rm pol}\Big(\big|\mathscr{M}^K_{\rm PV}\big|^2+\big|\mathscr{M}^K_{\rm PC}\big|^2\Big)\,, \tag{5}$$

$$\begin{aligned}\sum_{\rm pol}\big|\mathscr{M}^K_{\rm PV}\big|^2 \;=\;& 4\big\{|F_1|^2z^2+|F_2|^2y^2+|F_3|^2x^2+2\,|G(x,y,z)|^2\\ &+{\rm Re}\big[F_1^*F_2\,yz+F_2^*F_3\,xy+F_3^*F_1\,xz+2\big(F_1^*z+F_2^*y+F_3^*x\big)G(x,y,z)\big]\big\}xyz\,,\\ \sum_{\rm pol}\big|\mathscr{M}^K_{\rm PC}\big|^2 \;=\;& 4\big\{\big(|\mathscr{F}_1|^2+|\mathscr{H}_1|^2\big)z^2+\big(|\mathscr{F}_2|^2+|\mathscr{H}_2|^2\big)y^2+\big(|\mathscr{F}_3|^2+|\mathscr{H}_3|^2\big)x^2+2\,|\mathscr{G}(x,y,z)|^2\\ &+{\rm Re}\big[\big(\mathscr{F}_1^*+\mathscr{H}_1^*\big)\big(\mathscr{F}_2+\mathscr{H}_2+2\mathscr{G}(x,y,z)/y\big)yz\\ &\quad+\big(\mathscr{F}_2^*+\mathscr{H}_2^*\big)\big(\mathscr{F}_3+\mathscr{H}_3+2\mathscr{G}(x,y,z)/x\big)xy\\ &\quad+\big(\mathscr{F}_3^*+\mathscr{H}_3^*\big)\big(\mathscr{F}_1+\mathscr{H}_1+2\mathscr{G}(x,y,z)/z\big)xz\big]\big\}xyz\,,\end{aligned}$$

where the 3! accounts for the 3 photons being identical particles, $s_{mn}=(k_m+k_n)^2$, $F_1=F(x,y,z)$, $F_2=F(z,x,y)$, $F_3=F(y,z,x)$, and similarly for $\mathscr{F}_{1,2,3}$ and $\mathscr{H}_{1,2,3}$. We note that the preceding formulas apply more generally to any other neutral pseudoscalar particle decaying into 3γ, and they also work for the decay of a neutral scalar particle if the PC and PV parts are interchanged.

To explore the leading contributions, we adopt a chiral-Lagrangian approach [6]. Accordingly, they are expected to arise from the relevant portions in the chiral expansion and yield terms in the functions F, G, $\mathscr{F}$, $\mathscr{G}$, and $\mathscr{H}$ with the lowest numbers of powers of the photon momenta k_i. Since there are in principle many contributions to the amplitude, from tree and loop diagrams, with unknown parameters, it suffices to consider just one representative and rely on dimensional-analysis arguments to evaluate its size.

Treating $K_{\rm L}\to 3\gamma$ first and ignoring *CP* violation, we can focus on $\mathscr{M}^K_{\rm PV}$. From the simplest formulas $F(u,v,w)=c_F\,(u-v)$ and $G(u,v,w)=c_G[(u-v)f(w)+(v-w)f(u)+(w-u)f(v)]$ fulfilling (4), with $c_{F,G}$ being constants and f any well-behaved function, we see that F and G contain at least 2 and 4 powers of k_i, respectively. Thus $M_{\rm PV}$ involves at least 7 powers of k_i.

To assess the leading contributions to $M_{\rm PV}$, we look at a weak chiral Lagrangian for standard-model strangeness-changing, $|\Delta S|=1$, transitions which is parity odd, has 7 derivatives, and couples K to 3γ in a gauge-invariant way. As is well known, such a chiral Lagrangian proceeds from the dominant left-handed chiral octet piece of the weak interactions of light quarks [6] and has to be invariant under the *CP* transformation combined with the switching of s and d quarks [7]. An example with the desired properties is

$$\begin{aligned}\mathscr{L}_{\rm PV} \;=\;& c_7\big\langle\xi^\dagger h\xi\,\big(\nabla^\alpha\mathscr{V}^{\mu\nu}\big)\big[\mathscr{U}^\rho\,\nabla_\alpha\mathscr{V}_{\rho\sigma}+\big(\nabla_\sigma\mathscr{V}_{\rho\alpha}\big)\mathscr{U}^\rho\big]\,\nabla^\sigma\mathscr{V}_{\mu\nu}\big\rangle\;+\;{\rm H.c.}\\ =\;& \frac{8\sqrt{2}\,c_7\,e^3}{27\,f_\pi}\,\partial^\alpha F^{\mu\nu}\big(\partial_\alpha F_{\rho\sigma}+\partial_\sigma F_{\rho\alpha}\big)\,\partial^\rho\bar{K}^0\,\partial^\sigma F_{\mu\nu}\;+\;\cdots\;+\;{\rm H.c.}\,,\end{aligned} \tag{6}$$

where c_7 is a constant, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the usual photon field strength tensor, and other details can be found in [4]. This translates into

$$F(u,v,w)\;=\;\frac{32\sqrt{2}\;ic_7\,e^3}{27\,f_\pi}(u-v)\,,\qquad G(u,v,w)\;=\;0\,. \tag{7}$$

Assuming *CP* conservation and adopting the convention $K_{\rm L}=\big(K^0+\bar{K}^0\big)/\sqrt{2}$, we then obtain

$$\sum_{\rm pol}|\mathscr{M}(K_{\rm L}\to 3\gamma)|^2\;=\;\frac{|128\,c_7|^2\,e^6}{729\,f_\pi^2}\big(x^2y^2+y^2z^2+x^2z^2-xyz^2-xy^2z-x^2yz\big)xyz\,. \tag{8}$$

Since it is not yet possible to compute c_7 rigorously from the quark-level parameters, we estimate it with the aid of naive dimensional analysis [8]. Thus we get the order-of-magnitude value

$$c_7 \sim \frac{G_F \lambda_C f_\pi^4}{\sqrt{2}\Lambda^8} \simeq 1.0\times 10^{-9}\ \mathrm{GeV}^{-6}\,, \tag{9}$$

where $\lambda_C = 0.22$ is the Cabibbo mixing parameter and Λ represents the scale at which the chiral Lagrangian approach breaks down, which suggests we set $\Lambda = m_\rho = 775\,\mathrm{MeV}$ [1]. The resulting branching ratio is $\mathscr{B}(K_L \to 3\gamma) \sim 7.4\times 10^{-17}$.

As for $K_S \to 3\gamma$, the amplitude is dominated by $\mathscr{M}_{PC}^K$, and we can pick the leading-order form $\mathscr{F}(u,v,w) \sim \mathscr{H}(u,v,w) = \tilde{c}(u-v)$ with $\tilde{c}$ being a constant and $\mathscr{G} = 0$, satisfying (4). Hence the situation is similar to that of $\mathscr{M}_{PV}^K$ with F and G in (7). More precisely, making a comparison of $\Sigma_{\rm pol}|\mathscr{M}_{PC}^K|^2$ and $\Sigma_{\rm pol}|\mathscr{M}_{PV}^K|^2$ above for the two cases, respectively, one can see that their decay distributions have the same functional dependence on x, y, and z. It follows that $\Gamma(K_S \to 3\gamma)$ can be expected to be roughly of the same order as $\Gamma(K_L \to 3\gamma)$. Interestingly, the measured rates of their 2γ counterparts are also of similar order, $\Gamma(K_S \to 2\gamma) \sim 2.7\,\Gamma(K_L \to 2\gamma)$ [1]. In view of $\mathscr{B}(K_L \to 3\gamma)$ in the last paragraph, we can therefore predict that $\mathscr{B}(K_S \to 3\gamma) \sim 1\times 10^{-19}$.

In conclusion, we have revisited the rare kaon decay $K \to 3\gamma$, which is expected to be much suppressed because its amplitude has a large number of angular momentum suppression factors. We construct a general form of the amplitude which adheres to the requisites of gauge invariance and Bose symmetry and includes both parity-conserving and parity-violating components. In addition, we provide an expression for the squared amplitude, summed over the photon polarizations, which can be useful to produce a Dalitz plot distribution of the decay. These results are applicable generally to the decay of any spinless particle into 3γ. More specifically, we explore the leading-order contributions to the amplitudes for $K_{L,S} \to 3\gamma$ in the standard model by means of a chiral-Lagrangian technique along with dimensional-analysis reasoning. This finally leads us to branching ratios that are bigger by a few orders of magnitude than those calculated before, but still tiny. Nevertheless, any experimental findings on $\mathscr{B}(K_{L,S} \to 3\gamma)$ significantly exceeding our predictions would likely signal the effects of new physics.

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