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TMD Evolution at Moderate Hard Scales

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We summarize some of our recent work on non-perturbative transverse momentum dependent (TMD) evolution, emphasizing aspects that are necessary for dealing with moderately low scale processes like semi-inclusive deep inelastic scattering.

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1. TMD factorization and non-perturbative evolution

The purpose of this talk is to summarize results recently presented in Ref. [1]. We will discuss the Collins-Soper-Sterman (CSS) form of TMD factorization in the updated version presented in Ref. [2]. (See Ref. [3] for a general overview and for references.) For these proceedings, the relevant aspects of the TMD factorization theorems are the following:

- The unpolarized cross section for a process like Drell-Yan scattering is expressible as

$$\frac{d\sigma}{d^4q\, d\Omega} = \frac{2}{s}\sum_j \frac{d\hat{\sigma}_{j\bar{j}}(Q,\mu\to Q;\alpha_s(Q))}{d\Omega}\int d^2\mathbf{b}\; e^{i\mathbf{q}_\mathrm{T}\cdot\mathbf{b}}\; \tilde{F}_{j/A}(x_A,\mathbf{b};Q^2,Q)\; \tilde{F}_{\bar{j}/B}(x_B,\mathbf{b};Q^2,Q) + \text{large } q_\mathrm{T} \text{ “Y-term” correction}\,. \tag{1.1}$$

  where $d\hat{\sigma}_{j\bar{j}}/d\Omega$ is a hard partonic cross section and $\tilde{F}(x,\mathbf{b};Q^2,Q)$ is a TMD parton distribution function (PDFs) in coordinate space evaluated with a hard scale Q.

- Collins-Soper (CS) evolution applied to an individual TMD PDF leads to

$$\frac{\partial}{\partial \ln Q}\ln \tilde{F}_{j/A}(x_A,\mathbf{b}_\mathrm{T};Q^2,Q) = \tilde{K}(b_\mathrm{T};Q) + b_\mathrm{T}\text{ Independent Terms} \tag{1.2}$$

  The “b_T Independent Terms” only affect the *normalization* of $\tilde{F}$ but not its shape.

- The kernel $\tilde{K}(b_\mathrm{T};Q)$ is is strongly universal. At small b_T its b_T-dependence is perturbatively calculable with $1/b_\mathrm{T}$ acting as a hard scale. At large b_T its b_T-dependence is non-perturbative.

- For all b_T, $\tilde{K}(b_\mathrm{T};Q)$ obeys the renormalization group (RG) equation:

$$\frac{d}{d\ln\mu}\tilde{K}(b_\mathrm{T};\mu) = -\gamma_K(\alpha_s(\mu))\,. \tag{1.3}$$

At small b_T, one hopes to exploit perturbation theory with $1/b_\mathrm{T}$ as a hard scale to calculate $\tilde{K}(b_\mathrm{T};Q)$ while at large b_T a non-perturbative parametrization is needed. In the non-perturbative region, one hopes to exploit the strong universality of $\tilde{K}(b_\mathrm{T};Q)$ to make predictions. One needs a prescription to demarcate what constitutes large and small b_T. To smoothly interpolate between the two regions, one imposes a gentle cutoff on large b_T. A common choice of cutoff function is

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) = \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1+b_\mathrm{T}^2/b_\mathrm{max}^2}}\,. \tag{1.4}$$

Then an RG scale defined as $\mu_{b_*}\equiv C_1/b_*$ approaches C_1/b_T at small b_T and C_1/b_max at large b_T. We can separate $\tilde{K}(b_\mathrm{T};Q)$ into a large b_T part and a small b_T part by adding and subtracting $\tilde{K}(b_*;Q)$ in Eq. (1.2):

$$\frac{\partial}{\partial \ln Q}\ln \tilde{F}_{j/A}(x_A,\mathbf{b}_\mathrm{T};Q^2,Q) = \tilde{K}(b_*;Q) + \left[\tilde{K}(b_\mathrm{T};Q) - \tilde{K}(b_*;Q)\right] + b_\mathrm{T}\text{ Independent Terms}\,. \tag{1.5}$$

The $g_K(b_T;b_{max})$ function is defined as the term $\tilde{K}(b_T;Q) - \tilde{K}(b_*;Q)$, so that

$$\frac{\partial}{\partial \ln Q} \ln \tilde{F}_{j/A}(x_A, \mathbf{b}_T; Q^2, Q) = \tilde{K}(b_*;Q) - g_K(b_T;b_{max}) + b_T \text{ Independent Terms}. \tag{1.6}$$

By definition, the right side of Eq. (1.6) is exactly independent of b_{max}. From Eq. (1.3), $g_K(b_T;b_{max})$ is also exactly independent of Q. The Q dependence in each of the terms in the definition of $g_K(b_T;b_{max})$ cancels. We can apply Eq. (1.3) to exploit RG improvement in the calculation of $\tilde{K}(b_*;Q)$:

$$\tilde{K}(b_*;Q) = \tilde{K}(b_*;\mu_{b_*}) - \int_{\mu_{b_*}}^{Q} \frac{d\mu'}{\mu'} \gamma_K(\alpha_s(\mu')). \tag{1.7}$$

So, the evolution of the *shape* of $\tilde{F}_{j/A}(x_A, \mathbf{b}_T; Q^2, Q)$ is is given by

$$\frac{\partial}{\partial \ln Q} \ln \tilde{F}_{j/A}(x_A, \mathbf{b}_T; Q^2, Q)$$
$$= \tilde{K}(b_*;\mu_{b_*}) - \int_{\mu_{b_*}}^{Q} \frac{d\mu'}{\mu'} \gamma_K(\alpha_s(\mu')) - g_K(b_T;b_{max}) + b_T \text{ Independent Terms}. \tag{1.8}$$

The partial derivative symbol means x_A is to be held fixed. The $g_K(b_T;b_{max})$ function inherits the universality properties of $\tilde{K}(b_T;\mu)$. In particular, it is related to the *vacuum* expectation value of a relatively simple Wilson loop. It is independent of any details of the process and is even the same if the PDF $\tilde{F}_{j/A}(x_A, \mathbf{b}_T; Q^2, Q)$ is replaced with a fragmentation function. Thus we say that $g_K(b_T;b_{max})$ is "strongly" universal; see the graphic in Fig. 1. The $g_K(b_T;b_{max})$ function is often called the "non-perturbative" part of the evolution since it can contain non-perturbative elements. This is a slight misnomer, however, since $g_K(b_T;b_{max})$ can contain perturbative contributions as well. Indeed, at very small b_T it is entirely perturbatively calculable, though suppressed by powers of b_T/b_{max}, according to its definition in Eq. (1.5).

2. Large b_T behavior

A common choice for non-perturbative parametrizations of $g_K(b_T;b_{max})$ is a power-law form. These tend to yield reasonable success in fits that involve at least moderately high scales Q [4]. However, extrapolations of those fits to lower values of Q (such as those corresponding to many current SIDIS experiments) appear to appear to produce evolution that is far too rapid [5, 6]. In this talk, we carefully examine the underlying physics issues surrounding non-perturbative evolution and, on the basis of those considerations, we will propose a form for $g_K(b_T;b_{max})$ that accommodates both large and small Q behavior.

We will first write down our proposed ansatz for $g_K(b_T;b_{max})$ and then spend the remainder of the talk discussing its justifications. Our proposal is

$$g_K(b_T;b_{max}) = g_0(b_{max}) \left(1 - \exp\left[- \frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0(b_{max}) b_{max}^2} \right] \right), \tag{2.1}$$

where

$$g_0(b_{max}) = g_0(b_{max,0}) + \frac{2C_F}{\pi} \int_{C_1/b_{max,0}}^{C_1/b_{max}} \frac{d\mu'}{\mu'} \alpha_s(\mu'). \tag{2.2}$$

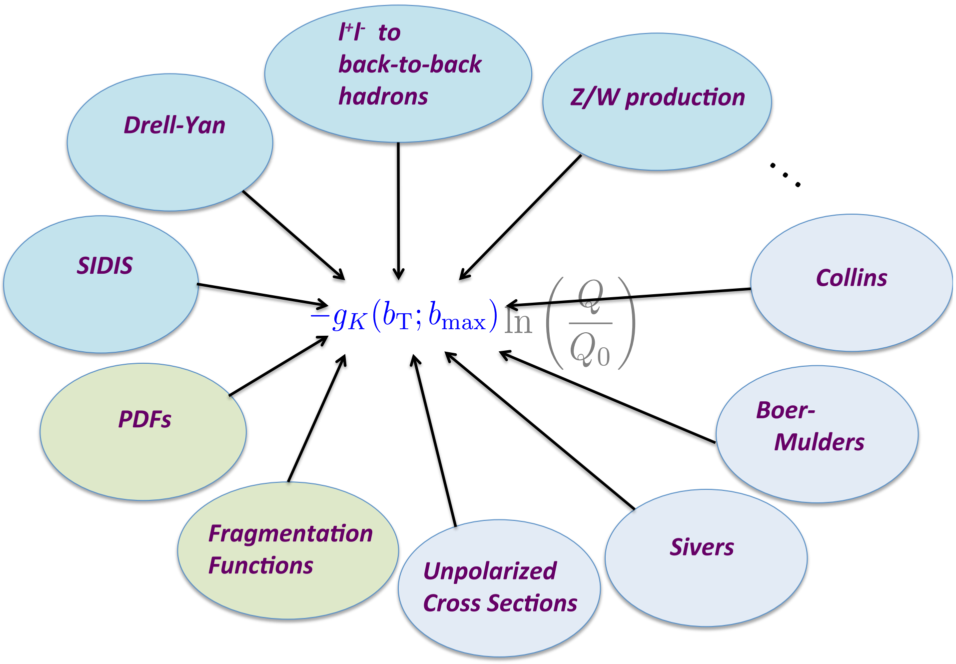

Figure 1: Strong universality of the non-perturbative evolution parametrized by $g_K(b_T;b_{max})$. The $-g_K(b_T;b_{max})\ln(Q/Q_0)$ combination appears exponentiated in the evolved cross section expression.

The only parameter of the model is $g_0(b_{max})$ and it varies with b_{max} according to Eq. (2.2). $b_{max,0}$ is a boundary value for g_0 relative to which other values are determined.

First, note that the a small b_T/b_{max} expansion of Eq. (2.1) gives

$$g_K(b_T;b_{max}) = \frac{C_F}{\pi}\frac{b_T^2}{b_{max}^2}\alpha_s(\mu_{b_*}) + O\left(\frac{b_T^4 C_F^2 \alpha_s(\mu_{b_*})^2}{b_{max}^4 \pi^2 g_0(b_{max})}\right), \tag{2.3}$$

while an expansion of the exact definition of $-g_K(b_T;b_{max})$ in Eq. (1.5) is

$$\begin{aligned} g_K(b_T;b_{max}) &= -\tilde{K}(b_T;\mu_{b_*};\alpha_s(\mu_{b_*})) + \tilde{K}(b_*;\mu_{b_*};\alpha_s(\mu_{b_*})) \\ &= \frac{C_F}{\pi}\frac{b_T^2}{b_{max}^2}\alpha_s(\mu_{b_*}) + O\left(\frac{b_T^4}{\pi^2 b_{max}^4}\alpha_s(\mu_{b_*})^2\right) \end{aligned} \tag{2.4}$$

So, the exact definition and Eq. (2.1) match in the small b_T limit.

3. Conditions on $g_K(b_T;b_{max})$

Our description of the large b_T limit of correlation functions like $\tilde{F}(x_A,\mathbf{b}_T;Q^2,Q)$ is motivated by the general observation that the analytic properties of correlation functions imply an exponential coordinate dependence, with a possible power-law fall-off, for the large b_T limit. That is, neglecting perturbative contributions,

$$\tilde{F}(x_A, \mathbf{b}_T; Q^2, Q) \overset{b_T \to \infty}{\sim} \frac{1}{{b_T}^{\alpha}} e^{-m b_T} , \tag{3.1}$$

with m and α independent of Q. See, for example, Ref. [7]. Therefore, from Eq. (1.2), $\tilde{K}(b_T; Q)$ must approach a b_T-independent constant at large b_T.

The set of requirements on $g_K(b_T; b_{\max})$ is

1. $\tilde{K}(b_T; \mu_{b_*}) \overset{b_T \to 0}{=} \tilde{K}(b_T; C_1/b_T)$ is calculable entirely in perturbation theory with C_1/b_T playing the role of a hard scale.

2. $\tilde{K}(b_T; Q)$ approaches a constant at $b_T/b_{\max} \to \infty$. The constant can be Q-dependent, but the Q-dependence can be calculated perturbatively for all b_T from Eq. (1.3).

3. Because of item 2, $g_K(b_T; b_{\max})$ must approach a constant at large b_T, but the constant depends on $b_{\max}$.

4. At small b_T, $g_K(b_T; b_{\max})$ is a power series in $(b_T/b_{\max})^2$ with perturbatively calculable coefficients, as in Eqs. (2.3,2.4).

5. By definition, the right side of Eq. (1.8) is independent of $b_{\max}$ and this should be preserved as much as possible in the functional form that parametrizes $g_K(b_T; b_{\max})$. For small b_T, this means

$$\underset{b_T \ll b_{\max}}{\text{asy}} \frac{\mathrm{d}}{\mathrm{d}b_{\max}} g_K(b_T; b_{\max}) \Big|_{\text{parametrized}} = \underset{b_T \ll b_{\max}}{\text{asy}} \frac{\mathrm{d}}{\mathrm{d}b_{\max}} g_K(b_T; b_{\max}) \Big|_{\text{truncated PT}} \tag{3.2}$$

where "parametrized" refers to a specific model of $g_K(b_T; b_{\max})$ while "truncated PT" refers to a truncated perturbative expansion. Eqs. (2.3,2.4) satisfy this requirement through order $\alpha_s(\mu_{b_*})$.

6. At large b_T, $b_{\max}$-independence of the exact $\tilde{K}(b_T, \mu)$ implies that, to a useful approximation,

$$\frac{\mathrm{d}}{\mathrm{d}\ln b_{\max}} g_K(b_T = \infty; b_{\max}) = \left[\frac{\mathrm{d}\tilde{K}(b_{\max}; C_1/b_{\max})}{\mathrm{d}\ln b_{\max}} - \gamma_K(\alpha_s(C_1/b_{\max})) \right]_{\text{truncated PT}} , \tag{3.3}$$

as obtained from Eq. (1.7) and the definition of g_K. Equation (2.2) ensures that Eq. (2.1) satisfies Eq. (1.8) so long as everything is calculated only to order $\alpha_s(\mu_{b_*})$. Enforcing both Eq. (3.2) and Eq. (3.3) simultaneously means $g_K(b_T; b_{\max})$ will produce a $b_{\max}$ independent contribution to $\tilde{K}(b_T; Q)$ for all b_T except perhaps for an intermediate region at the border between perturbative and non-perturbative b_T-dependence. The residual $b_{\max}$ dependence there can be reduced by calculating higher orders and refining knowledge of non-perturbative behavior.

For a much more detailed discussion of these considerations, see Sect. VII of Ref. [1]. Equation (2.1) is one of the simplest models that satisfies all 6 of these properties simultaneously.

4. Conclusion

In Sect. 3 we enumerated properties that a model of $g_K(b_T;b_{max})$ needs tp ensure basic consistency in a calculation $\tilde{K}(b_T;Q)$. A simple parametrization was proposed in Sect. 2.

Note that a quadratic $(b_T/b_{max})^2$ dependence at small b_T emerges naturally from (2.1), but with a perturbatively calculable coefficient. Furthermore, the dependence is not exactly quadratic because the coefficients contain logarithmic b_T dependence through $\alpha_s(\mu_{b_*})$.

In a process dominated by very large b_T, Sect. 3 and Eq. (2.1) predict an especially simple evolution for the low-Q cross section. Namely, the cross section scales as $(Q/Q_0)^a$ where a is combination of $g_K(\infty,b_{max})$ and perturbatively calculable quantities. (See Eq. (85,86) of Ref. [1].)

Future phenomenological work should include efforts to constrain g_0. Because of its strongly universal nature, this offers a relatively simple way to test TMD factorization.

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References

[1] J. Collins and T. Rogers, "Understanding the large-distance behavior of transverse-momentum-dependent parton densities and the Collins-Soper evolution kernel," Phys. Rev. D **91**, no. 7, 074020 (2015) doi:10.1103/PhysRevD.91.074020 [arXiv:1412.3820 [hep-ph]].

[2] J. Collins, "Foundations of perturbative QCD," (Cambridge monographs on particle physics, nuclear physics and cosmology. 32)

[3] T. C. Rogers, "An Overview of Transverse Momentum Dependent Factorization and Evolution," arXiv:1509.04766 [hep-ph].

[4] A. V. Konychev and P. M. Nadolsky, "Universality of the Collins-Soper-Sterman nonperturbative function in gauge boson production," Phys. Lett. B **633**, 710 (2006) doi:10.1016/j.physletb.2005.12.063 [hep-ph/0506225].

[5] P. Sun and F. Yuan, "Transverse momentum dependent evolution: Matching semi-inclusive deep inelastic scattering processes to Drell-Yan and W/Z boson production," Phys. Rev. D **88**, no. 11, 114012 (2013) doi:10.1103/PhysRevD.88.114012 [arXiv:1308.5003 [hep-ph]].

[6] C. A. Aidala, B. Field, L. P. Gamberg and T. C. Rogers, "Limits on transverse momentum dependent evolution from semi-inclusive deep inelastic scattering at moderate Q," Phys. Rev. D **89**, no. 9, 094002 (2014) doi:10.1103/PhysRevD.89.094002 [arXiv:1401.2654 [hep-ph]].

[7] P. Schweitzer, M. Strikman and C. Weiss, "Intrinsic transverse momentum and parton correlations from dynamical chiral symmetry breaking," JHEP **1301**, 163 (2013) doi:10.1007/JHEP01(2013)163 [arXiv:1210.1267 [hep-ph]].