PROCEEDINGS OF SCIENCE

Spin-Orbit Coupling in an Unpolarized Heavy Nucleus

Matthew Sievert[*] [†]
Brookhaven National Lab
E-mail: msievert@bnl.gov

We decompose the transverse-momentum-dependent parton distribution functions (TMD's) of an unpolarized heavy nucleus in terms of the TMD's of its nucleons. By considering the symmetries of the nuclear wave functions, we parameterize the possible spin-orbit coupling terms which can occur. In doing so, we find that these spin-orbit coupling terms, together with multiple rescattering on spectator nucleons, are responsible for the mixing of various TMD's between the nucleons and the nucleus. Indeed, the same spin-orbit coupling term is responsible for TMD mixing in multiple sectors, making it possible to explicitly test the predictions of this theory and, in principle, directly extract the strength of the spin-orbit coupling from experiment.

QCD Evolution 2015
May 26-30, 2015
Jefferson Lab (JLAB), Newport News Virginia, USA

[*]Speaker.

[†]This material is supported in part by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under Award Number DE-SC0004286 and in part under DOE Contract No. DE-SC0012704.

PoS(QCDEV2015)034

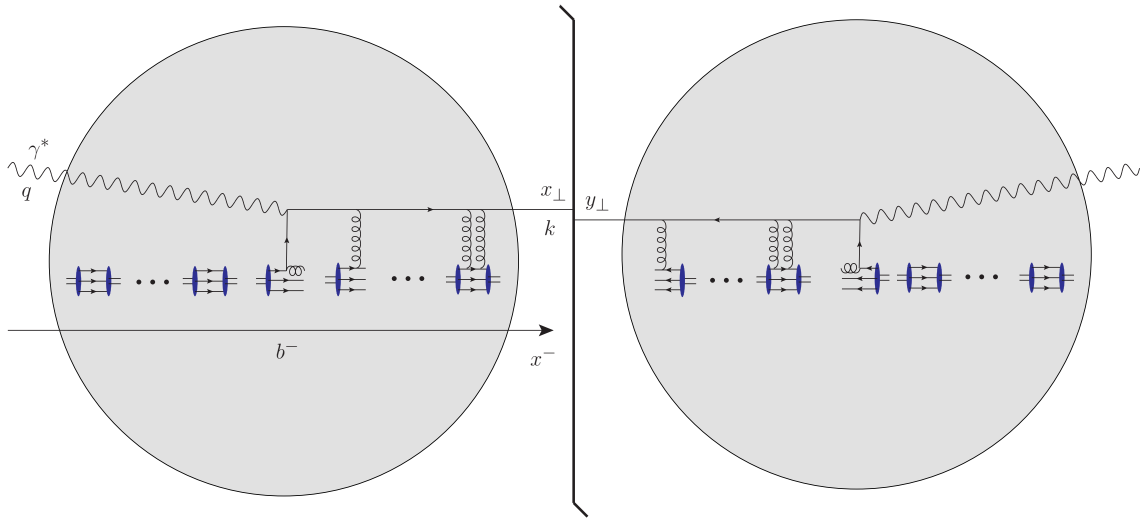

Figure 1: SIDIS cross section as a square of the scattering amplitude explicitly illustrating the x^--ordering of the nucleons in the nucleus. The solid vertical line denotes the final-state cut.

1. Quasi-Classical Factorization in a Heavy Nucleus

This article distills the main conclusions of our paper [1]; for further details we refer the interested reader to that paper.

1.1 TMD's of a Heavy Nucleus

The transverse-momentum-dependent (TMD) quark correlator in a hadronic state $|h(P,S)\rangle$ with momentum P and spin S is defined by

$$\phi_{\alpha\beta}(x,\underline{k};P,S) \equiv \frac{g_{+-}}{(2\pi)^3} \int d^{2-}r\, e^{ik\cdot r} \langle h(P,S)|\, \overline{\psi}_\beta(0)\, \mathscr{U}[0,r]\, \psi_\alpha(r)\, |h(P,S)\rangle_{r^+=0}\,, \tag{1.1}$$

where α,β are Dirac indices, the separation of the quark fields is $r^\mu = (0^+, r^-, \underline{r})$, and the "staple-shaped" gauge link $\mathscr{U}[0,r]$ extends to future/past light-front infinity depending on the process. The notation $d^{2-}r$ is shorthand for $d^2r\,dr^-$, and the light-front coordinates are defined by $x^\pm \equiv \sqrt{\frac{g^{+-}}{2}}(x^0 \pm x^3)$, where $g^{+-} = 1$ and $g^{+-} = 2$ are two common choices of metric. The momentum fraction $x \equiv \frac{k^+}{P^+}$, and we use the notation $\underline{k} \equiv (k_\perp^1, k_\perp^2)$ to indicate a two-vector in the transverse plane with magnitude $k_T \equiv |\underline{k}|$. We will work in a frame such that the hadron h moves predominantly in the x^+ direction.

At leading twist, the correlator (1.1) can be expanded into 8 independent TMD's [2, 3] as

$$\begin{aligned}\phi(x,\underline{k};P,S) = &\left(f_1 - \frac{\underline{k}\times\underline{S}}{m} f_{1T}^\perp\right)\left[\frac{1}{2}g_{+-}\gamma^-\right] + \left(S_L g_1 + \frac{\underline{k}\cdot\underline{S}}{m} g_{1T}\right)\left[\frac{1}{2}g_{+-}\gamma^5\gamma^-\right] + \\ &+ \left(S_\perp^i h_{1T} + \frac{k_\perp^i}{m} S_L h_{1L}^\perp + \frac{k_\perp^i}{m}\frac{(\underline{k}\cdot\underline{S})}{m} h_{1T}^\perp\right)\left[\frac{1}{2}g_{+-}\gamma^5\gamma_{\perp i}\gamma^-\right] + \left(\frac{k_\perp^i}{m} h_1^\perp\right)\left[\frac{i}{2}g_{+-}\gamma_{\perp i}\gamma^-\right],\end{aligned} \tag{1.2}$$

where $(\underline{x}\times\underline{y}) \equiv \epsilon_T^{ij} x_\perp^i y_\perp^j \equiv x_\perp^1 y_\perp^2 - x_\perp^2 y_\perp^1$ and m is the mass of the hadron h.

As was discussed in [4], the cross-section for semi-inclusive deep inelastic scattering (SIDIS) on a heavy nucleus with A nucleons such that $\alpha_s^2 A^{1/3} \sim \mathscr{O}(1)$ can be decomposed as

$$\frac{d\sigma^{\gamma^*+A\to q+X}}{d^2k\,dy} = A\, g_{+-} \sum_\sigma \int \frac{dp^+\, d^2p\, db^-}{(2\pi)^3} \int d^2x\, d^2y\, W_\sigma(p,b;P,S) \times \int \frac{d^2k'}{(2\pi)^2}\, e^{-i(\underline{k}-\underline{k}')\cdot(\underline{x}-\underline{y})}\, \frac{d\hat{\sigma}^{\gamma^*+N\to q+X}}{d^2k'\,dy}(p,q)\, D_{xy}[\infty^-,b^-], \tag{1.3}$$

where

$$W_\sigma(\overline{p},b;P,S) \equiv \frac{1}{2(2\pi)^3} \int \frac{d^{2+}(p-p')}{\sqrt{p^+p'^+}}\, e^{-i(p-p')\cdot b} \times \sum_{\mathbb{X}} \langle A(P,S)|\, N(p',\sigma);\mathbb{X}\rangle\, \langle N(p,\sigma);\mathbb{X}|\, A(P,S)\rangle \tag{1.4}$$

is the Wigner distribution of nucleons inside the nucleus, and the correlator

$$D_{xy}[\infty^-,b^-] \equiv \frac{1}{N_c} \left\langle \mathrm{Tr}\left[V_x[\infty^-,b^-]\, V_y^\dagger[\infty^-,b^-]\right]\right\rangle, \tag{1.5}$$

with N_c the number of colors, describes the multiple scattering on spectator nucleons through the (future-pointing) semi-infinite Wilson lines

$$V_x[\infty^-,b^-] \equiv \mathscr{P} \exp\left[ig \int_{b^-}^{\infty^-} dz^-\, g_{+-} A^{+a}(0^+,z^-,\underline{x})\, T^a\right]. \tag{1.6}$$

To leading order in $A^{1/3}$, only the symmetric part of the Wilson line correlator contributes [4], which for a nucleus of uniform density is given by

$$S_{xy}[\infty^-,b^-] \equiv \frac{1}{2}\left(D_{xy}[\infty^-,b^-] + D_{yx}[\infty^-,b^-]\right) = \exp\left[-\frac{1}{4}|x-y|_T^2\, Q_s^2\left(|\tfrac{x+y}{2}|_T\right) \left(\frac{R^-(|\frac{x+y}{2}|_T) - b^-}{2R^-(|\frac{x+y}{2}|_T)}\right) \ln\frac{1}{|x-y|_T \Lambda}\right], \tag{1.7}$$

where $R^-(b_T)$ is the longitudinal radius of the nucleus at transverse position b_T, Q_s is the saturation scale, and Λ is an infrared cutoff. This decomposition of the cross-section translates, in Bjorken kinematics at moderate x, to a quasi-classical factorization of the TMD correlator Φ^A of the nucleus in terms of the corresponding correlator ϕ^N of the nucleons:

$$\Phi^A_{\alpha\beta}(x,\underline{k};P,S) = A\, \frac{g_{+-}}{(2\pi)^5} \sum_\sigma \int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\underline{k}-\underline{k}'-\hat{x}\underline{p})\cdot\underline{r}} \times W_\sigma(p,b;P,S)\, \phi^N_{\alpha\beta}(\hat{x},\underline{k}';p,\sigma)\, S_{(r_T,b_T)}^{[\infty^-,b^-]}, \tag{1.8}$$

where $\underline{r} \equiv \underline{x} - \underline{y}$, $\underline{b} \equiv \frac{1}{2}(\underline{x}+\underline{y})$, and $\hat{x} \equiv \frac{P^+}{p^+} x$ is the momentum fraction of the quark relative to the nucleon.

1.2 Covariant Spin Decompositions

The Wigner distribution for nucleons with an arbitrary spin state $|S\rangle$ in an unpolarized nucleus

$$W(\overline{p},b,S) \equiv \frac{1}{2(2\pi)^3} \int \frac{d^{2+}(\delta p)}{\sqrt{p^+ p'^+}} e^{-i\delta p \cdot b} \langle A(P)|\, N(p',S)\rangle \, \langle N(p,S)|\, A(P)\rangle \tag{1.9}$$

can be decomposed [5] in terms of light-front helicity states $|\pm\rangle$ using

$$\begin{aligned} |S\rangle\langle S| &\overset{R.F.}{=} \frac{1}{2} \Big[|+\rangle \;\; |-\rangle \Big] \left\{ \Big[\mathbf{1} \Big] + \vec{S}_{R.F.} \cdot \Big[\vec{\sigma} \Big] \right\} \begin{bmatrix} \langle +| \\ \langle -| \end{bmatrix} \\ &= \frac{1}{2} \Big[|+\rangle \;\; |-\rangle \Big] \left\{ \Big[\mathbf{1} \Big] - S_\mu(p) \Big[\hat{\sigma}^\mu(p) \Big] \right\} \begin{bmatrix} \langle +| \\ \langle -| \end{bmatrix}. \end{aligned} \tag{1.10}$$

The canonical spin vector $S^\mu(p)$ is obtained by boosting the spin vector $\vec{S}_{R.F.}$ from the rest frame (R.F.) with a single rotationless boost [6]:

$$\begin{aligned} S^\mu(p) &\equiv \left(\frac{\vec{S}_{R.F.} \cdot \vec{p}}{m}, \vec{S}_{R.F.} + \frac{\vec{p}}{m} \left[\frac{\vec{p} \cdot \vec{S}_{R.F.}}{E+m} \right] \right) \\ &\overset{R.F.}{=} \left(0, \vec{S}_{R.F.} \right) = \left(0, S_\perp^1, S_\perp^2, \lambda \right), \end{aligned} \tag{1.11}$$

and we can make a similar boost of the Pauli matrices

$$\begin{aligned} \hat{\sigma}^\mu(p) &\equiv \left(\frac{\vec{\sigma} \cdot \vec{p}}{m}, \vec{\sigma} + \frac{\vec{p}}{m} \left[\frac{\vec{p} \cdot \vec{\sigma}}{E+m} \right] \right) \\ &\overset{R.F.}{=} (0, \vec{\sigma}) = \left(0, \sigma_\perp^1, \sigma_\perp^2, \sigma_\perp^3 \right), \end{aligned} \tag{1.12}$$

so that the Lorentz invariant product $-S_\mu(p)[\hat{\sigma}^\mu(p)]$ corresponds to the product of three vectors $\vec{S}_{R.F.} \cdot [\vec{\sigma}]$ in the rest frame.

In this way, we obtain a spin density matrix over the light-front helicities λ, λ'

$$W_{\lambda\lambda'}(\overline{p},b) \equiv \frac{1}{2(2\pi)^3} \int \frac{d^{2+}(\delta p)}{\sqrt{p^+ p'^+}} e^{-i\delta p \cdot b} \langle A(P)|\, N(p',\lambda)\rangle \, \langle N(p,\lambda')|\, A(P)\rangle . \tag{1.13}$$

$$= W_{unp}(\bar{p},b) \Big[\mathbf{1} \Big]_{\lambda\lambda'} - \hat{W}_{pol,\mu}(\bar{p},b) \Big[\hat{\sigma}^\mu(\bar{p}) \Big]_{\lambda\lambda'}, \tag{1.14}$$

in terms of which any spin state $|S\rangle$ can be constructed:

$$W(\bar{p},b,S) = W_{unp}(\bar{p},b) - S_\mu(\bar{p}) \hat{W}^\mu_{pol}(\bar{p},b), \tag{1.15}$$

and the polarized and unpolarized distributions are obtained from the traces

$$\begin{aligned} W_{unp}(\overline{p},b) &= \frac{1}{2} \sum_{\lambda\lambda'} \Big[W_{\lambda\lambda'}(\bar{p},b)\, \mathbf{1}_{\lambda'\lambda} \Big] = \frac{1}{2} \mathrm{Tr}[W(\bar{p},b)] \\ \hat{W}^\mu_{pol}(\bar{p},b) &= \frac{1}{2} \sum_{\lambda\lambda'} \Big[W_{\lambda\lambda'}(\bar{p},b)\, \hat{\sigma}^\mu_{\lambda'\lambda}(\bar{p}) \Big] = \frac{1}{2} \mathrm{Tr}[W(\bar{p},b)\, \hat{\sigma}^\mu(\bar{p})]. \end{aligned} \tag{1.16}$$

PoS(QCDEV2015)034

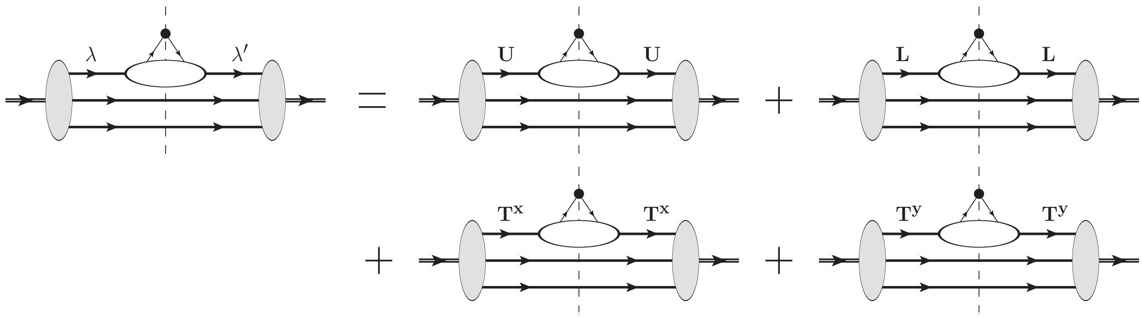

Figure 2: Summing over the 4 independent components of the (2×2) matrices $[\phi]_{\lambda\lambda'}$, $[W]_{\lambda'\lambda}$ corresponds to summing over the 4 possible intermediate polarizations of the nucleons: U = unpolarized, L = longitudinally polarized, T^j = transversely polarized in the $\hat{j}$ direction.

Similarly, we can define a spin density matrix representation of the TMD correlator

$$\phi_{\lambda\lambda'}(x,\underline{k}) \equiv \frac{g_{+-}}{(2\pi)^3} \int d^{2-}r\, e^{ik\cdot r} \left\langle N(P,\lambda)\right| \overline{\psi}_\beta(0)\, \mathscr{U}[0,r]\, \psi_\alpha(r) \left|N(P,\lambda')\right\rangle . \tag{1.17}$$

$$= \phi_{unp}(x,\underline{k}) \left[\mathbf{1} \right]_{\lambda\lambda'} - \hat{\phi}_{pol,\mu}(x,\underline{k}) \left[\hat{\sigma}^\mu \right]_{\lambda\lambda'} \tag{1.18}$$

$$\phi(x,\underline{k};S) = \phi_{unp}(x,\underline{k}) - S_\mu \hat{\phi}^\mu_{pol}(x,\underline{k}), \tag{1.19}$$

where

$$\phi_{unp}(x,\underline{k}) = \frac{1}{2}\sum_{\lambda\lambda'} \left[\phi_{\lambda\lambda'}(x,\underline{k})\, \mathbf{1}_{\lambda'\lambda} \right] = \frac{1}{2}\mathrm{Tr}[\phi(x,\underline{k})]$$

$$\hat{\phi}^\mu_{pol}(x,\underline{k}) = \frac{1}{2}\sum_{\lambda\lambda'} \left[\phi_{\lambda\lambda'}(x,\underline{k})\, \hat{\sigma}^\mu_{\lambda'\lambda} \right] = \frac{1}{2}\mathrm{Tr}[\phi(x,\underline{k})\, \hat{\sigma}^\mu]. \tag{1.20}$$

The sum over nucleon spins in the quasi-classical factorization formula (1.8) can then be implemented using these density matrices

$$\begin{aligned}\Phi^A(x,\underline{k};P) &= A\frac{g_{+-}}{(2\pi)^5} \sum_{\lambda\lambda'} \int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\underline{k}-\underline{k}'-\hat{x}\underline{p})\cdot\underline{r}} \\ &\qquad \times\ W_{\lambda\lambda'}(p,b;P)\, \phi^N_{\lambda'\lambda}(\hat{x},\underline{k}';p)\, S^{[\infty^-,b^-]}_{(r_T,b_T)} \\ &= \frac{2A\, g_{+-}}{(2\pi)^5} \int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\underline{k}-\underline{k}'-\hat{x}\underline{p})\cdot\underline{r}} \\ &\quad \times \left(W_{unp}(p,b;P)\, \phi_{unp}(\hat{x},\underline{k}';p) - \hat{W}_{pol,\mu}(p,b;P) \hat{\phi}^\mu_{pol}(\hat{x},\underline{k}';p) \right) S^{[\infty^-,b^-]}_{(r_T,b_T)},\end{aligned} \tag{1.21}$$

as represented in Fig. 2. The intermediate nucleons can be either unpolarized, or have a polarization in the longitudinal or transverse directions.

2. TMD's of an Unpolarized Nucleus

Let us decompose the TMD's of a heavy nucleus into the corresponding TMD's of the nucleons, considering for simplicity an unpolarized nucleus, but which may contain polarized nucleons.

By tracing (1.2) with a Dirac matrix Γ, $\phi^{[\Gamma]} \equiv \frac{1}{2}\mathrm{Tr}[\phi\Gamma]$, we select a linear combination of TMD's associated with a particular quark spin. We can write these projections for the TMD's of a polarized or unpolarized particle (nucleon or nucleus) as

$$\begin{aligned}
\phi_{unp}^{[\Gamma]} &= \left(f_1\right)\left[\frac{1}{4}g_{+-}\mathrm{Tr}[\gamma^-\Gamma]\right] + \left(\frac{k_\perp^i}{m}h_1^\perp\right)\left[\frac{i}{4}g_{+-}\mathrm{Tr}[\gamma_{\perp i}\gamma^-\Gamma]\right] \\
\phi_{pol}^{0\,[\Gamma]} &\overset{R.F.}{=} 0 \\
\phi_{pol}^{3\,[\Gamma]} &\overset{R.F.}{=} \left(g_1\right)\left[\frac{1}{4}g_{+-}\mathrm{Tr}[\gamma^5\gamma^-\Gamma]\right] + \left(\frac{k_\perp^i}{m}h_{1L}^\perp\right)\left[\frac{1}{4}g_{+-}\mathrm{Tr}[\gamma^5\gamma_{\perp i}\gamma^-\Gamma]\right] \\
\phi_{pol,\perp}^{j\,[\Gamma]} &\overset{R.F.}{=} \left(-\frac{k_\perp^i}{m}\varepsilon_T^{ij}f_{1T}^\perp\right)\left[\frac{1}{4}g_{+-}\mathrm{Tr}[\gamma^-\Gamma]\right] + \left(\frac{k_\perp^j}{m}g_{1T}\right)\left[\frac{1}{4}g_{+-}\mathrm{Tr}[\gamma^5\gamma^-\Gamma]\right] \\
&\quad + \left(\delta^{ij}h_{1T} + \frac{k_\perp^i k_\perp^j}{m^2}h_{1T}^\perp\right)\left[\frac{1}{4}g_{+-}\mathrm{Tr}[\gamma^5\gamma_{\perp i}\gamma^-\Gamma]\right].
\end{aligned} \tag{2.1}$$

For an unpolarized nucleus, only $\Phi_{unp}^{A\,[\Gamma]}$ exists,

$$\Phi_{unp}^{A\,[\Gamma]}(x,\underline{k}) = \left(f_1^A(x,k_T)\right)\left[\frac{1}{4}g_{+-}\mathrm{Tr}[\gamma^-\Gamma]\right] + \left(\frac{k_\perp^i}{M_A}h_1^{\perp A}(x,k_T)\right)\left[\frac{i}{4}g_{+-}\mathrm{Tr}[\gamma_{\perp i}\gamma^-\Gamma]\right] \tag{2.2}$$

$$\begin{aligned}
&= \frac{2A\,g_{+-}}{(2\pi)^5}\int d^{2+}p\,d^{2-}b\,d^2r\,d^2k'\,e^{-i(\underline{k}-\underline{k}'-\hat{x}\underline{p})\cdot\underline{r}} \\
&\times\left(W_{unp}(p,b)\,\phi_{unp}^{[\Gamma]}(\hat{x},\underline{k}') - \hat{W}_{pol,\mu}(p,b)\hat{\phi}_{pol}^{\mu\,[\Gamma]}(\hat{x},\underline{k}')\right)S_{(r_T,b_T)}^{[\infty^-,b^-]},
\end{aligned} \tag{2.3}$$

and is related to the projections $\phi^{[\Gamma]}$ of the nucleons by the quasi-classical factorization formula (1.21). Note that even though the nucleus is unpolarized, there may be contributions $\phi_{pol}^{[\Gamma]}$ from polarized nucleons.

Because the Wigner distributions are constructed purely from the light-front wave functions of the nucleus (without the multiple scattering effects embodied in the Wilson lines), they are constrained by a number of symmetries, namely parity and time-reversal invariance

$$W(\vec{p},\vec{b},\vec{S}) \overset{P}{=} W(-\vec{p},-\vec{b},\vec{S}) \overset{T}{=} W(-\vec{p},\vec{b},-\vec{S}), \tag{2.4}$$

as well as rotational invariance in the rest frame. These symmetries strongly constrain the functional form of the Wigner distributions of polarized and unpolarized nucleons:

$$W_{unp}(\vec{p},\vec{b}) = W_{unp}\left[\vec{p}^2,\vec{b}^2,(\vec{p}\cdot\vec{b})^2\right] \tag{2.5}$$

$$\vec{S}\cdot\vec{W}_{pol}(\vec{p},\vec{b}) = \left(\vec{S}\cdot(\vec{b}\times\vec{p})\right)W_{pol}\left[\vec{p}^2,\vec{b}^2,(\vec{p}\cdot\vec{b})^2\right]. \tag{2.6}$$

We see that the symmetries constrain the only possible spin-orbit coupling to be of the form $\vec{L}_N\cdot\vec{S}_N$, with $\vec{L}_N = \vec{b}\times\vec{p}$ the orbital angular momentum of nucleons in the nucleus and $\vec{S}_N$ the nucleon spin.

Although the Wigner distributions contain 3-dimensional rotation symmetry in the rest frame, the TMD correlator (2.3), embodying a high-energy collision process like SIDIS, contains only 2-dimensional rotation symmetry about the beam axis (which defines the direction of the Wilson lines). Since the transverse position $\underline{b}$ of the struck nucleon is integrated out in obtaining the TMD correleator, we can – without loss of generality – replace $b_\perp^i b_\perp^j \Rightarrow \frac{1}{2} b_T^2 \delta^{ij}$ in the distributions. This further simplifies the structure of the Wigner distributions to

$$W(\alpha, \underline{p}; b^-, \underline{b}; \lambda, \underline{S}) \Rightarrow W_{unp}\left[p_T^2, b_T^2; (\alpha - \tfrac{1}{A})^2, (P^+ b^-)^2\right] + \left(\tfrac{g_{+-} P^+ b^-}{M_A}\right) (\underline{p} \times \underline{S})\, W_{OAM}\left[p_T^2, b_T^2; (\alpha - \tfrac{1}{A})^2, (P^+ b^-)^2\right], \tag{2.7}$$

where we have changed variables from b_z and p_z to b^- and $\alpha = \frac{p^+}{P^+}$. Equivalently we can write the components of the distributions explicitly as

$$\begin{aligned}
W_{unp}(p,b) &= W_{unp}\left[p_T^2, b_T^2; (\alpha - \tfrac{1}{A})^2, (P^+ b^-)^2\right] \\
\hat{W}_{pol}^0(p,b) &\overset{R.F.}{=} 0 \\
\hat{W}_{pol}^3(p,b) &\overset{R.F.}{=} 0 \\
\hat{W}_{pol,\perp}^j(p,b) &\overset{R.F.}{=} \left(\tfrac{g_{+-} P^+ b^-}{M_A}\right) p_\perp^i \epsilon_T^{ij} W_{OAM}\left[p_T^2, b_T^2; (\alpha - \tfrac{1}{A})^2, (P^+ b^-)^2\right].
\end{aligned} \tag{2.8}$$

The interplay of the possible distributions (2.8) of nucleons in the nucleus with the associated distributions (2.1) of quarks within the nucleon generates unique channels which relate the TMD's of the nucleus to the TMD's of the nucleons.

2.1 The Unpolarized Quark Distribution f_1^A

The distribution f_1^A of unpolarized quarks within the unpolarized nucleus is obtained by projecting on $\Gamma = \gamma^+$. Using this in (2.2) and (2.3), together with (2.1) and (2.8), gives the explicit decomposition

$$f_1^A(x, k_T) = \frac{2A\, g_{+-}}{(2\pi)^5} \int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\underline{k} - \underline{k}' - \hat{x}\underline{p}) \cdot \underline{r}}\, S_{(r_T, b_T)}^{[\infty^-, b^-]} \times \left(W_{unp}(p,b)\, f_1^N(\hat{x}, k_T') - \tfrac{g_{+-}}{M_A m_N} (P^+ b^-)(\underline{p} \cdot \underline{k}')\, W_{OAM}(p,b)\, f_{1T}^{\perp N}(\hat{x}, k_T') \right), \tag{2.9}$$

which constructs f_1^A through two distinct channels. The trivial channel obtains unpolarized quarks from unpolarized nucleons through f_1^N, which are distributed symmetrically throughout the nucleus. The novel channel obtains unpolarized quarks from transversely polarized nucleons through the nucleonic Sivers function $f_{1T}^{\perp N}$. In this channel, the antisymmetric motion of quarks inside the nucleon couples to the antisymmetric orbital motion of the nucleons inside the nucleus through the kinematic factor $(P^+ b^-)(\underline{p} \cdot \underline{k}')$. Such a channel is possible because of the presence of $\vec{L}_N \cdot \vec{S}_N$ coupling in the nuclear structure.

2.2 The Boer-Mulders Distribution $h_1^{\perp A}$

The Boer-Mulders distribution $h_1^{\perp A}$ of transversely-polarized quarks within an unpolarized nucleus is obtained by projecting on $\Gamma = \gamma^+ \gamma_\perp^j \gamma^5$. Using this in (2.2) and (2.3), together with (2.1) and (2.8) gives the explicit decomposition

$$\varepsilon_T^{ji} \frac{k_\perp^i}{M_A} h_1^{\perp A}(x,k_T) = \frac{2A\, g_{+-}}{(2\pi)^5} \int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\underline{k}-\underline{k}'-\hat{x}\underline{p})\cdot \underline{r}} S_{(r_T,b_T)}^{[\infty^-,b^-]}$$
$$\times \left[\varepsilon_T^{ji} \frac{k_\perp'^i}{m_N} W_{unp}(p,b)\, h_1^{\perp N}(\hat{x},k_T') + \tfrac{g_{+-}}{M_A}(P^+b^-) p_\perp^i \varepsilon_T^{i\ell} W_{OAM}(p,b) \right.$$
$$\left. \times \left(\delta^{j\ell} h_{1T}^N(\hat{x},k_T') + \frac{k_\perp'^j k_\perp'^\ell}{m_N^2} h_{1T}^{\perp N}(\hat{x},k_T') \right) \right]. \tag{2.10}$$

which we can rewrite as

$$h_1^{\perp A}(x,k_T) = \frac{2A\, g_{+-}}{(2\pi)^5} \frac{M_A}{k_T^2} \int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\underline{k}-\underline{k}'-\hat{x}\underline{p})\cdot \underline{r}} S_{(r_T,b_T)}^{[\infty^-,b^-]}$$
$$\times \left(\frac{(\underline{k}\cdot\underline{k}')}{m_N} \left[W_{unp}(p,b)\, h_1^{\perp N}(\hat{x},k_T') \right] - \tfrac{g_{+-}}{M_A}(P^+b^-)(\underline{p}\cdot\underline{k}) \left[W_{OAM}(p,b)\, h_1^N(\hat{x},k_T') \right] \right.$$
$$\left. - \tfrac{g_{+-}}{M_A}(P^+b^-) \frac{k_T'^2}{m_N^2} \left(\frac{(\underline{p}\times\underline{k}')(\underline{k}\times\underline{k}')}{k_T'^2} - \frac{1}{2}(\underline{p}\cdot\underline{k}) \right) \left[W_{OAM}(p,b)\, h_{1T}^{\perp N}(\hat{x},k_T') \right] \right). \tag{2.11}$$

where

$$h_1^N(\hat{x},k_T') \equiv h_{1T}^N(\hat{x},k_T') + \frac{k_T'^2}{2m_N^2} h_{1T}^{\perp N}(\hat{x},k_T') \tag{2.12}$$

is the nucleonic transversity.

As with the decomposition of the unpolarized quark distribution f_1^A, there is a trivial channel which obtains the transversely-polarized quarks from the intrinsic Boer-Mulders distribution $h_1^{\perp N}$ of unpolarized nucleons, distributed symmetrically throughout the nucleus. But now there are two nontrivial channels by which transversely-polarized quarks can be obtained from transversely-polarized nucleons which orbit inside the nucleus due to the $\vec{L}_N \cdot \vec{S}_N$ coupling. If the quark and nucleon transverse polarizations are parallel, then the quarks arise from the nucleonic transversity distribution h_1^N with the kinematic factor $(P^+b^-)(\underline{p}\cdot\underline{k})$. If the quark and nucleon transverse polarizations are perpendicular, then the quarks arise from the nucleonic pretzelosity distribution $h_{1T}^{\perp N}$ with the different kinematic factor $(P^+b^-)[\frac{(\underline{p}\times\underline{k}')(\underline{k}\times\underline{k}')}{k_T'^2} - \frac{1}{2}(\underline{p}\cdot\underline{k})]$.

3. Conclusions

By studying the TMD's of a heavy nucleus, we make the substructure of the TMD's amenable to first-principles calculation because the multiple scattering contained in the gauge link $\mathscr{U}$ is dominated by independent rescattering on spectator nucleons. These effects can be resummed and calculated explicitly. In this way, it becomes possible to decompose the TMD structure of a heavy nucleus in terms of the TMD's of the nucleons, their motion within the nucleus, and the calculable effects of multiple scattering.

A generic feature which arises by considering such a decomposition is the possibility of TMD mixing. One has not only the possibility of building up a nuclear TMD f^A from the same nucleonic TMD f^N with p_T-broadening due to multiple rescattering; one has also the possibility of building up nuclear TMD's from different nucleonic TMD's. In the case of an unpolarized nucleus, this occurs in (2.9) due to the admixture of the nucleonic Sivers function $f_{1T}^{\perp N}$ into the nuclear quark distribution f_1^A. (The converse – the admixture of f_1^N into the nuclear Sivers function $f_{1T}^{\perp A}$ – was previously explored in [4].) It also occurs in (2.11) due to the admixture of the nucleonic transversity h_1^N and pretzelosity $f_{1T}^{\perp N}$ distributions into the nuclear Boer-Mulders function $h_1^{\perp A}$.

This mixing connects the nuclear and nucleonic TMD's with opposite time-reversal symmetry; in this case the $\vec{L}_N \cdot \vec{S}_N$ coupling generates the preferred directions, while the multiple scattering provides the necessary time-reversal breaking. Interestingly, the same spin-orbit coupling term is responsible for TMD mixing in multiple sectors; in this case, the $\vec{L}_N \cdot \vec{S}_N$ coupling parameterized by W_{OAM} is responsible for the TMD mixing in both the f_1^A sector as well as the $h_1^{\perp A}$ sector. This in principle provides a testable prediction of the theory: one could attempt to extract the spin-orbit coupling W_{OAM} directly by measuring the admixture of $f_{1T}^{\perp N}$ into f_1^A, which then provides a prediction for the admixture of h_1^N and $h_{1T}^{\perp N}$ into $h_1^{\perp A}$. All of these are generic features which should always occur when both multiple scattering and spin-orbit coupling are present. These conclusions should also hold for the more complex case of a polarized nucleus with polarized nucleons, and possibly even for the case of generalized transverse-momentum-dependent parton distributions (GTMD's) [7]. This latter possibility would imply that a small number of spin-orbit coupling structures could be responsible for a large number of mixings simultaneously in the TMD sector and in the GPD sector.

Finally, this approach is well-suited for phenomenological and modeling applications. One starts with an input for the Wigner distribution of nucleons in the nucleus. For instance, one could choose a uniform distribution of static nucleons,

$$W_{unp}(\vec{p},\vec{b}) \stackrel{R.F.}{=} \frac{3\pi^2}{R^3}\theta(R^2-\vec{b}^2)\delta^3(\vec{p}) \tag{3.1}$$

and model the nucleonic TMD's with those of a quark target

$$f_1^N(x,k_T) = \frac{\alpha_s C_F}{2\pi^2 k_T^2}\frac{1+x^2}{1-x}; \tag{3.2}$$

one then recovers the quark distribution at small-x previously calculated with other techniques in [8]. Similarly one could model a nontrivial spin-orbit coupling term

$$W_{OAM}(p,b) \approx \xi\,\frac{g^{+-}(2\pi)^3}{2A}\,\rho(\underline{b},b^-)\,\frac{e^{-p_T^2/m_N^2}}{\pi m_N^2}\,\delta\left(p^+ - \frac{P^+}{A}\right), \tag{3.3}$$

and compute the subsequent correction to the nuclear TMD's. For serious phenomenological application, one could take the Wigner distributions of nucleons in the nucleus from wave functions obtained in realistic nuclear structure calculations, combine this with experimental data on the TMD's of the proton, and obtain a prediction for the TMD's of nuclei.

Thus, this work opens the door to much more theoretical and phenomenological investigation. Another crucial component which will be necessary for a serious attempt at phenomenology is the inclusion of quantum evolution effects at both small and moderate x. We explore aspects of these issues in a separate article in this Proceedings, in our paper [1], and in additional forthcoming work.

Acknowledgments

This material is supported in part by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under Award Number DE-SC0004286 and in part under DOE Contract No. DE-SC0012704.

References

[1] Y. V. Kovchegov and M. D. Sievert, arXiv:1505.01176 [hep-ph].

[2] D. Boer and P. J. Mulders, Phys. Rev. D **57**, 5780 (1998) [hep-ph/9711485].

[3] P. J. Mulders and R. D. Tangerman, Nucl. Phys. B **461**, 197 (1996) [Nucl. Phys. B **484**, 538 (1997)] [hep-ph/9510301].

[4] Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D **89**, no. 5, 054035 (2014) [arXiv:1310.5028 [hep-ph]].

[5] M. Diehl and P. Hagler, Eur. Phys. J. C **44**, 87 (2005) [hep-ph/0504175].

[6] E. Leader and C. Lorcé, Phys. Rept. **541**, 163 (2014) [arXiv:1309.4235 [hep-ph]].

[7] C. Lorcé, Private Communication

[8] K. Itakura, Y. V. Kovchegov, L. McLerran and D. Teaney, Nucl. Phys. A **730**, 160 (2004) [hep-ph/0305332].