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Theoretical uncertainties and dependence on the C_1, C_2, C_3 parameters in the CSS formalism in Drell-Yan and SIDIS

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We study the theoretical uncertainties of the transverse momentum resummed cross sections in the Collins Soper Sterman formalism related to the scale parameters C_1, C_2 and C_3.

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Perturbative theories allow to solve a problem expressing its solution in power series of some small scale. In this way an approximate solution can be found by truncating the expansion at some fixed order in that scale. This is the case of perturbative QCD (pQCD) where perturbative methods are extensively applied. Evaluating the errors due to the truncation of the expansions is non trivial and empirical methods are used. One common procedure is to vary the scale and to obtain an uncertainty band which reflects the error associated with the (arbitrary) choice of that scale. Let us consider, for instance, the Drell-Yan (DY) or the Deep Inelastic Scattering (DIS) cross sections: they depend on the factorization scale through logarithmic terms of the type $\log(Q/\mu_F)$ where Q^2 is the virtuality of the photon (i.e. the hard scale of the process). To "optimize" the expansion, a common choice is to set $\mu_F = Q$. However a full non-truncated calculation at all orders must not depend on this scale. The truncation induces a (spurious) dependence on μ_F. The size of this dependence is an empirical measurement of the quality of the truncation. Conventionally, theoretical uncertainties are evaluated by varying μ_F in the range $Q/2 < \mu_F < 2Q$ and showing a theoretical error band for the results. some order, the truncation induces a dependence on this scale. The size of this dependence is an empirical measurement of the quality of the truncation. As an example we can consider the Drell-Yan (DY) or the Deep Inelastic Scattering (DIS) cross sections in which logs of the type $\log(Q/\mu_F)$ appear, where μ_F is the factorization scale and Q the mass of the photon (i.e. the hard scale of the process). To "optimize" the expansion a common choise is to set $\mu_F = Q$. Here we will adopt a similar approach for the study of the transverse momentum resummed cross sections in the Collins Soper Sterman (CSS)[1] formalism where several scales appear. Quantifying the theoretical uncertainties can help to identify the perturbative regions in the calculation, and to estimate the relevance of the non-perturbative parameters extracted from data fitting. Following Refs. [1, 2] we can write the transverse momentum dependent DY cross section as:

$$\begin{aligned}\frac{d\sigma}{dQ^2dydq_T^2} &= \pi\sigma_0^{DY}\left\{\int\frac{d^2b_Te^{iq_T\cdot b_T}}{(2\pi)^2}W^{DY}(x_1,x_2,b_*,Q,C_1,C_2,C_3)F_{NP}^{DY}(x_1,x_2,b_T,Q)\right\}\\ &+Y(x_1,x_2,q_T,Q,C_4),\end{aligned} \tag{1}$$

where the first term on the right hand side is the resummed cross section and the second term is the so called Y-factor (or matching factor). Here Q, y and q_T are respectively the mass, the rapidity and the transverse momenta of the virtual boson (i.e. of the dilepton) in the hadron-hadron c.m. frame while x_1 and x_2 are the parton momentum fractions. The factor σ_0 is given by:

$$\sigma_0^{DY} = \frac{4\pi\alpha^2}{9Q^2s} \tag{2}$$

where s is the the square root of c.m. energy in hadron-hadron c.m. frame. Eq. (1) holds also for Z_0 production with proper modifications, see Refs. [1, 2] for further details.

The quantity W^{DY} is the perturbatively calculated part of the resummed cross section and is given by:

$$W^{DY}(x_1,x_2,b_T,Q,C_1,C_2,C_3) = \sum_{j=q,\bar{q}}e_j^2\sum_{i,k}\exp\left[S(b_T,Q,C_1,C_2)\right]\left[C_{ji}\otimes f_i\right]\left[C_{\bar{j}k}\otimes f_k\right] \tag{3}$$

where:

$$[C_{ji} \otimes f_i] = \int_x^1 \frac{dz}{z} C_{ji}(z, b, \mu = C_3/b_T, C_1, C_2) f_i(x/z, \mu = C_3/b_T) \tag{4}$$

represents the convolution of the Wilson coefficients C_{ij} and the unpolarized collinear parton distributions (PDFs) for the parton i, $f_i(x,\mu)$ and

$$S(b_T, Q, C_1, C_2) = -\int_{C_1^2/b_T^2}^{C_2^2 Q^2} \frac{d\kappa^2}{\kappa^2} \left[A(\alpha_s(\kappa), C_1) \ln\left(\frac{C_2^2 Q^2}{\kappa^2}\right) + B(\alpha_s(\kappa), C_1, C_2) \right] \tag{5}$$

is the so called perturbative Sudakov form factor. Finally F_{NP} is the non-perturbative Sudakov form factor necessary to model the large (non-perturbative) b_T-region. C_1, C_2, C_3 and C_4 are constant parameters related to the relevant scales that appear in the resummed cross section and in the Y-factor, namely:

$$\mu_1(b_T) = \frac{C_1}{b_T} \quad \mu_2(Q) = C_2 Q \quad \mu_3(b_T) = \frac{C_3}{b_T} \quad \mu_4(Q) = C_4 Q\,. \tag{6}$$

It is clear from Eq. (6) that the scale μ_1 and μ_3 become soft at large b_T. For this reason, in the perturbative parts of the resummed cross section, W, b_T is replaced by b_*:

$$b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}\,, \tag{7}$$

according to the so-called b_* prescription, see Ref. [1]. Analyzing Eq. (5) we can see that the scale $\mu_1 = \frac{C_1}{b_T}$ appear as lower limit of integration in the Sudakov form factor. It is, in fact, the soft scale at which the perturbative resummation starts. The scale $\mu_2 = C_2 Q$ is instead the hard scale up to which the integration is performed. Similarly, $\mu_4 = C_4 Q$ is the hard scale in the Y-factor. The scale $\mu_1 = \frac{C_3}{b_T}$ is the soft scale at which the PDFs are calculated and convoluted to the Wilson coefficients. Notice that in the original CSS paper [1] $\mu_1 \equiv \mu_3$, i.e. there is no distinction between these two scales.

The functions A, B in Eq. (17) and C in Eq.(4) can be expanded in powers of the strong coupling, α_s:

$$A(\alpha_s(\mu)) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\mu)}{\pi}\right)^n A^{(n)} \tag{8}$$

$$B(\alpha_s(\mu)) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\mu)}{\pi}\right)^n B^{(n)} \tag{9}$$

$$C_{ji}(z, \alpha_s(\mu)) = \delta_{ij}\delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\mu)}{\pi}\right)^n C_{ij}^{(n)}(z) \tag{10}$$

$$\tag{11}$$

and depend on the parameters C_1, C_2 and C_3. If we want to calculate the Sudakov form factor at Next to Leading Log accuracy (NLL) we need the coefficients $A^{(1)}$, $A^{(2)}$ and $B^{(1)}$:

$$A^{(1)}(C_1) = C_F, \quad A^{(2)}(C_1) = \frac{C_F}{2} \left[\left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_G - \frac{5}{9} N_f - \beta_0 \ln(b_0/C_1) \right] \tag{12}$$

$$B^{(1)}(C_1, C_2) = -\frac{C_F}{2} [3 + 4\ln(C_2 b_0/C_1)] \tag{13}$$

where

$$b_0 = 2\exp(-\gamma_E) \tag{14}$$

and γ_E is the Euler-gamma. The Wilson coefficients at Next to Leading Order (NLO) in α_s are given by:

$$\begin{aligned} C^{(1)}_{jk}(z,b,\mu,C_1,C_2) &= \delta_{jk}\frac{C_F}{2}\Bigg\{(1-z)-2\ln(\mu b/b_0)\left[\frac{1+z^2}{1-z}\right]_+ \\ &+ \delta(1-z)\left[-2\ln^2\left(\frac{C_1}{b_0C_2}\exp(-3/4)\right)+\frac{\pi^2}{2}-\frac{23}{8}\right]\Bigg\} \end{aligned} \tag{15}$$

and

$$C^{(1)}_{jg}(z,b,\mu) = T_F\Bigg\{[z(1-z)]-\ln(\mu b/b_0)[z^2+(1-z)^2]\Bigg\} \tag{16}$$

with $\mu \equiv \mu_3(b) = C_3/b$. Looking at Eqs. (13), (15) and (16) we can see that several logs containing the scales in Eq. (6) appears. Similarly to the pQCD all this logs can be set to one, "optimizing" [1] the perturbative expansion, with a smart choice of the parameters C_1, C_2, C_3 and C_4. The canonical choice is $C_1 = C_3 = b_0$, $C_2 = C_4 = 1$.

Similarly for Semi-Inclusive Deep Inelastic Scattering we can write [3, 4]:

$$\begin{aligned} \frac{d\sigma}{dx\,dy\,dz\,dq_T^2} &= \pi\sigma_0^{DIS}\Bigg\{\int\frac{d^2\mathbf{b_T}e^{i\mathbf{q_T}\cdot\mathbf{b_T}}}{(2\pi)^2}W^{SIDIS}(x,z,b_*,Q,C_1,C_2,C_3)F_{NP}^{SIDIS}(x,z,b_T,Q) \\ &+Y^{SIDIS}(x,z,q_T,Q,C_4)\,, \end{aligned} \tag{17}$$

where q_T is the virtual photon momentum in the frame where the incident nucleon N and the produced hadron h are head to head, and

$$\sigma_0^{DIS} = \frac{4\pi\alpha_{\rm em}^2}{sxy^2}\left(1-y+\frac{y^2}{2}\right), \tag{18}$$

where x is the Bijorken variable and y the DIS inelasticity. Notice that, for SIDIS, it is common to quote the results as function of the transverse momentum $\mathbf{P_T}$ of the final detected hadron, h, in the γ^*N c.m. frame, rather than to the virtual photon momentum $\mathbf{q_T}$, in the Nh c.m. frame. They are related by the hadronic momentum fraction z through the expression $|\mathbf{P}_T| = z|\mathbf{q_T}|$, so that

$$\frac{d\sigma}{dx\,dy\,dz\,dP_T^2} = \frac{d\sigma}{dx\,dy\,dz\,dq_T^2}\frac{1}{z^2}. \tag{19}$$

The perturbative part of the resummed cross section, W^{SIDIS} is given by:

$$W(x,z,b_T,Q,C_1,C_2,C_3) = \sum_{j=q,\bar{q}} e_j^2 \sum_{i,k}\exp\left[S(b_T,Q,C_1,C_2)\right]\left[C^{in}_{ji}\otimes f_i\right]\left[C^{out}_{kj}\otimes D_k\right] \tag{20}$$

where S is the Sudakov form factor, C^{in} and C^{out} the Wilson coefficients, $f_i(x,\mu)$ the collinear PDF and $D_k(z,\mu)$ are the usual collinear fragmentation functions for the parton k. In principle, in the CSS approach, the Sudakov and the Wilson coefficient are process dependent. However it can be shown that at NLL the DY ans SIDIS Sudakov form factor are the same while for the Wilson coefficients we have:

$$C_{jk}^{(1)in}(z,b,\mu,C_1,C_2) = \delta_{jk}\frac{C_F}{2}\Bigg\{(1-z)-2\ln(\mu b/b_0)\left[\frac{1+z^2}{1-z}\right]_+ + \delta(1-z)\left[-2\ln^2\left(\frac{C_1}{b_0C_2}\exp(-3/4)\right)-\frac{23}{8}\right]\Bigg\} \tag{21}$$

$$C_{jg}^{(1)in}(z,b,\mu) = T_F\Big\{[z(1-z)]-\ln(\mu b/b_0)[z^2+(1-z)^2]\Big\} \tag{22}$$

$$C_{jk}^{(1)out}(z,b,\mu,C_1,C_2) = \delta_{jk}\frac{C_F}{2}\Bigg\{(1-z)-2\ln\left(\frac{\mu b}{b_0 z}\right)\left[\frac{1+z^2}{1-z}\right]_+ + \delta(1-z)\left[-2\ln^2\left(\frac{C_1}{b_0C_2}\exp(-3/4)\right)-\frac{23}{8}\right]\Bigg\} \tag{23}$$

$$C_{gj}^{(1)out}(z,b,\mu) = \frac{C_F}{2}\left[z-2\ln\left(\frac{\mu b}{b_0 z}\right)\frac{1+(1-z)^2}{z}\right] \tag{24}$$

with $\mu \equiv \mu_3(b) = C_3/b$. As mentioned in the introduction, we want to evaluate the theoretical errors in the resummed cross section. We adopt here the conventional procedure of pQCD. Therefore we evaluate our cross sections varying the coefficients C_1, C_2 and C_3 in the ranges:

$$b_0/2 < C_1 < 2b_0 \tag{25}$$

$$1/2 < C_2 < 2 \tag{26}$$

$$b_0/2 < C_3 < 2b_0\,. \tag{27}$$

We show the results varying one scale and taking the others fixed to the value of the canonical choice. As the distinction between C_1 and C_3 is not so widely accepted, we also show the results calculated taking $C_1 = C_3$. Since here we do not take into account the matching factor Y we will not vary the coefficient C_4.

When we evaluate theoretical errors at some fixed order we should not pay attention to the absolute size, rather we should compare these errors with the subsequent order in the expansion or with the experimental errors. Here we will follow the latter approach.

In order to make a comparison of our theoretical resummed cross section and DY data we need an explicit form of the non perturbative function F_{NP}^{DY}. We choose the so called BLNY parametrization [5]:

$$F_{NP}^{DY}(x_1,x_2,Q) = \exp\left\{\left[-g_1-g_2\ln(Q/(2Q_0))-g_1g_3\ln(100x_1x_2)\right]b_T^2\right\} \tag{28}$$

where the parameters g_1, g_2 and g_3 are those extracted in Ref. [5]:

$$g_1 = 0.21\,\mathrm{GeV}^2\,,\quad g_2 = 0.68\,\mathrm{GeV}^2\,,\quad g_3 = -0.6 \tag{29}$$

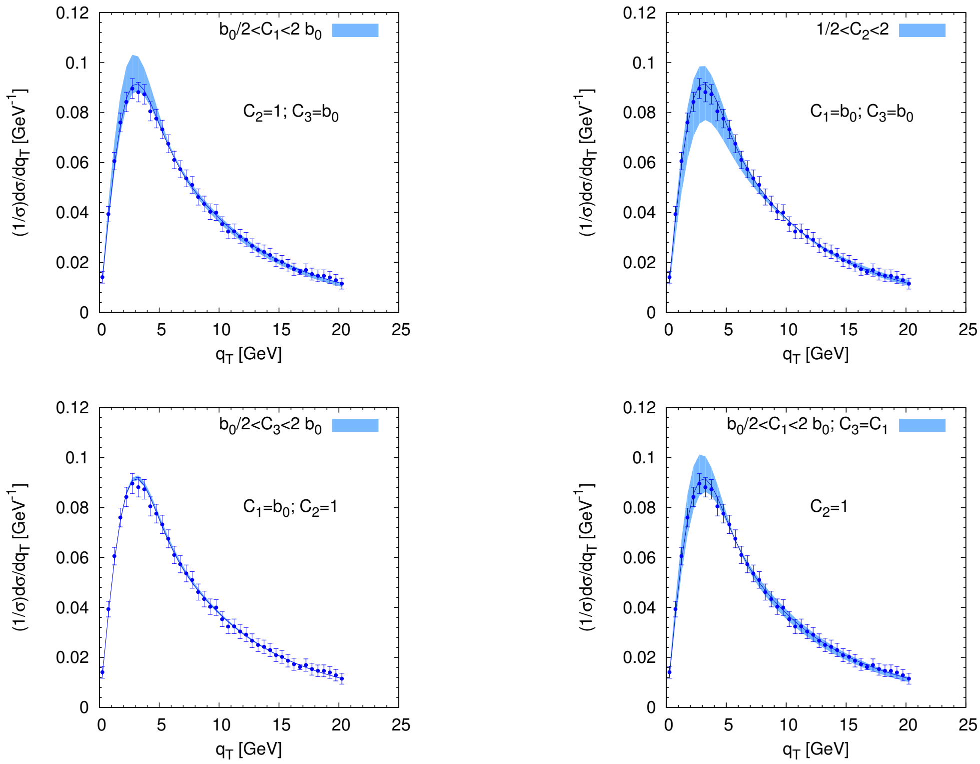

Figure 1: CDF Run II data [6] compared to the resummed cross section calculated using the non-perturbative parameters in as in Ref. [5] and by varying C_1 (left upper plot), C_2 (right upper plot), C_3 (left lower plot) and $C_1 = C_3$ (right lower plot).

and

$$b_{max} = 0.5\,\text{GeV}^{-1}, \quad Q_0 = 1.6\,\text{GeV}\,. \tag{30}$$

These parameters describe the DY cross section for the canonical choice $C_1 = C_3 = b_0, C_2 = C_4 = 1$. As paradigm in our study, here we consider two experiments at rather different energies: the CDF run II [6], at $\sqrt{s} = 1.96$ TeV and $Q = M_{Z_0}$ and the Fermilab E288 [7] at $\sqrt{s} = 23.6$ GeV and $5 < Q < 6$ GeV. We can see from Fig. 1 that at Tevatron energies, at large q_T, the cross section is largely unaffected by scale errors. At large transverse momentum the calculation is mainly perturbative and Fig. 1 shows that a NLL-NLO computation is, in this region, sufficiently accurate. On the contrary at small q_T the size of the band increases. This means that here we are approaching the non-perturbative region where the perturbative part cannot be trusted anymore. It is evident that if we were to refit the data using a different choice of parameters C_1, C_2 and C_3 we would obtain slightly different values of the non perturbative parameters. At smaller energies like in Fig. 2 the error band is large over the whole range explored and in this case refitting would give rather different parameters.

Current available SIDIS data as functions of transverse momenta come from low energy experiments, namely: HERMES ($\sqrt{s} \simeq 7$ GeV) and COMPASS($\sqrt{s} \simeq 18$ GeV). Since the center of mass energy is very small it is not clear to which extent CSS resummation can be applied. In

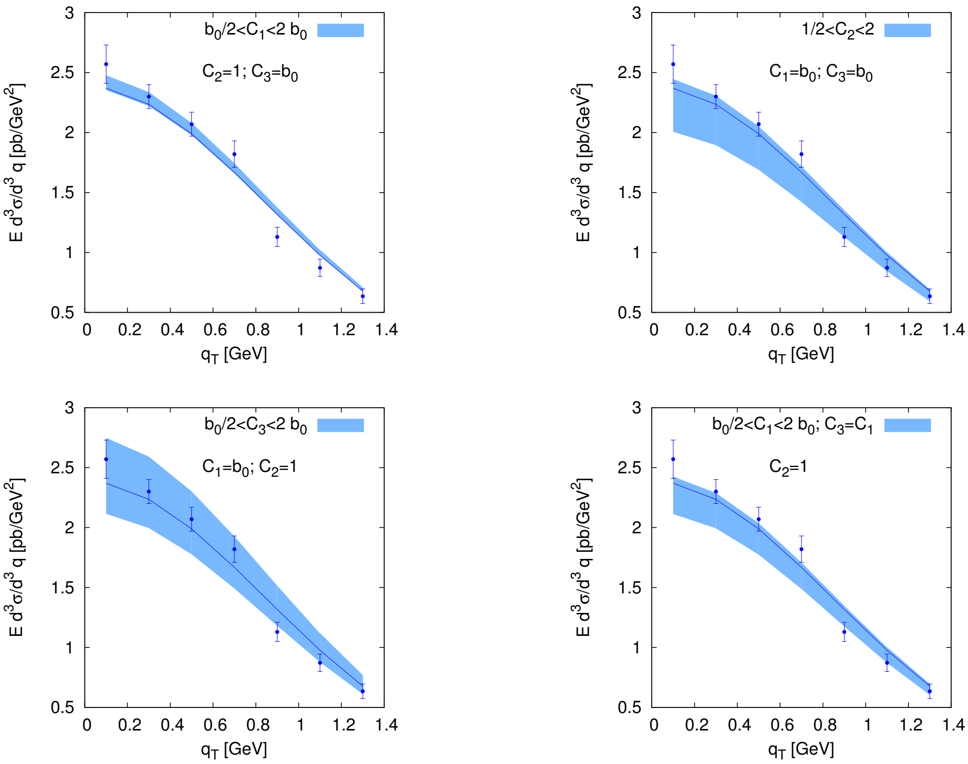

Figure 2: Fermilab E288 data [7] at $\sqrt{s} = 23.6$ GeV and $5 < Q < 6$ GeV compared to the resummed cross section calculated using the non-perturbative parameters in as in Ref. [5] and by varying C_1 (left upper plot), C_2 (right upper plot), C_3 (left lower plot) and $C_1 = C_3$ (right lower plot).

fact, for these experiments the bulk of the data is at low $Q \simeq 1-2$ GeV and low $P_T \lesssim 1.5$ GeV ($q_T \sim P_T/z$). In particular, it seems that current phenomenological implementations fail to catch the proper normalization of the data [8]. In this work we do not intend to make a comprehensive study of these data, therefore we will simply perform a study of the theoretical errors for HERMES multidimensional multiplicity data of pion off proton target [9] in a particular bin corresponding to $\langle Q^2 \rangle = 9.21$ GeV^2 (the last bin), $\langle z \rangle = 0.35$ and $\langle x \rangle = 0.34$. To compare theory and data, for this bin, we find a theoretical curve which describes the data in an acceptable way. To this aim we consider the following non-perturbative function:

$$F_{NP}^{SIDIS}(x,z,Q) = \exp\left\{\left[-\frac{g_1 + g_{1f}/z^2}{2} - g_2 \ln(Q/(2Q_0)) - g_1 g_3 \ln(10x)\right] b_T^2\right\} \tag{31}$$

where g_1, g_2 and g_3 are the BLNY [5] parameters, Eq. (29), and b_{max} and Q_0 are those in Eq. (30), while g_{1f} is a free parameter for the fragmentation. Introducing a normalization factor N, we find that the values:

$$g_{1f} \simeq 0.15 \text{ GeV}^2 \qquad N \simeq 2.5 \tag{32}$$

can reproduce the data well. Fig 3 shows how this HERMES bin is described with and without normalization. Now we can perform our study on the theoretical errors varying C_1, C_2 and C_3 as we

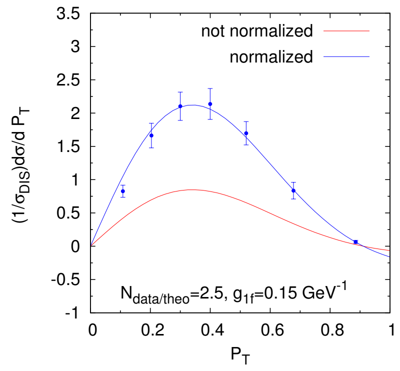

Figure 3: HERMES multidimensional multiplicity data of pion off proton target [9] at $\langle Q^2 \rangle = 9.21$ GeV^2, $\langle z \rangle = 0.35$ and $\langle x \rangle = 0.34$. The red and blue lines are the resummed cross section and the normalized resummed cross section, respectively.

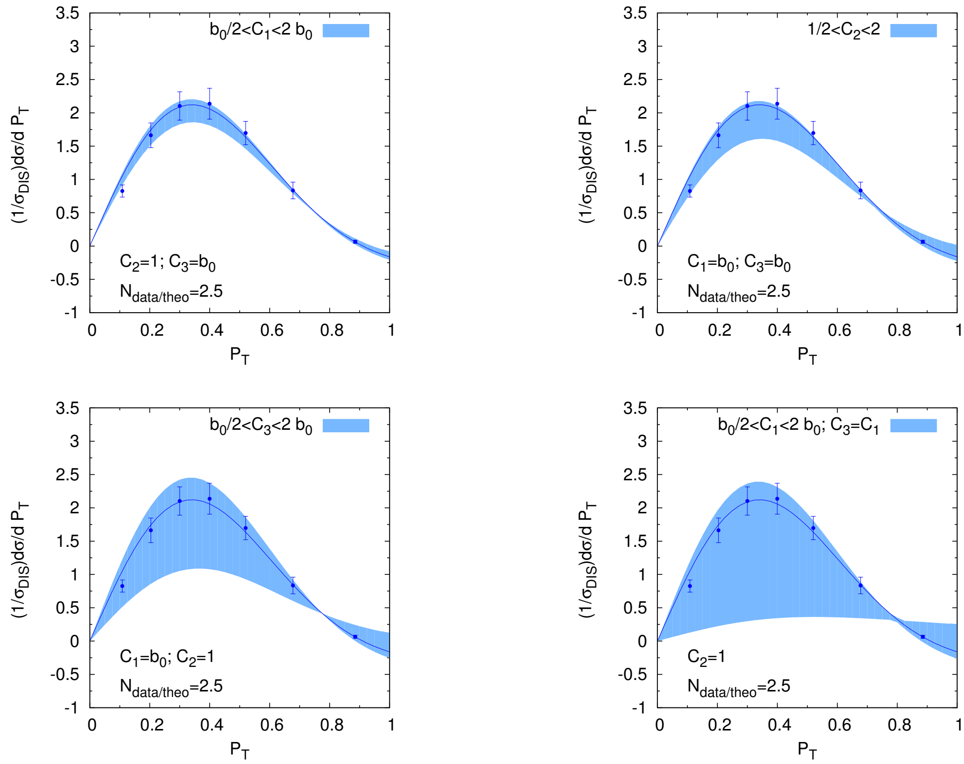

Figure 4: HERMES multidimensional multiplicity data of pion off proton target [9] at $\langle Q^2 \rangle = 9.21$ GeV^2, $\langle z \rangle = 0.35$ and $\langle x \rangle = 0.34$ compared to the resummed cross section by varying C_1 (left upper plot), C_2 (right upper plot), C_3 (left lower plot) and $C_1 = C_3$ (right lower plot)

did for the DY case. The results can be seen in Fig 4. As expected, in this case the theoretical error is very large. The cross section covers a completely non-perturbative regime, and the perturbative part is actually calculated in a region where it is not perturbative at all. The variation of C_2, for instance, reflects the fact that the perturbative Sudakov strongly changes, changing the upper integration limit. Again, this is expected because we are calculating quantities at relatively small scales. Notice that the bin that we choose is the last in Q^2: this means that the majority of data are at even lower Q^2. The main point here is that the theoretical errors strongly affect the normalization of the data. So it is not surprising that they cannot be properly normalized. Whether a NNLL calculation could attenuate this problem remains to be investigate. Another important warning is that we are blindingly applying the resummation formalism. However one should not forget that, if on one side q_T is very small, on the other side we are stretching the resummation approach in a region where $q_T \sim Q$. In fact, only a few points of the HERMES data fulfill the condition $q_T \lll Q$ ($q_T \sim P_T/z$): if we strictly fulfill this condition, probably the few points left are not enough to allow to perform a fit of the non-perturbative part. Indeed, a proper description of this region would require a matching factor (Y-factor). However we found that the matching is actually rather difficult [10] and it could, in principle, be subject to similar theoretical errors.

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