

Chiral-odd GPDs in large- N_c QCD

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We study the flavor structure of the nucleon's chiral-odd generalized parton distributions (transversity GPDs) in the large- N_c limit of QCD. It is found that in the distributions H_T and \tilde{E}_T the flavor-nonsinglet component $u-d$ is leading in the $1/N_c$ expansion, while in E_T and \tilde{H}_T it is the flavor-singlet component $u+d$. This pattern is consistent with the flavor structure extracted from hard exclusive π^0 and η electroproduction data, assuming that the processes are dominated by the twist-3 mechanism involving the chiral-odd pseudoscalar meson distribution amplitude.

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1. Introduction

Generalized parton distributions (or GPDs) unify the concepts of parton density and elastic form factor and enable a comprehensive description of the nucleon's quark and gluon single-particle structure in QCD; see Refs. [1, 2, 3, 4] for a review. At the leading-twist level the nucleon's quark structure is described by 4 chiral-even (quark helicity-conserving) and 4 chiral-odd (quark helicity-flipping) GPDs [5]. The chiral-even GPDs reduce to the usual unpolarized and polarized quark parton distribution functions (PDFs) in the limit of zero momentum transfer. These GPDs appear in the amplitudes of hard exclusive processes such as deeply virtual Compton scattering and exclusive meson production with longitudinal photon polarization, for which QCD factorization theorems have been derived, and can be accessed experimentally in this way [6, 7, 8, 9]. The chiral-odd GPDs reduce to the quark transversity PDFs in the limit of zero momentum transfer. Relating these GPDs to hard exclusive processes has proved to be more challenging, as they decouple from vector meson production at leading twist in all orders in perturbative QCD due to the chirality requirements for massless fermions [10]. The chiral-odd GPDs have been associated with the diffractive electroproduction of two vector mesons at leading twist [11], but the process is difficult to measure [12, 13, 14], and no data are available at present. Recent work suggests that pseudoscalar meson electroproduction (π^0, η, π^+) may be dominated by a twist-3 mechanism involving the chiral-odd nucleon GPDs and the chiral-odd distribution amplitude, which originates from the dynamical breaking of chiral symmetry in QCD [15, 16, 17, 18]. While no factorization theorem exists at this level, the pseudoscalar production amplitudes have been calculated in the modified hard scattering approach of Ref. [16, 17], which implements suppression of large-size $q\bar{q}$ configurations in the meson through the QCD Sudakov form factor. The results agree well with the π^0 and η electroproduction data from the JLab CLAS experiment at 6 GeV incident energy, regarding both the absolute cross sections and the L/T ratio as inferred from the azimuthal-angle-dependent response functions [19, 20]. This has opened the prospect of more detailed studies of the chiral-odd GPDs in further measurements of pseudoscalar meson electroproduction with the JLab 12 GeV Upgrade.

To further explore this possibility, it is necessary to gain more insight into the properties of the chiral-odd GPDs and their relation to other measures of nucleon structure. Contrary to the chiral-even GPDs, in the chiral-odd case neither the zero-momentum transfer limit of the GPDs (transversity PDFs) nor the local operator limit of the GPD (form factor of local tensor operator) correspond to structures that are easily measurable, so that little useful information can be obtained in this way. The transversity PDFs can be extracted from observables in semi-inclusive deep-inelastic scattering; see Refs. [21, 22, 23] and references therein. The form factors of local tensor operators, which constrain the lowest x -moment of the chiral-odd GPDs, have been calculated in lattice QCD [24] and in the chiral quark-soliton model [25, 26, 27]. The x -dependent chiral-odd GPDs have been studied in quark bound-state models of nucleon structure [28, 29, 30]. Besides these estimates not much is known about the properties of the chiral-odd GPDs.

Here we report about a study of the flavor structure of the chiral-odd GPDs in QCD in the limit of a large number of colors (large- N_c limit) [31]. The large- N_c limit represents a rigorous approach to QCD in the non-perturbative domain and leads to model-independent relations governing meson and nucleon structure at hadronic energies [32]. In the large- N_c limit QCD becomes semi-classical,

and baryons can be described by mean field solutions in terms of meson fields [33]. It is assumed that the qualitative properties of QCD, such as the dynamical breaking of chiral symmetry in the ground state, are preserved as the limit is taken [34]. While the dynamics at large N_c remains complex and cannot be solved exactly, and the form of the mean field solution is not known, important insights can be obtained by exploiting known symmetry properties of the mean field [35, 36]. In this way one obtains relations for baryon mass splittings, meson-baryon coupling constants, electromagnetic and axial form factors, and other observables, which are generally in good agreement with observations, see Ref. [37] for a review. In the matrix elements of quark bilinear operators (vector or axial vector currents, tensor operators) the large- N_c limit identifies leading and subleading spin-flavor components. The approach can be extended to parton densities [38, 39], where e.g. it suggests a large flavor asymmetry of the polarized antiquark distribution $\Delta\bar{u} - \Delta\bar{d}$ as seems to be supported by the recent RHIC W^\mp production data [40, 41].

Our study generalizes previous results on large- N_c relations for chiral-even GPDs [1], quark transversity distributions [42, 43, 44], and matrix elements of local chiral-odd operators [25, 26, 27] and relies on the formal apparatus developed in these earlier works (see Sec.3 of Ref. [1]). In this note we review the basic properties of chiral-odd GPDs (Sec. 2), describe the $1/N_c$ expansion of the chiral-odd GPDs (Sec. 3), and compare the pattern with the pseudoscalar meson electroproduction data (Sec. 4).

2. Chiral-odd GPDs

GPDs parametrize the non-forward nucleon matrix elements of QCD light-ray operators of the general form [1, 2, 3, 4]

$$\mathcal{M}(\Gamma) = P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle N(p', \lambda') | \psi(-\frac{z}{2}) \Gamma \psi(\frac{z}{2}) | N(p, \lambda) \rangle \Big|_{z^+=0, \vec{z}_T=0}, \quad (2.1)$$

where $P \equiv \frac{1}{2}(p' + p)$ is the average nucleon 4-momentum, z is the displacement of the quark fields, and the 4-vectors are described by their light-cone components $z^\pm = (z^0 \pm z^3)/\sqrt{2}$, $\vec{z}_T = (z^1, z^2)$, etc. Γ denotes a generic matrix in spinor and flavor indices and defines the spin-flavor quantum numbers of the operator. In the chiral-odd case the spinor matrix is of the form $\Gamma = i\sigma^{+j}$, and the matrix element is parametrized as [5]

$$\begin{aligned} \mathcal{M}(i\sigma^{+j}T^q) = \bar{u}(p', \lambda') & \left[i\sigma^{+j} H_T^q + \frac{P^+\Delta^j - \Delta^+P^j}{M_N^2} \tilde{H}_T^q \right. \\ & \left. + \frac{\gamma^+\Delta^j - \Delta^+\gamma^j}{2M_N} E_T^q + \frac{\gamma^+P^j - P^+\gamma^j}{M_N} \tilde{E}_T^q \right] u(p, \lambda). \end{aligned} \quad (2.2)$$

The GPDs $H_T^q = H_T^q(x, \xi, t)$, etc., are functions of the quark plus momentum fraction x , the plus momentum transfer $\xi = -\Delta^+/(2P^+) = (p - p')^+/(p + p')^+$, and the invariant momentum transfer $t = \Delta^2 = (p' - p)^2$. (For brevity we do not indicate the dependence of the GPDs on the normalization scale.) Here we consider GPDs corresponding to a given quark flavor $q = (u, d)$, and T^q denotes the corresponding projector on quark flavor indices.

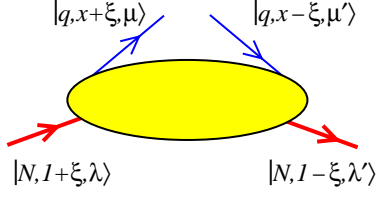


Figure 1: Representation of GPDs in the region $\xi < x < 1$ as nucleon–quark helicity amplitudes. In the nucleon and quark states (denoted as N, q) the second label denotes the fraction of the light-cone plus momentum P^+ carried by the particle, and the third label denotes the light-cone helicity.

As their chiral-even counterparts, the chiral-odd GPDs satisfy certain symmetry relations in ξ due to time reversal invariance,

$$\text{GPD}(x, -\xi, t) = \begin{cases} +\text{GPD}(x, \xi, t) & \text{for GPD} = H_T^q, \tilde{H}_T^q, E_T^q, \\ -\text{GPD}(x, \xi, t) & \text{for GPD} = \tilde{E}_T^q. \end{cases} \quad (2.3)$$

Their integrals over x (or first moments) coincide with the form factors of the local tensor operator $\bar{\psi}(0)i\sigma^{\mu\nu}\psi(0)$,

$$\begin{aligned} \int dx H_T^q(x, \xi, t) &= F_T^q(t), \\ \int dx \tilde{H}_T^q(x, \xi, t) &= \tilde{F}_T^q(t), \\ \int dx E_T^q(x, \xi, t) &= E_T^q(t), \\ \int dx \tilde{E}_T^q(x, \xi, t) &= 0. \end{aligned} \quad (2.4)$$

The vanishing of the first moment of \tilde{E}_T^q is a consequence of the antisymmetry in ξ , Eq. (2.3); its higher moments are non-zero. More generally, the higher x -moments of the chiral-odd GPDs are polynomials in ξ (generalized tensor form factors).

In the limit of zero momentum transfer (forward limit) the chiral-odd GPD H_T^q reduces to the transversity PDF $h_1^q(x)$,

$$H_T^q(x, \xi = 0, t = 0) = h_1^q(x). \quad (2.5)$$

Its first moment is known as the nucleon’s tensor charge. Because the local tensor operator is not a conserved current, the tensor charge is scale-dependent and cannot directly be related to low-energy properties of the nucleon.

The partonic interpretation of the GPDs is described in Refs. [1, 2, 3, 4]. Our study of the large- N_c limit of the chiral-odd GPDs relies essentially on the representation of the GPDs as partonic helicity amplitudes [45]. This representation most naturally appears in the region $\xi < x < 1$, where the GPDs describe the amplitude for the “emission” by the nucleon of a quark with plus momentum fraction $x + \xi$ and subsequent “absorption” of a quark with $x - \xi$ (see Fig. 1). (In the region $-1 < x < -\xi$ the GPDs describe the emission and absorption of an antiquark, while in $-\xi < x < \xi$ they describe the emission of a quark–antiquark pair by the nucleon. We do not need to consider these regions explicitly in the subsequent arguments, as the N_c -scaling of the GPDs is uniform in x .) The initial and final nucleon are described by light-cone helicity spinors, which are obtained from rest-frame spinors (polarized along the z -direction) through a longitudinal and transverse boost; these spinors transform in a simple manner under Lorentz transformations and

have a clear connection to polarization states in the rest frame. The helicity amplitudes are then defined as

$$A_{\lambda'\mu',\lambda\mu}^q = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle N(P',\lambda') | \mathcal{O}_{\mu'\mu}^q | N(P,\lambda) \rangle \Big|_{z^+=0, \bar{z}_T=0}, \quad (2.6)$$

where $\lambda(\lambda')$ are the light-front helicity of the initial (final) nucleon and $\mu(\mu')$ those of the initial (final) quark. Without loss of generality one may choose a frame in which the nucleon 3-momenta \vec{p}, \vec{p}' are in the x-z-plane. The chiral-odd light-ray operator associated with quark helicity flip is given by [5]

$$\mathcal{O}_{+-}^q = \psi_q(-\frac{\bar{z}}{2}) \frac{i}{4} \sigma^{+1} (1 - \gamma_5) \psi_q(\frac{\bar{z}}{2}). \quad (2.7)$$

The helicity amplitudes are related to chiral odd GPDs as

$$A_{++,+}^q = \delta_k \left(\tilde{H}_T^q + \frac{1-\xi}{2} (E_T^q + \tilde{E}_T^q) \right), \quad (2.8a)$$

$$A_{+,-,-}^q = \delta_k \left(\tilde{H}_T^q + \frac{1+\xi}{2} (E_T^q - \tilde{E}_T^q) \right), \quad (2.8b)$$

$$A_{++,-}^q = \sqrt{1-\xi^2} \left(H_T^q + \delta_k^2 \tilde{H}_T^q - \frac{\xi^2}{1-\xi^2} E_T^q + \frac{\xi}{1-\xi^2} \tilde{E}_T^q \right), \quad (2.8c)$$

$$A_{-+,-}^q = \sqrt{1-\xi^2} \delta_k^2 \tilde{H}_T^q, \quad (2.8d)$$

where the ‘‘kinematic’’ prefactor δ_k is defined as

$$\delta_k = \text{sign} \left(P^+ \Delta^1 - \Delta^+ P^1 \right) \frac{\sqrt{t_0 - t}}{2M_N}, \quad -t_0 = \frac{4M_N^2 \xi^2}{1 - \xi^2}, \quad (2.9)$$

in which $-t_0$ is the minimal value of $-t$ for the given value of ξ .

In the $1/N_c$ expansion it is convenient to work with linear combinations of the helicity amplitudes Eqs. (2.8a-2.8d) that have a simple interpretation in terms of spin transitions in the nucleon rest frame. We define them as

$$A_0^q = \frac{1}{2} \left(A_{++,+}^q + A_{+,-,-}^q \right) \propto \delta_{\lambda\lambda'}, \quad (2.10a)$$

$$A_1^q = \frac{1}{2} \left(A_{++,-}^q + A_{-+,-}^q \right) \propto \sigma_{\lambda\lambda'}^1, \quad (2.10b)$$

$$A_2^q = \frac{1}{2} \left(A_{+,-,-}^q - A_{-+,-}^q \right) \propto \sigma_{\lambda\lambda'}^2, \quad (2.10c)$$

$$A_3^q = \frac{1}{2} \left(A_{++,+}^q - A_{+,-,-}^q \right) \propto \sigma_{\lambda\lambda'}^3, \quad (2.10d)$$

where $\sigma_{\lambda\lambda'}^a$ are the Pauli spin matrices. In the nucleon rest frame, these combinations describe spin transitions in which the matrix element of the spin operator has components along the x, y and z axes.

3. $1/N_c$ expansion of chiral-odd GPDs

In the large- N_c the nucleon mass scales as $M_N \sim N_c$, while the nucleon size remains stable, $R \sim N_c^0$. The $1/N_c$ expansion of GPDs is performed in a class of frames where the initial and

final nucleon move with 3-momenta $p^k, p'^k \sim N_c^0$ ($k = 1, 2, 3$) and have energies $p^0, p'^0 = M_N + O(1/N_c)$, which implies an energy and momentum transfer $\Delta^0 \sim N_c^{-1}, \Delta^k \sim N_c^0$, and thus

$$\Delta^i \sim N_c^0 \quad (i = 1, 2), \quad \xi \sim N_c^{-1}, \quad |t| \sim N_c^0. \quad (3.1)$$

In the partonic variable x one considers the parametric region

$$x \sim N_c^{-1}, \quad (3.2)$$

corresponding to non-exceptional longitudinal momenta of the quarks and antiquarks relative to the slowly moving nucleon, $xM_N \sim R^{-1} \sim N_c^0$. Likewise, it is assumed that the normalization scale of the light-ray operator is $\sim N_c^0$, so that the typical quark transverse momenta are $\sim N_c^0$. Equation (3.2) corresponds to the intuitive picture of a nucleon consisting of N_c “valence” quarks and a “sea” of $O(N_c)$ quark-antiquark pairs, with each quark/antiquark carrying on average a fraction $\sim 1/N_c$ of the nucleon momentum. Altogether, the N_c -scaling relations for GPDs can then be expressed in the form

$$\text{GPD}(x, \xi, t) \sim N_c^k \times \text{Function}(N_c x, N_c \xi, t), \quad (3.3)$$

where the scaling exponent k depends on the spin-flavor structure and can be established on general grounds, while the scaling function on the right-hand-side does not explicitly depend on N_c and can only be determined in specific dynamical models.

A general method for the $1/N_c$ expansion of nucleon matrix elements of quark bilinear operators has been given in Ref. [1]. It uses the fact that the large- N_c nucleon is described by a classical field with a certain spin-isospin symmetry, and that the nucleon states of definite spin, isospin, and momentum are obtained through quantization of the (iso-) rotational and translational zero modes [35]. In this classical picture the nucleon matrix element of the quark bilinear operator is obtained by taking the expectation value of the operator in the static classical field, allowing for collective (iso-) rotations, and computing the transition matrix element between collective wave functions corresponding to the desired nucleon spin-isospin and momentum states. While the expectation value of the operator in the classical field cannot be calculated from first principles and remains unknown, its symmetry properties are unambiguously determined by the spin-isospin symmetry of the classical field and the quantum numbers of the quark bilinear operator. Altogether, this allows one to establish the N_c -scaling of the different spin-isospin components of the matrix element without calculating the coefficients accompanying the powers of $1/N_c$.

We use the method of Ref. [1] to establish the N_c -scaling of the nucleon-quark helicity amplitudes and the chiral-odd GPDs. It is natural to work with the combinations Eqs. (2.10a–2.10d), which correspond to definite spin transitions in the frame where the nucleons are moving slowly, and which exhibit homogeneous scaling in $1/N_c$. When expressing the helicity amplitudes in terms of the GPDs, we use that in the large- N_c limit the kinematic factor $\delta_k \sim N_c^{-1}$ in (2.9) and the variable ξ become

$$\delta_k = \frac{\Delta^1}{2M_N}, \quad \xi = -\frac{\Delta^3}{2M_N}. \quad (3.4)$$

In this way we obtain in leading order of the $1/N_c$ expansion the relations

$$N_c^2 \sim A_0^{u+d} = \frac{\delta_k}{2} \left(E_T^{u+d} + 2\tilde{H}_T^{u+d} \right), \quad (3.5a)$$

$$N_c^2 \sim A_1^{u-d} = \frac{1}{2} \left(H_T^{u-d} + \xi \tilde{E}_T^{u-d} \right), \quad (3.5b)$$

$$N_c^2 \sim A_2^{u-d} = \frac{1}{2} \left(H_T^{u-d} + \xi \tilde{E}_T^{u-d} \right), \quad (3.5c)$$

$$N_c^2 \sim A_3^{u-d} = \frac{\delta_k}{2} \left(\tilde{E}_T^{u-d} \right), \quad (3.5d)$$

which are to be understood in the sense of Eq. (3.3), and in which we indicate only the scaling exponent. In subleading order of the $1/N_c$ expansion,

$$N_c \sim A_0^{u-d} = \frac{\delta_k}{2} \left(E_T^{u-d} - \xi \tilde{E}_T^{u-d} + 2\tilde{H}_T^{u-d} \right), \quad (3.6a)$$

$$N_c \sim A_1^{u+d} = \frac{1}{2} \left(H_T^{u+d} - \xi^2 E_T^{u+d} + \xi \tilde{E}_T^{u+d} + 2\delta_k^2 \tilde{H}_T^{u+d} \right), \quad (3.6b)$$

$$N_c \sim A_2^{u+d} = \frac{1}{2} \left(H_T^{u+d} - \xi^2 E_T^{u+d} + \xi \tilde{E}_T^{u+d} \right), \quad (3.6c)$$

$$N_c \sim A_3^{u+d} = \frac{\delta_k}{2} \left(\tilde{E}_T^{u+d} - \xi E_T^{u+d} \right). \quad (3.6d)$$

From Eqs. (3.5a—3.6c) we can read off the large- N_c flavor structure of the GPDs, namely

$$H_T^{u-d} \sim N_c^2, \quad H_T^{u+d} \sim N_c, \quad (3.7)$$

$$E_T^{u+d} \sim N_c^3, \quad E_T^{u-d} \sim N_c^2, \quad (3.8)$$

$$\tilde{H}_T^{u+d} \sim N_c^3, \quad \tilde{H}_T^{u-d} \sim N_c^2, \quad (3.9)$$

$$\tilde{E}_T^{u-d} \sim N_c^3, \quad \tilde{E}_T^{u+d} \sim N_c^2. \quad (3.10)$$

These relations generalize previous results for the N_c -scaling of the nucleon's quark transversity PDF h_1^q [obtained as the forward limit of H_T^q , Eq. (2.5)] [42, 43, 44] and the tensor form factors [obtained as the first x -moment, Eq. (2.4)] [26, 27].

The scaling relations Eq. (3.7) reveal several interesting properties of the chiral-odd GPDs. First, one sees that in H_T and \tilde{E}_T the flavor-nonsinglet component is leading and the flavor-singlet one subleading, while in E_T and \tilde{H}_T the order is opposite.

Second, one notices that the amplitudes A_1^{u-d} and A_2^{u-d} are degenerate in leading order of the $1/N_c$ expansion; see Eqs. (3.5b, 3.5c). This happens because the mean-field picture implied by the large- N_c limit makes no distinction between different transverse polarization directions in leading order of the $1/N_c$ expansion. The equality $A_1^{u-d} = A_2^{u-d}$ implies that the nucleon-quark double helicity-flip amplitude vanishes, $A_{-,+}^{u-d} = 0$, i.e., the amplitude in which the quark has helicity *opposite* to the nucleon in the initial state and *both* helicities are flipped in the final state. If we view the nucleon as a system composed of the active quark and a “remnant,” the double-flip transition involves a change of the relative orbital angular momentum by two units, $\Delta L = 2$ [5]. Among all the nucleon-quark helicity amplitudes (including also the chiral-even ones) this is the only amplitude

that is (i) double-flip, and (ii) vanishes exactly in the leading order of the large- N_c limit. It merits further study whether this observation could be explained more directly in light of the mechanical picture implied by the large- N_c limit.

Third, one notices in Eqs. (3.5a—3.6d) that the combination $E_T^q + 2\tilde{H}_T^q \equiv \bar{E}_T^q$ emerges naturally from the large- N_c expansion of the chiral-odd GPDs. The differences between the individual GPDs E_T^q and \tilde{H}_T^q are suppressed in $1/N_c$. Interestingly, this combination appears also in the amplitude for pseudoscalar meson production [17], meaning that the virtual photon cannot “distinguish” between E_T^q and \tilde{H}_T^q .

4. Comparison with pseudoscalar meson production data and outlook

It is interesting to compare our results with preliminary data from the JLab CLAS exclusive pseudoscalar meson production experiments [19, 20] (cf. comments in Sec. 1). Analysis of the azimuthal-angle dependent response functions shows that $|\sigma_{LT}| \ll |\sigma_{TT}|$, which indicates dominance of the twist-3 amplitudes, involving the chiral-odd GPDs H_T^q and $\bar{E}_T = E_T^q + 2\tilde{H}_T^q$, over the twist-2 amplitudes involving the chiral-even GPD \tilde{E}^q . A preliminary flavor decomposition was performed assuming dominance of the twist-3 amplitudes and combining the data on π^0 and η production, in which the u and d quark GPDs enter with different relative weight. Results show opposite sign of the exclusive amplitudes $\langle H_T^u \rangle$ and $\langle H_T^d \rangle$, which is consistent with the leading appearance of the flavor-nonsinglet H_T^{u-d} in the $1/N_c$ expansion. (Here $\langle \dots \rangle$ denotes the integral over x of the GPD, weighted with the meson wave function, hard process amplitude, and Sudakov form factor [17].) The results also suggest same sign of $\langle \bar{E}_T^u \rangle$ and $\langle \bar{E}_T^d \rangle$, which is again consistent with the leading appearance of the flavor-singlets E_T^{u+d} and \tilde{H}_T^{u+d} in the $1/N_c$ expansion. These findings should be interpreted with several caveats: (a) the errors in the experimental extraction of $\langle H_T^q \rangle$ and $\langle \bar{E}_T^q \rangle$ are substantial; (b) the $1/N_c$ expansion predicts only the scaling behavior, not the absolute magnitude of the individual flavor combinations, cf. Eq. (3.3).

It is encouraging that the flavor structure of the amplitudes extracted from the π^0 and η electroproduction data is consistent with the pattern predicted by the $1/N_c$ expansion. Our findings further support the idea that pseudoscalar meson production at $x_B \gtrsim 0.1$ and $Q^2 \sim \text{few GeV}^2$ is governed by the twist-3 mechanism involving the chiral-odd GPDs. It would be interesting to calculate the chiral-odd GPDs in dynamical models that consistently implement the N_c -scaling, such as the chiral quark-soliton model. Such calculations would allow one to calculate also the scaling functions in the large- N_c relations, Eq. (3.3), and supplement the scaling studies with dynamical information.

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