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Extracting the η' meson mass from gluonic correlators in lattice QCD

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Calculation of the η' meson mass is a notoriously difficult problem, as it requires evaluation of the disconnected diagram which is costly and noisy. In this work, we use a gluonic operator to extract the eta-prime state after smearing the link variables through the Wilson flow. With this choice, one can avoid a large cancellation of pion contribution between the connected and disconnected diagrams. We obtain the η' meson mass on lattices with three different lattice spacings and two physical volumes, which allow us to estimate its continuum and large volume limits.

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1. Introduction

The η' meson is an interesting particle among the low-energy hadrons in QCD. While it would be a pseudo Nambu Goldstone (pNG) boson if it is related to a spontaneous breaking of the axial $U(1)$ symmetry, its mass $m_{\eta'}$ ($= 958$ MeV) is much heavier than other pNG bosons like pion or kaon. This $U(1)$ problem [1, 2] is one of the physical evidences of the chiral anomaly, breaking of the symmetry at quantum level.

The axial $U(1)$ anomaly relates its violation to the topological feature of the gluon fields. More directly Witten [3] and Veneziano [4] computed the mass of the η' meson in the large N_c (number of colors) limit, as a function of the topological susceptibility in QCD.

In QCD with $N_c = 3$ and dynamical light quarks, however, the argument of Witten and Veneziano is no longer valid. It is not the η' meson but the pion that controls the topological susceptibility. This was confirmed in our previous lattice QCD simulations where we kept the chiral symmetry (nearly) exact [5, 6, 7]. We found that the topological susceptibility is proportional to the light sea quark masses, consistent with the prediction from chiral perturbation theory, $\chi_t = \frac{\Sigma}{\sum_i 1/m_i}$, where Σ denotes the chiral condensate, and m_i the i-th light quark mass. In particular, χ_t vanishes in the limit of massless up and down quarks, reflecting the long-range dynamics of the pion field.

It is then interesting to ask what happens to the η' meson with $N_c = 3$. Since the effect of the anomaly is stronger than that of the large-N_c limit, the η' meson should be more sensitive to the topological fluctuation of the gluon field, while it must be insensitive to χ_t. This implies a non-trivial double-scale structure in the topological excitation of gluon field: it creates the η' meson at short distances, while it is connected to the pion at long distances.

In this work, we perform $2+1$-flavor lattice QCD simulation, and show that the two-point function of the topological charge density at short distances gives a mass consistent with the experimental value of the η' meson mass. Since we have computed χ_t using the same correlation functions (see [7] for the details), our result clearly shows the double-scale structure of the topological property of gauge fields. Our results were already presented in a paper [8]. In this article, we review the main part of it.

Not only being theoretically interesting, but our work also provides a practically useful method to calculate the η' meson mass. Direct lattice computation of the η' meson mass has been challenging because of the disconnected diagram of quarks, which appears from the Wick contraction of the fermion [9, 10, 11, 12, 13]. This is numerically expensive and statistically noisy.

Using a gluonically defined operator of the flavor singlet pseudoscalar, we can avoid the computational cost of stochastically evaluating the disconnected diagram. Our gluonic definition of the topological charge density does not require any inversion of the Dirac operator. Moreover, our method avoids the contamination from the pions. In the conventional fermionic approach, one calculates both of the connected and disconnected diagrams of quark fields, both of which have the pion propagation and cancel with each other. A large statistics is required for the cancellation before extracting the η' meson physics. Since the purely gluonic definition of the topological charge density

$$q(x) = \frac{1}{32\pi^2}\varepsilon_{\mu\nu\rho\sigma}\mathrm{Tr}F^{\mu\nu}_{\mathrm{cl}}F^{\rho\sigma}_{\mathrm{cl}}(x), \tag{1.1}$$

where $F^{\mu\nu}_{\mathrm{cl}}$ denotes the field strength tensor of the gluon field defined through the so-called clover term made by four plaquettes, does not directly couple to the pions, its correlator is free from the pion's fluctuation.

Note here that the sum of Eq. (1.1) over the lattice volume gives the global topological charge up to discretization effects. In order to reduce the cut-off effects, we *cool down* the link variables using the Yang-Mills (YM) gradient flow [14]. At a flow time t, it amounts to smoothing the gauge fields in a range of the length $\sqrt{8t}$. It was shown that the topological charge Q defined through (1.1) converges to an integer value at a sufficiently large flow time [15, 16]. This smearing procedure eliminates short-distance noises and also suppresses the noise at longer distances.[1]

In order to extract the η' meson mass, the YM gradient flow time should not be too long not to destroy the correlation of the η' propagation. Assuming a Gaussian form of the smoothing effect, Bruno *et al.* [17] estimated the size of distortion of the correlator as

$$\Delta\langle q(x)q(y)\rangle \sim e^{-(|x-y|/\sqrt{8t}-m_{\eta'}\sqrt{8t})^2}\frac{m_{\eta'}(8t)^{3/2}}{2\sqrt{\pi}|x-y|^2}. \tag{1.2}$$

In our analysis below, we use the reference flow time around $\sqrt{8t} = 0.2$ fm for the fit range $|x-y| >$ 0.6 fm, for which we estimate the above correction to be less than 1% for $m_{\eta'} \simeq 1$ GeV.

2. Lattice setup

In our simulations, the Symanzik gauge action and the $2+1$-flavor Möbius domain-wall fermion action are employed to generate gauge configurations [23, 24, 25]. For the Dirac operator, three steps of stout smearing of the link variables are performed. Our main lattice QCD simulations are performed on two different lattice volumes $L^3 \times T = 32^3 \times 64$ and $48^3 \times 96$, for which we set β = 4.17 and 4.35, respectively. The lattice cut-off $1/a$ is estimated to be 2.4 GeV (for $\beta = 4.17$) and 3.6 GeV (for $\beta = 4.35$), using the input $\sqrt{t_0} = 0.1465$ fm [26] where the reference YM gradient flow time t_0 defined by $t^2\langle E\rangle|_{t=t_0} = 0.3$ [14] with the energy density E of the gluon field, is used. These two lattices have a similar physical volume size $L \sim 2.6$ fm. We set the strange quark mass m_s at around its physical point, and use 3–4 values of the up and down quark mass m_{ud} for each m_s. Our lightest pion mass is around 230 MeV with our smallest value of am_{ud} = 0.0035 at β = 4.17. In order to control the systematics due to finite volume sizes and lattice spacings, we also perform simulations on a larger lattice $48^3 \times 96$ (at $\beta = 4.17$ and $m_\pi \sim 230$ MeV), and a finer lattice $64^3 \times 128$ (at $\beta = 4.47$ [$1/a \sim 4.5$ GeV] and $m_\pi \sim 285$ MeV). For each parameter set, we sample 500–1000 gauge configurations from 10000 molecular dynamics (MD) time. We find that the residual mass in the Möbius domain-wall fermion formalism is kept smaller than ~ 1 MeV [27] by choosing L_s = 12 at β = 4.17 and L_s = 8 at β = 4.35 (and 4.47).

On each generated configuration, we perform 500–1,000 steps of the YM gradient flow (using the conventional Wilson gauge action) with a step-size $a^2\Delta t$ =0.01. At every 20–30 steps, we store $q(x)$ and measure its correlator using the Fast Fourier Transform (FFT) technique.

We find that the two-point function $\langle q(x)q(y)\rangle$ at our target distance $|x-y| \sim 0.7$ fm always shows a shorter autocorrelation time than 10 MD time, while that of the global topological charge,

[1] A similar method was tried in a quenched study to extract the "pseudoscalar glueball mass" [18]. Other types of smearings were tried in previous works to probe topological structure of the QCD vacuum [19, 20, 21, 22].

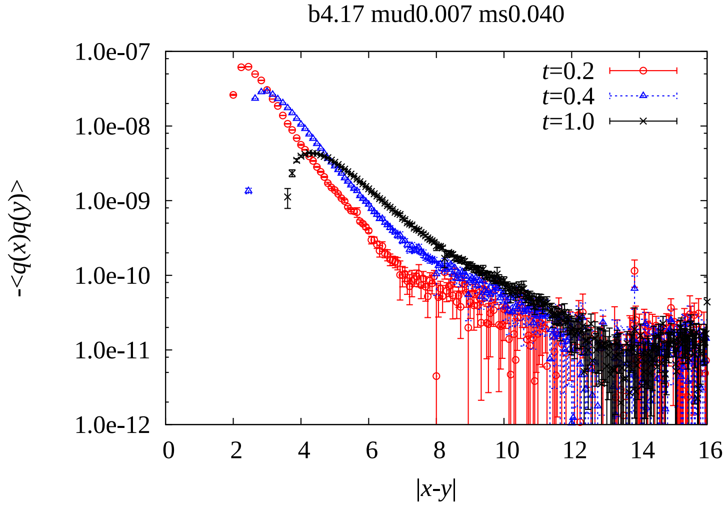

Figure 1: The correlator $-\langle q(x)q(y)\rangle$ at the flow times a^2t = 0.2 (circles), 0.4 (triangles) and 1.0 (crosses). Data at $\beta = 4.17$, $am_{ud} = 0.007$ and $am_s = 0.040$ are presented.

$Q = \sum_x q(x)$ is $O(100)$ or higher MD time at $\beta = 4.35$. This is a good evidence that the η' meson physics is *decoupled* [28] from the physics of the global topological charge. In the following analysis, we estimate the statistical error by the jackknife method after binning the data in 140–200 MD time.

3. Numerical result

Figure 1 shows the topological charge density correlator $C(|x-y|) = -\langle q(x)q(y)\rangle$ at three different flow times. Using FFT, the rotationally symmetric data points are averaged. As the flow-time increases, the statistical fluctuation of the correlator becomes milder, while the region at small $|x-y|$ is distorted. We therefore need to find a region of t where the correlator has sufficiently small noises while it is not spoiled by the smearing of the YM gradient flow.

To extract the mass $m_{\eta'}$, we fit our data to the function of a single boson propagation:

$$f(r,m_{\eta'}) = A\frac{K_1(m_{\eta'}r)}{r}, \tag{3.1}$$

where $r = |x-y|$, K_1 is the modified Bessel function and A is an unknown constant, which depends on the flow time t. The fitting range, is determined by inspecting a local “effective mass” $m_{\mathrm{eff}}(r)$, a solution of $f(r+\Delta r, m_{\mathrm{eff}}(r))/f(r, m_{\mathrm{eff}}(r)) = C(r+\Delta r)/C(r)$, where we set $\Delta r = 1/2$. A reasonable plateau is found for $m_{\mathrm{eff}}(r)$ around $r \sim$ 8–12 ($>$ 0.6 fm) at $t = 1$ ($\sqrt{8t} \sim 0.2$ fm).

Figure 2 shows the obtained values of the η' meson mass as a function of $\sqrt{8t}$. The data around $\sqrt{8t} \sim 0.2$ fm are stable, while a large distortion is found at larger smearing lengths $\sqrt{8t} \gtrsim$ 0.3 fm. We take the data at $\sqrt{8t} =$ 0.2–0.25 fm (filled symbols in Fig. 2) for our results.

We plot the results in Fig. 3 as a function of the square of the pion mass m_π. The results look insensitive to the quark masses, as well as to V and a. We therefore perform a global fit of our data to a linear function $m_{\eta'}^{\mathrm{phys}} + C_a a^2 + C_{ud}[m_\pi^2 - (m_\pi^{\mathrm{phys}})^2] + C_s[(2m_K^2 - m_\pi^2) - \{2(m_K^{\mathrm{phys}})^2 - (m_\pi^{\mathrm{phys}})^2\}]$,

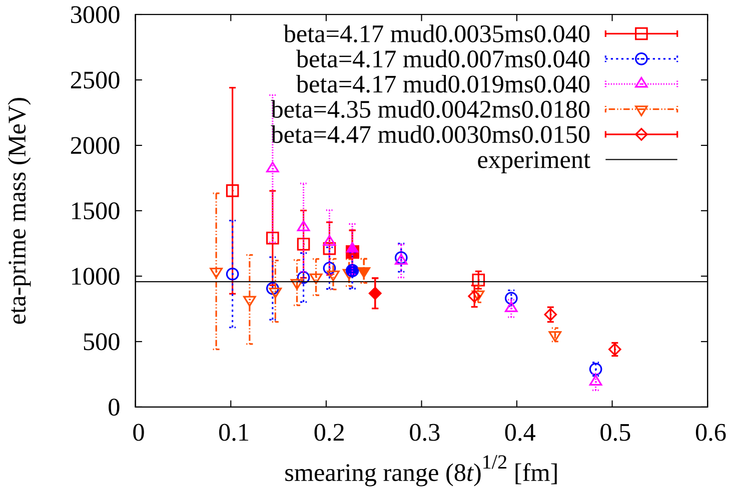

Figure 2: The flow-time dependence of the η' meson mass. The data at various sea quark masses and β values are shown, as specified in the legend. The filled symbols represent our data taken for the central values.

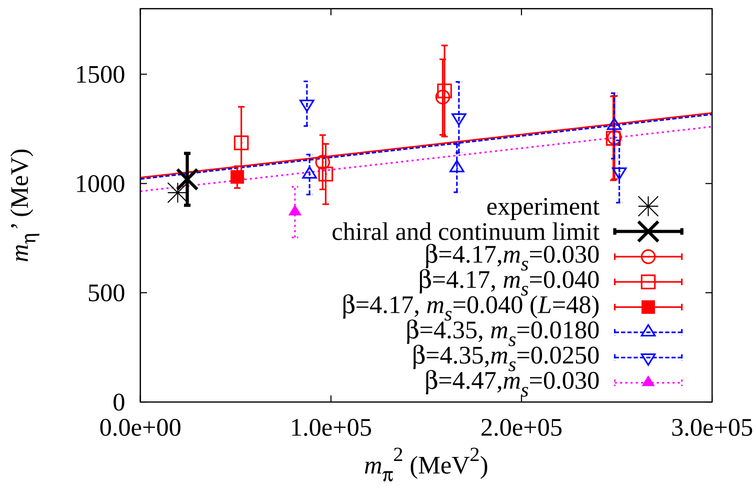

Figure 3: The extracted η' meson mass from each gauge ensemble. The three fit lines (representing the same linear fit function at three values of a) are shown for higher m_s's at each β.

where $m_{\eta'}^{\text{phys}}$, C_a, C_{ud}, and C_s are free parameters. Here, $m_{\pi/K}^{\text{phys}}$ denotes the experimental value of the pion/kaon mass. As shown by the lines (which are shown for higher m_s only) in Fig. 3, we find that our linear function fits the lattice data reasonably well with χ^2/(degrees of freedom) ~ 1.6.

In Ref. [8] we reported the study of various systematic effects, including the long auto-correlation of the global topological charge ($< 1\%$), finite volume effects ($\sim 10^{-6}$), the mixing with the η meson ($\sim +5\%$), and the chiral and continuum extrapolations ($\pm 8\%$).

Our final result at the physical point is

$$m_{\eta'} = 1019(119)(^{+97}_{-86}) \quad \text{MeV}, \tag{3.2}$$

which agrees well with the experimental value $m_{\eta'} = 957.78(6)$ MeV [32]. Here the first error is statistical and the second is the systematic error from the mixing with the η meson and the chiral and continuum extrapolations (added in quadrature).

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