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X(3872) and Y(4140) using diquark-antidiquark operators with lattice QCD

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We discuss a recent lattice study of charmonium-like mesons with $J^{PC} = 1^{++}$ and three quark contents $\bar{c}c\bar{d}u$, $\bar{c}c(\bar{u}u + \bar{d}d)$ and $\bar{c}c\bar{s}s$, where the latter two can mix with $\bar{c}c$. In this quantum channel, the long known exotic candidate, X(3872), resides. This simulation employs $N_f = 2$, $m_\pi = 266$ MeV and a large basis of $\bar{c}c$, two-meson and diquark-antidiquark interpolating fields, with diquarks in both anti-triplet and sextet color representations. It aims at the possible signatures of four-quark exotic states. Along the way, we discuss the relations between the diquark-antidiquark operators and the two-meson operators via the Fierz transformations.

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1. Introduction

Existence of hadrons with exotic flavor quantum numbers and exotic nature has been an interesting open question in hadron spectroscopy. Studies in this direction achieved a boost with the discovery of charged resonances $Z_c(3900)^+$ [1] and $Z(4430)^\pm$ [2, 3] that confirms the existence of hadrons composed of two quarks and two antiquarks. Many of the other exotic excitations in the charmonium sector, classified as so-called the XYZ mesons, also appear to have significant four-quark Fock components.

In this talk, we present the results from our recent lattice investigation of charmonium spectrum in the $J^{PC} = 1^{++}$ quantum channel with three quark contents: $\bar{c}c\bar{d}u$, $\bar{c}c(\bar{u}u+\bar{d}d)$ and $\bar{c}c\bar{s}s$, where the latter two channels have $I=0$ and can mix with $\bar{c}c$ [4]. These calculations were aimed at a first principles study of $X(3872)$ and $Y(4140)$ and other possible hadrons in these channels.

Experimentally, the quantum numbers of $X(3872)$ have been confirmed to be $J^{PC} = 1^{++}$, while its isospin, I, remains unsettled. This is mainly due to its nearly equal branching fraction to either isospin decay channel, $I = 0$ ($X(3872) \to J/\psi\,\omega$) and $I = 1$ ($X(3872) \to J/\psi\,\rho$) [5], and lack of evidence for existence of any charged partner states [6]. Other XYZ candidates with $C=+1$ that could possibly have $J^{PC} = 1^{++}$ include $X(3940)$ [7], $Z(4050)^\pm$ [3] and $Z(4250)^\pm$ [3]. The signatures for the $Y(4140)$ with charge parity $C=+1$ in the $J/\psi\phi$ invariant mass [8], indicates the existence of exotic hadrons with hidden strangeness. However, the quantum numbers for most of these excitations are undetermined. A detailed review on these can be found in Refs. [9].

Several theoretical calculations based on phenomenology have been performed, which suggests a variety of interpretations for these exotic observations : mesonic molecules, hybrid mesons, tetraquarks, cusp phenomena, etc. Detailed reports on these investigations can be found in the Refs. [9]. Lattice QCD promises a first principles approach to study these systems so as to establish their fundamental nature. A lattice candidate for $X(3872)$ with $I=0$ was first reported in Ref. [10], where a combination of $\bar{c}c$ as well as $D\bar{D}^*$ and $J/\psi\omega$ interpolators was used. Recently, another lattice investigation using the HISQ action also reported a lattice candidate for $X(3872)$ using $\bar{c}c$ and $D\bar{D}^*$ interpolating fields [11] supporting the previous observation. There has been no evidence from lattice studies supporting existence of the $Y(4140)$ resonance, even though there have been lattice calculations of $J/\psi\phi$ scattering in search of the $Y(4140)$ resonance [12].

In this lattice investigation, we make a dynamical study involving diquark-antidiquark interpolators along with several two-meson and $\bar{c}c$ kind of interpolators, so as to explore the significance of tetraquark Fock components in the charmonium spectrum with $J^{PC} = 1^{++}$. We consider the color structures $\mathcal{G} = \bar{3}_c, 6_c$ for the diquarks. This study addresses the following questions: What are the effects of diquark-antidiquark interpolators on the established lattice candidate for $X(3872)$ and the charmonium spectrum itself? Do we observe additional levels for charged or neutral $X(3872)$ with $I=1$? Do we see additional levels for $Y(4140)$ using operators with hidden strangeness? Do we find signatures for other possible exotic states in the channels being probed?

The paper is organized as follows. Section 2 discusses the details of the lattice ensemble, the interpolators and the technologies used in this investigation. In Section 3, the relation between diquark-antidiquark and two-meson interpolators via Fierz transformations is discussed. In Section 4 and Section 5 the results and conclusions are presented.

2. Lattice methodology

$N_f = 2$ dynamical gauge configurations with $m_\pi \simeq 266$ MeV [13] were used in these calculations. Other relevant details of the gauge ensemble are described in Table 1. The sea and valence quarks were realized with the tree-level improved Wilson clover action. The absence of a strange sea prevents $\bar{c}c\bar{s}s$ intermediate states in the $\bar{c}c(\bar{u}u+\bar{d}d)$ sector, in accordance with treating these two $I = 0$ sectors separately in our study. Charm quarks are tuned by equating the spin averaged kinetic mass of the $1S$ charmonium to its physical value. We quote our spectrum in terms of $E_n = E_n^{lat} - m_{s.a.}^{lat} + m_{s.a.}^{exp}$ ($m_{s.a.} = \frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$), which will be compared with the experiments.

Lattice size	κ	β	$N_{\rm cfgs}$	m_π [MeV]	a [fm]	L [fm]
$16^3 \times 32$	0.1283	7.1	280	266(3)(3)	0.1239(13)	1.98

Table 1: Details of the gauge field ensemble used.

Altogether 22 interpolators, including $\bar{c}c$, two-meson and diquark-antidiquark ($[\bar{c}\bar{q}][cq]$) interpolators, with $J^{PC} = 1^{++}$ (T_1^{++} irrep of octahedral group, O_h) and total momentum zero were constructed for the three cases of our interest. Using these interpolators, the relevant non-interacting two-meson levels with $J^{PC} = 1^{++}$ and total momentum zero are

- $I = 0$; $\bar{c}c(\bar{u}u+\bar{d}d)$ and $\bar{c}c$; $E \lesssim 4.2$ GeV
  $D(0)\bar{D}^*(0)$, $J/\psi(0)\omega(0)$, $D(1)\bar{D}^*(-1)$, $J/\psi(1)\omega(-1)$, $\eta_c(1)\sigma(-1)$, $\chi_{c1}(0)\sigma(0)$
- $I = 1$; $\bar{c}c\bar{d}u$; $E \lesssim 4.2$ GeV
  $D(0)\bar{D}^*(0)$, $J/\psi(0)\rho(0)$, $D(1)\bar{D}^*(-1)$, $J/\psi(1)\rho(-1)$, $\chi_{c1}(1)\pi(-1)$, $\chi_{c0}(1)\pi(-1)$
- $I = 0$; $\bar{c}c\bar{s}s$ and $\bar{c}c$; $E \lesssim 4.3$ GeV
  $D_s(0)\bar{D}_s^*(0)$, $J/\psi(0)\phi(0)$, $D_s(1)\bar{D}_s^*(-1)$, $J/\psi(1)\phi(-1)$.

The flavor sectors $\bar{c}c(\bar{u}u+\bar{d}d)$ and $\bar{c}c\bar{s}s$ are considered separately in this calculation. We assume that either of these sectors have negligible coupling to two-meson levels from other. It is to be noted that that $Y(4140)$ has been experimentally observed only in the $J/\psi\phi$ final state with valence strange content, but it has not been observed in $D\bar{D}^*$ and $J/\psi\omega$ final states. With this assumption the resulting spectrum would be less dense, making the identification of the eigenstates easier. Furthermore, we assume that the valence strange content could uncover hints on the existence of the charm-strange exotics, if they exists, although the dynamical strange quarks are absent in this ensemble.

We show the two-meson non-interacting energies by the horizontal lines in our plots. They are calculated from energies of single hadrons determined on the same ensemble [14, 15, 16] using $E^{n.i.}_{M_1(\mathbf{n})M_2(-\mathbf{n})} = E_1(p) + E_2(p)$, $p = \frac{2\pi|\mathbf{n}|}{L}$, $\mathbf{n} \in N^3$. Some hadrons in the list above (e.g. ρ and σ) are resonances and a proper simulation should consider three meson interpolators. Such a calculation has not been performed in practice yet. Hence we follow an approximation in which the energy of the low-lying state from single hadron correlations are used to determine the two-meson non-interacting levels. We also exclude the relevant non-resonant three meson levels in the energy range of our interest, as we expect these three meson levels would be absent without explicit inclusion of respective interpolators in the operator basis.

We compute the full coupled correlation functions with our interpolator set using the 'Distillation' method [17]. In this study, we exclude Wick contraction diagrams, where the charm quark lines are disconnected between the source and sink, and we take into account all other Wick contractions. The eigensystem are extracted using the well-established generalized eigenvalue problem [18] and the energies are extracted asymptotically from two-exponential fits to the eigenvalues. All quoted statistical errors are obtained using single elimination jackknife analysis. The existence of possible exotic states is studied by analyzing the number of energy levels, their positions and overlaps with the considered lattice operators $\langle\Omega|O_j|n\rangle$. Based on experience of identifying additional levels corresponding to ρ [14], $K^*(892)$ [19], $D_0^*(2400)$ [16], $K_0^*(1430)$ [20] and $X(3872)$ [10], we expect an additional energy level if an exotic state is of similar origin.

3. Fierz relations

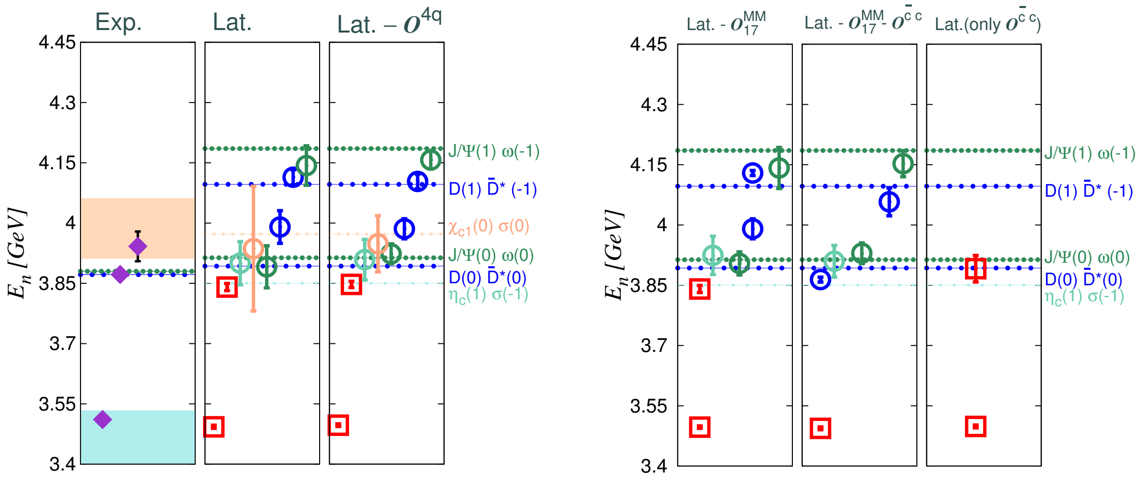
The diquark-antidiquark operators $[\bar{c}\bar{q}]_{3_c}[cq]_{\bar{3}_c}$ and $[\bar{c}\bar{q}]_{\bar{6}_c}[cq]_{6_c}$ are linearly related to the two-meson currents $(\bar{c}c)_{1_c}(\bar{q}q)_{1_c}$ and $(\bar{c}q)_{1_c}(\bar{q}c)_{1_c}$ via Fierz rearrangement [21]. We show an example of such a relation between one of our $[\bar{c}\bar{q}][cq]$ operators and two-meson operators in eq. 3.1, which is derived for local quarks. The Fierz relations suggest that O^{4q} and O^{MM} are linearly dependent, even though our quark fields are smeared and mesons are projected to definite momentum. Note that the first and second terms in the Fierz expansion (3.1) represent $D\bar{D}^*$, while the seventh term is similar to the $\chi_{c1}\ \sigma$ for $I=0$. Hence we expect significant correlations between these operators.

$$
\begin{aligned}
O^{4q}_{3_c} &= [\bar{c}\, C\gamma_5\, \bar{u}]_{\bar{3}_c}[c\ \gamma_i C\ u]_{3_c} + [\bar{c}\, C\gamma_i\ \bar{u}]_{\bar{3}_c}[c\ \gamma_5 C\ u]_{3_c} + \{u\to d\} \qquad (3.1)\\
&= -\frac{(-1)^i}{2}\{\ (\bar{c}\ \gamma_5\ u)(\bar{u}\ \gamma_i\ c) - \ (\bar{c}\ \gamma_i u)(\bar{u}\ \gamma_5\ c) + \ (\bar{c}\ \gamma^\nu\gamma_5\ u)(\bar{u}\ \gamma_i\gamma_\nu\ c)|_{i\neq\nu}\\
&\quad -\ (\bar{c}\ \gamma_i\gamma_\nu\ u)(\bar{u}\ \gamma^\nu\gamma_5\ c)|_{i\neq\nu}\} + \frac{(-1)^i}{2}\{\ (\bar{c}\ c)(\bar{u}\ \gamma_i\gamma_5\ u) + \ (\bar{c}\ \gamma_i\gamma_5\ c)(\bar{u}\ u)\\
&\quad -(\bar{c}\ \gamma^\nu c)(\bar{u}\ \gamma_i\gamma_\nu\gamma_5\ u)|_{i\neq\nu}\ -\ (\bar{c}\ \sigma^{\alpha\beta}\ c)(\bar{u}\ \sigma_{\alpha\beta}\gamma_i\gamma_5\ u)|_{i\neq(\alpha<\beta)}\} + \ \{u\to d\}
\end{aligned}
$$

4. Results

Figure 1 and Figure 2 show the discrete energy spectrum, which is the main result from this lattice calculation. The lattice energy levels are indicated by circles, while the horizontal lines indicate the energies of two meson states in the non-interacting limit. We identify the eigenstates that have dominant overlaps with two-meson scattering interpolators based on the spectral overlaps $\langle 0|O_i|n\rangle$ and additional criteria described in Ref. [4]. The corresponding circles are represented with the same colors as the lines for the non-interacting two meson states. The remaining states, that are not attributed to the two-meson states, are represented by red squares. These figures also compare the spectra obtained from two interpolator sets, one with the diquark-antidiquark operators and other without. In all three flavor sectors, we see almost negligible effect of $[\bar{c}\bar{q}][cq]$ on the low lying states, while we do observe an improvement in the signals for higher lying states in the basis without $[\bar{c}\bar{q}][cq]$.

Figure 1 shows the I=0 charmonium spectrum with $J^{PC}=1^{++}$ and u/d valence quarks. We identify the levels related to $X(3872)$ as $n=2$ (red squares) and $n=6$ (blue circle). One of the two levels remains absent when $D\bar{D}^*$ and O^{4q} are used and $O^{\bar{c}c}$ is not, as is evident from the first



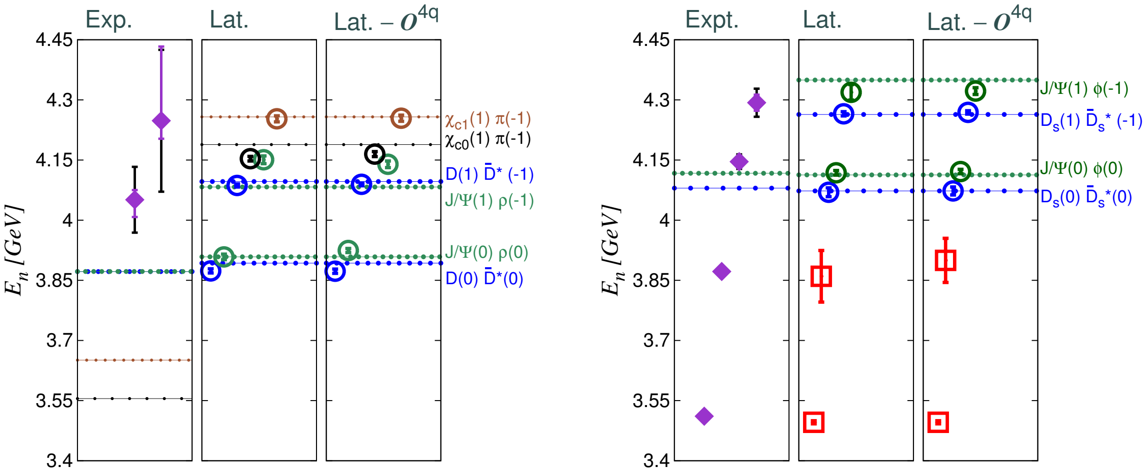
(a) $I = 0: \ \bar{c}c(\bar{u}u + \bar{d}d)$ & $\bar{c}c$ (b) $I = 0: \ \bar{c}c(\bar{u}u + \bar{d}d)$ & $\bar{c}c$

Figure 1: The I=0 spectrum with $J^{PC} = 1^{++}$ with u/d valence quarks. The energies are shown as $E_n = E_n^{lat} - m_{s.a.}^{lat} + m_{s.a.}^{exp}$. The two-meson non-interacting levels and experimental thresholds are displayed as horizontal lines, where the colored bands indicate the σ width. (a) The middle block shows the discrete spectrum determined from our lattice simulation, while the right-hand block shows the spectrum we obtained with the $[\bar{c}\bar{q}][cq]$ operators excluded from our analysis. The left-hand block shows the physical thresholds and possible experimental candidates χ_{c1}, $X(3872)$ and $X(3940)$. The violet error-bars for experimental candidates show the uncertainties in the energy and the black error-bars show its width. (b) The left block shows the spectrum from interpolator basis containing all kinds of operators. The middle block shows the spectrum after excluding $\bar{c}c$ kind of operators. The right hand side block is the spectrum extracted purely from $\bar{c}c$ kind of operators. The $O_{17}^{MM} = \chi_{c1}(0)\sigma(0)$ is excluded from the basis to achieve better signals and clear comparison.

and second panel from left of Figure 1(b). This indicates that the importance of $\bar{c}c$ interpolators for lattice candidate of $X(3872)$, while the $[\bar{c}\bar{q}][cq]$ structure alone does not produce it. Furthermore, it also indicates the significance of $\bar{c}c$ and $D\bar{D}^*$ operators in determining the position of these two levels, while the O^{4q} doesn't have any significant implications on them. We extract the DD^* scattering matrix S(E) at two energy values $E_{n=2,6}$ using Lüscher's relation. The scattering matrix is interpolated near the threshold and a pole just below threshold is found [4]. The results indicate a shallow bound state immediately below $D\bar{D}^*$ threshold, interpreted as experimentally observed $X(3872)$. The extracted mass and binding energy of $X(3872)$ indicate that it is insensitive to the $[\bar{c}\bar{q}][cq]$ interpolators. The mass of $X(3872)$ was determined along these lines for the first time in Ref. [10], where this channel was studied in a smaller energy range on the same ensemble without $[\bar{c}\bar{q}][cq]$ interpolators. All other extracted levels are identified with different two meson scattering channels.

Figure 2(a) shows the I=1 spectrum with $J^{PC} = 1^{++}$ and quark content $\bar{c}c\bar{d}u$. All the eigenstates have dominant overlap with the two-meson interpolators. The spectrum shows very little influence on the inclusion of $[\bar{c}\bar{q}][cq]$, which is evident from Figure 2(a). Our results do not give evidence for a charged or neutral $X(3872)$ with $I = 1$ or other charged exotic mesons like $Z_c(4050)^+$ and $Z_c(4250)^+$.

Figure 2(b) shows the I=0, $J^{PC} = 1^{++}$ charmonium spectrum with hidden strange quarks.



(b) $I = 1:\ \bar{c}c\bar{u}d$ (c) $I = 0:\ \bar{c}c\bar{s}s$ & $\bar{c}c$

Figure 2: (a) The I=1 spectrum with $J^{PC} = 1^{++}$ and (b) the I=0 spectrum with $J^{PC} = 1^{++}$ with hidden strange valence quarks. The experimental candidates shown are (a) $Z_c^+(4050)$ and $Z_c^+(4250)$ and (b) χ_{c1}, $X(3872)$, $Y(4140)$ and $Y(4274)$. For further details see Figure 1.

We identify the two low lying states represented by squares to be $\chi_{c1}(1P)$ and the level related to $X(3872)$. The remaining four levels are identified with the $D_s\bar{D}_s^*$ and $J/\psi\phi$ scattering levels based on overlap factors and behavior of the spectrum on omitting these operators. Thus we find no levels that could be related to the $Y(4140)$ or any other exotic structure below 4.2 GeV. Note that existence and the quantum numbers of most of the XYZ's are not yet settled from experiments. Therefore it is possible that the absence of additional levels in our studies is due to the fact that we explored the channel $J^P = 1^{++}$ only.

5. Conclusions

In this talk, we report the results from our lattice investigation of charmonium spectra with $J^{PC} = 1^{++}$ and three different quark contents: $\bar{c}c\bar{d}u$, $\bar{c}c(\bar{u}u + \bar{d}d)$ and $\bar{c}c\bar{s}s$, where the later two can mix with $\bar{c}c$. These calculations were performed on $N_f = 2$ dynamical gauge configurations with $m_\pi \simeq 266$ MeV. Using a large number of interpolators, including $[\bar{c}\bar{q}]_{3_c}[cq]_{\bar{3}_c}$, $[\bar{c}\bar{q}]_{\bar{6}_c}[cq]_{6_c}$, $(\bar{c}q)_{1_c}(\bar{q}c)_{1_c}$, $(\bar{c}c)_{1_c}(\bar{q}q)_{1_c}$ and $(\bar{c}c)_{1_c}$, we extract the spectra up to 4.2 GeV. We identify and extract the lattice estimate for χ_{c1} and $X(3872)$, while all the remaining eigenstates are related to the expected two-meson scattering channels. The $\bar{c}c$ Fock component in $X(3872)$ appears to be more important than the $[\bar{c}\bar{q}][cq]$, since we find a candidate for $X(3872)$ only if $\bar{c}c$ interpolating fields are used. No additional levels were observed in the $I = 1$ spectra with quark content $\bar{c}c\bar{d}u$, which could have implied lattice candidate for charged or neutral $X(3872)$. Future simulations with broken isospin could be crucial for this channel. We also do not find a candidate for $Y(4140)$ or any other exotic charmonium-like structure. Our search for the exotic states assumes an appearance of an additional energy eigenstate on the lattice, which is a typical manifestation for conventional hadrons. Further analytic work is needed to establish whether this working assumption applies also for several coupled channels and all exotic structures of interest.

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References

[1] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **110**, 252001 (2013); Z. Q. Liu *et al.* [Belle Collaboration], Phys. Rev. Lett. **110**, 252002 (2013).

[2] S. K. Choi *et al.* [BELLE Collaboration], Phys. Rev. Lett. **100**, 142001 (2008).

[3] R. Mizuk *et al.* [BELLE Collaboration], Phys. Rev. D **80**, 031104 (2009).

[4] M. Padmanath, C. B. Lang and S. Prelovsek, Phys. Rev. D **92**, 034501 (2015).

[5] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).

[6] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **71**, 031501 (2005).

[7] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **94**, 182002 (2005); B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **101**, 082001 (2008).

[8] T. Aaltonen *et al.* [CDF], Phys. Rev. Lett. **102**, 242002 (2009).

[9] S. L. Olsen, Front. Phys. **10**, 101401 (2015); N. Brambilla *et al.*, Eur. Phys. J. C **74**, no. 10, 2981 (2014); X. Liu, Chin. Sci. Bull. **59**, 3815 (2014).

[10] S. Prelovsek and L. Leskovec, Phys. Rev. Lett. **111**, 192001 (2013).

[11] S. H. Lee *et al.* [Fermilab Lattice and MILC Collaborations], arXiv:1411.1389 [hep-lat].

[12] S. Ozaki and S. Sasaki, Phys. Rev. D **87**, 014506 (2013).

[13] A. Hasenfratz, R. Hoffmann and S. Schaefer, Phys. Rev. D **78**, 054511 (2008); Phys. Rev. D **78**, 014515 (2008).

[14] C. B. Lang *et al.*, Phys. Rev. D **84**, 054503 (2011) [Erratum-ibid. D **89**, 059903 (2014)].

[15] C. B. Lang *et al.*, JHEP **1404**, 162 (2014); Phys. Rev. D **86**, 054508 (2012).

[16] D. Mohler, S. Prelovsek and R. M. Woloshyn, Phys. Rev. D **87**, 034501 (2013).

[17] M. Peardon *et al.* [HSC], Phys. Rev. D **80**, 054506 (2009).

[18] C. Michael, Nucl. Phys. B **259**, 58 (1985); M. Lüscher, Commun. Math. Phys. **104**, 177 (1986).

[19] S. Prelovsek *et al.*, Phys. Rev. D **88**, 054508 (2013).

[20] J. J. Dudek *et al.* [HSC], Phys. Rev. Lett. **113**, 182001 (2014).

[21] J. F. Nieves and P. B. Pal, Am. J. Phys. **72**, 1100 (2004).