Exploring possibly existing $qq\bar{b}\bar{b}$ tetraquark states with $qq = ud, ss, cc$

Antje Peters$^a$, Pedro Bicudo$^b$, Krzysztof Cichy$^{a,c}$, Björn Wagenbach$^a$, Marc Wagner$^a$

$^a$Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik, Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany

$^b$CFTP, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais, 1049-001 Lisboa, Portugal

$^c$Adam Mickiewicz University, Faculty of Physics, Umultowska 85, 61-614 Poznan, Poland

E-mail: peters@th.physik.uni-frankfurt.de, bicudo@tecnico.ulisboa.pt, kcichy@th.physik.uni-frankfurt.de, wagenbach@th.physik.uni-frankfurt.de, mwagner@th.physik.uni-frankfurt.de

We compute potentials of two static antiquarks in the presence of two quarks $qq$ of finite mass using lattice QCD. In a second step we solve the Schrödinger equation, to determine, whether the resulting potentials are sufficiently attractive to host a bound state, which would indicate the existence of a stable $qq\bar{b}\bar{b}$ tetraquark. We find a bound state for $qq = (ud - du)/\sqrt{2}$ with corresponding quantum numbers $I(J^P) = 0(1^+)$ and evidence against the existence of bound states with isospin $I = 1$ or $qq \in \{cc, ss\}$.
1. Motivation

A number of mesons observed in experiments like LHCb or Belle are not well understood. Those mesons have masses and quantum numbers, which are not typical for standard quark-antiquark states, but indicate an exotic four-quark structure. Prominent examples are the charged charmonium-like and bottomonium-like states $Z_c^\pm$ and $Z_b^\pm$ (cf. e.g. [1]). Their masses and decay products suggest the presence of a $c\bar{c}$ or $b\bar{b}$ pair, respectively. On the other hand their electric charge indicates additionally a light quark-antiquark pair $u\bar{d}$ or $d\bar{u}$. Those four-quark systems, in the following also referred to as tetraquarks, are expected to be studied in more detail in the near future by experimental collaborations. Therefore, a sound theoretical understanding of those systems is crucial and of great interest.

Here we summarize the main results of our recently published work [2], where we have studied four-quark systems with two heavy antiquarks $b\bar{b}$ and two lighter quarks $qq$ using lattice QCD and the Born-Oppenheimer approximation. First $b\bar{b}$ potentials in the presence of lighter quarks $qq$ are computed. Then the Schrödinger equation is solved using these potentials, where possibly existing bound states indicate stable tetraquarks. Other papers studying the same systems with similar methods include [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

2. Qualitative discussion of $qqb\bar{b}$ systems

At small $b\bar{b}$ separations the $b\bar{b}$ interaction is dominated by 1-gluon exchange. For a bound state the $b\bar{b}$ pair must, therefore, be in an attractive color triplet. Due to the Pauli principle and because we assume a spatially symmetric s-wave, $b\bar{b}$ has to form an antisymmetric color-spin-flavor combination and, hence, a symmetric spin combination, i.e. $b\bar{b}$ spin $j_b = 1$. Since the complete four-quark system is color neutral, the light quarks $qq$ must be in an antisymmetric color antitriplet. Again due to the Pauli principle $qq$ has to form an antisymmetric color-spin-flavor combination and, hence, a symmetric spin-flavor combination. Candidates for tetraquarks are, therefore, the (spin) scalar isosinglet and, hence, a symmetric spin-flavor combination. Those four-quark systems, in the following also referred to as tetraquarks, are expected to be studied in more detail in the near future by experimental collaborations. Therefore, a sound theoretical understanding of those systems is crucial and of great interest.

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At large $b\bar{b}$ separations the $b\bar{b}$ interaction is screened by the light quarks $qq$, i.e. the four quarks form a system of two heavy-light mesons. One expects stronger screening for increasing quark mass $m_q$, because the wave functions of the corresponding mesons $q\bar{b}$ are then more compact.

3. Lattice QCD computation of static antiquark-antiquark potentials

We extract potentials of two static antiquarks $Q\bar{Q}$ (approximating the two $\bar{b}$ quarks of the $qqb\bar{b}$ system) in the presence of two light quarks $qq$ from correlation functions

$$C(t,r) = \langle \Omega | \hat{Q}(t) \hat{Q}(0) | \Omega \rangle \propto \exp(-V(r)t).$$  

(3.1)

To be able to study flavor antisymmetric $qq$ combinations with $q = s$, we consider two hypothetical degenerate flavors with the mass of the $s$ quark, $s^{(1)}$ and $s^{(2)}$, and similarly for $q = c, c^{(1)}$ and $c^{(2)}$. 

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\( \mathcal{O} \) denotes a four-quark creation operator,
\[
\mathcal{O} = (c \Gamma_{AB}(c \Gamma_{CD}(Q_{C}(r_1)d_A^{(1)}(r_1))(Q_D(r_2)d_B^{(2)}(r_2))) - r = |r_1 - r_2|, \tag{3.2}
\]
where \( \Gamma \) is an appropriate combination of \( \gamma \) matrices accounting for defined quantum numbers light quark spin \( |j_z|\), parity \( P \) and \( P_t \) (cf. [8] for details). \( \Gamma \in \{(1 - \gamma_0)\gamma_5, (1 - \gamma_0)\gamma_5\} \) does not affect the resulting potential \( V(r) \), since the static quark spin is irrelevant. \( \mathcal{O} = \gamma_0\gamma_5 \) denotes the charge conjugation matrix. Note that operators like (3.2) generate overlap not only to mesonic molecule structures, but also to diquark-antidiquark structures [15, 16].

The asymptotic value of a potential and whether it is attractive or repulsive depends on the quantum numbers \((|j_z|, P, P_t)\) and, hence, on \( \Gamma \). In the following we are exclusively interested in attractive potentials between two ground state static-light mesons: the scalar isosinglet corresponding to \( \Gamma = (1 + \gamma_0)\gamma_5 \) and the vector isotriplet corresponding to \( \Gamma = (1 + \gamma_0)\gamma_j \).

Computations have been performed using two ensembles of gauge link configurations generated by the European Twisted Mass Collaboration (ETMC) with dynamical \( u/d \) quarks. Information on these ensembles can be found in Table 1 and [17, 18].

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>lattice size</th>
<th>( \mu_t )</th>
<th>( a ) in fm</th>
<th>( m_{\pi} ) in MeV</th>
<th># configurations</th>
</tr>
</thead>
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<tr>
<td>3.90</td>
<td>24(^3) x 48</td>
<td>0.00400</td>
<td>0.079</td>
<td>340</td>
<td>480</td>
</tr>
<tr>
<td>4.35</td>
<td>32(^3) x 64</td>
<td>0.00175</td>
<td>0.042</td>
<td>352</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 1: Ensembles of gauge link configurations (\( \beta \): inverse gauge coupling; \( \mu_t \): bare \( u/d \) quark mass in lattice units; \( a \): lattice spacing; \( m_{\pi} \): pion mass).

4. \( qq\bar{b}\bar{b} \) tetraquarks in the Born-Oppenheimer approximation

To determine an analytical expression for the \( \bar{Q}Q \) potential or equivalently \( b\bar{b} \) potential, we fit the ansatz
\[
V(r) = -\frac{\alpha}{r} \exp \left(-\frac{(r)}{d}\right) + V_0 \tag{4.1}
\]
with respect to \( \alpha, d \) and \( V_0 \) to the lattice QCD results obtained in the previous section. The constant \( V_0 \) accounts for twice the mass of the ground state static-light meson.

We insert the the analytical expression (4.1) in the Schrödinger equation for the radial coordinate of the two \( \bar{b} \) quarks (which we assume to be in an \( s \)-wave),
\[
\left(-\frac{1}{2\mu\frac{d^2}{dr^2}} + U(r)\right)R(r) = E_B R(r) \tag{4.2}
\]
with \( U(r) = V(r)|_{V_0=0} \) and \( \mu = m_{b}/2 \) and determine the lowest eigenvalue \( E_B \). If \( E_B < 0 \), the four quarks \( qq\bar{b}\bar{b} \) can form a tetraquark. If \( E_B > 0 \), there is no binding, i.e. the four-quark system will always be a system of two unbound \( B \) mesons. Notice that this so-called Born-Oppenheimer approximation is valid for \( m_q \ll m_b \), which is certainly the case for \( q \in \{u, d, s\} \) and at least crudely fulfilled for \( q = c \).

To quantify the systematic errors of different channels (scalar isosinglet and vector isotriplet, different light flavors \( q \in \{u, d, s, c\} \)), we perform a large number of fits varying the range of
temporal separations $t_{\text{min}} \leq t \leq t_{\text{max}}$ of the correlation function $C(t, r)$ (cf. eq. (3.1)), at which the lattice potential is read off, as well as the range of spatial $\bar{b}b$ separations $r_{\text{min}} \leq r \leq r_{\text{max}}$ considered in the $\chi^2$ minimizing fit of eq. (4.1) to the lattice potential. Details on this parameter variation can be found in [2]. For each set of input parameters $(t_{\text{min}}, t_{\text{max}}, r_{\text{min}}, r_{\text{max}})$ we determine $\alpha$, $d$ and $E_B$. Then we generate histograms for $\alpha$, $d$ and $E_B$ weighted according to the corresponding $\chi^2$/dof. The widths of these histograms are taken as systematic errors of $\alpha$, $d$ and $E_B$ [19], while the statistical errors are obtained via a jackknife analysis. In Figure 1 example histograms for the scalar isosinglet for $qq = ud$ are shown.

![Histograms for the scalar isosinglet for $qq = ud$. The red/green/blue bars indicate the statistical/systematic/combined errors.](image)

Figure 1: Histograms for the scalar isosinglet for $qq = ud$. The red/green/blue bars indicate the statistical/systematic/combined errors.

The resulting potentials fits for different channels, i.e. eq. (4.1) with corresponding values for $\alpha$ and $d$, are collected in Figure 2. The error bands represent the combined systematic and statistical errors. One can observe that the potentials are wider and deeper for lighter $qq$ quark masses. Moreover, the scalar channels are more attractive than the respective vector channels. Correspondingly, it turns out that there is a bound state only for the scalar isosinglet with $qq = ud$ with binding energy $-E_B = 93^{+47}_{-43}$ MeV, i.e. a bound state with around 2$\sigma$ confidence level.

In Figure 3 we present our results in an alternative graphical way. The three plots correspond to $u/d$, $s$ and $c$ light quarks $qq$, respectively. Each fit of eq. (4.1) to lattice potential results is represented by a dot (red: scalar channels; green: vector channels; crosses: $r_{\text{min}} = 2a$; boxes: $r_{\text{min}} = 3a$). The extensions of the point clouds represent the systematic uncertainties with respect to $\alpha$ and $d$. If a point cloud is localized above or left of the isoline with $E_B = -0.1$ MeV (essentially the binding threshold), the corresponding four quarks $qq\bar{b}\bar{b}$ cannot form a bound state. A localization below or right of that isoline is a strong indication for the existence of a tetraquark. Again there is clear evidence for a tetraquark state in the scalar $u/d$ channel. The scalar $s$ channel is slightly above, but rather close to the binding threshold. The scalar $c$ and all vector channels clearly do not host bound four-quark states.

5. Summary and outlook

We have found a $ud\bar{b}\bar{b}$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ (i.e. in the scalar isosinglet channel with $qq = ud$) with a confidence level of around 2$\sigma$. There seem to exist no
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Figure 2: Potentials fits for different channels (upper line: scalar isosinglet; lower line: vector isotriplet). The curves without an error band are copied from the respective other plots in the same line for easy comparison. Vertical lines indicate the available lattice $\bar{b}\bar{b}$ separations.

tetraquarks for the other channels.

In this work lattice QCD computations have been performed for light $u/d$ quarks corresponding to $m_\pi \approx 340 \text{MeV}$. We plan to repeat the analysis for at least another pion mass and then extrapolate to the physical point. It will then be most interesting to check, whether a bound state will also appear in the vector isotriplet channel with $qq = ud$. Another aspect is to investigate the structure of the found $I(J^P) = 0(1^+)$ tetraquark, i.e. to explore, whether it is rather a mesonic molecule or a diquark-antidiquark pair. We also plan to include corrections due to the heavy quark spins (for first preliminary results cf. [14]). Finally, one should study the experimentally more accessible, but theoretically more challenging case of $q\bar{q}b\bar{b}$ systems.

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Figure 3: Binding energy isolines $E_B = \text{constant}$ in the $\alpha$-$d$-plane for $u/d$, $s$ and $c$ light quarks $qq$ together with the fit results of eq. (4.1) to lattice potentials.
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References