

Glue Spin of the Proton *

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We report the progress on the lattice QCD calculation of the glue spin contribution to proton spin. This calculation is carried out with valence overlap fermion on 2+1 flavor DWF gauge configurations at two lattice spacings with the momentum of the frame in the kinematic range $0 \leq p^2 \leq 2 \text{ GeV}^2$. A mild frame dependence is observed. The matching and mixing with large-momentum effective field theory are in progress. The unrenormalized result at $p^2 = 4 \text{ GeV}^2$ with $O(a^2)$ correction gives $S_G = 0.13(3)$.

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Deep-inelastic experiments revealed that, contrary to the naive quark model picture, the quark spin contribution to proton spin is quite small, about 25% [1]. Recent analysis [2, 3] of the high-statistics 2009 STAR [4] and PHENIX [5] experiments at RHIC showed evidence of non-zero glue helicity in the proton, Δg . For $Q^2 = 10 \text{ GeV}^2$, the glue helicity distribution $\Delta g(x, Q^2)$ is found to be positive and away from zero in the momentum fraction region $0.05 \leq x \leq 1$ (specifically $0.05 \leq x \leq 0.2$, the region in which RHIC can determine $\Delta g(x)$ much better than the other regions). However, the results have very large uncertainty in the region $x \leq 0.05$.

Given the importance of Δg to address the origin of the proton spin, and the fact that large efforts are devoted to a precise experimental determination, a theoretical understanding and estimation of Δg is highly desired. In the infinite momentum frame (IMF), the total glue helicity which is the integral of the glue helicity distribution, i.e. $\Delta G = \int_0^1 \Delta g(x) dx$, is defined as [6],

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS | F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}_{\alpha,b}^+(0) | PS \rangle \quad (1)$$

where the light front coordinates are $\xi^\pm = (\xi^0 \pm \xi^3)/\sqrt{2}$. The proton plane wave state is written as $|PS\rangle$, with momentum P^μ and polarization S^μ . The light cone gauge-link $\mathcal{L}(\xi^-, 0)$ is in the adjoint representation. It connects the gauge field tensor and its dual, $\tilde{F}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\mu\nu} F_{\mu\nu}$, to construct a gauge invariant operator. After integrating the longitudinal momentum x , the light-cone operator for the matrix element has the following non-local expression [7, 8],

$$O_{\Delta G} = \left[\vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla^+} (\vec{\nabla} A^{+,b}) \mathcal{L}^{ba}(\xi^-, 0)) \right]^z \quad (2)$$

In principle, one cannot evaluate this expression in the light-front frame on the lattice due to its dependence on real time, given by ξ^- .

On the other hand, one can reach the glue helicity operator from the glue spin operator defined as

$$\vec{S}_g = \int d^3x \vec{E}^a \times \vec{A}_{phys}^a \quad (3)$$

where A_μ^{phys} comes from the decomposition scheme as proposed in [9, 10],

$$A_\mu = A_\mu^{phys} + A_\mu^{pure}. \quad (4)$$

In such a scheme, A^{phys} and A^{pure} transform homogeneously and inhomogeneously, with respect to gauge transformation,

$$\begin{aligned} A_\mu^{phys} &\rightarrow A_\mu'^{phys} = g(x) A_\mu^{phys} g^{-1}(x) \\ A_\mu^{pure} &\rightarrow A_\mu'^{pure} = g(x) A_\mu^{pure} g^{-1}(x) + \frac{i}{g_0} g(x) \partial_\mu g^{-1}(x), \end{aligned} \quad (5)$$

where g is the gauge transformation matrix and g_0 is the coupling constant. In order to have a unique solution, conditions are set as follows: the pure gauge part does not give rise to a field tensor by itself

$$F_{\mu\nu}^{pure} = \partial_\mu A_\nu^{pure} - \partial_\nu A_\mu^{pure} + ig_0 [A_\mu^{pure}, A_\nu^{pure}] = 0 \quad (6)$$

and A_μ^{phys} satisfies the non-Abelian transverse condition,

$$D_i A_i^{phys} = \partial_i A_i^{phys} - ig_0 [A_i, A_i^{phys}] = 0. \quad (7)$$

It is shown in [8] that when boosting the glue spin operator \vec{S}_g in Eq. (3) to IMF, the condition Eq. (7) corresponds to the light-cone gauge fixing condition $A_+^{phys} = 0$ and the forward matrix element of the longitudinal glue spin operator corresponds to the glue helicity, ΔG .

In contrast to the definition of the glue helicity operator in the IMF, the matrix element of the glue spin operator in the finite momentum frame is calculable in lattice QCD. Solutions for A_μ^{phys} and A_μ^{pure} satisfying Eqs. (5-7) can be obtained through a gauge link fixed in the Coulomb gauge under a gauge transformation $g_C(x)$,

$$U_\mu(x) = g_C(x) U_\mu^C(x) g_C^{-1}(x + a\hat{\mu}) \quad (8)$$

where $U^C(x)$ is fixed in the Coulomb gauge. One can confirm that the solution for A_μ^{phys} satisfying the gauge transformation law in Eq. (5) can be obtained as in Ref. [11]

$$\begin{aligned} A_\mu^{phys} &\equiv \frac{i}{ag_0} (U_\mu(x) - U_\mu^{pure}(x)) \\ &= g_C(x) A_\mu^C(x) g_C^{-1}(x) + \mathcal{O}(a), \end{aligned} \quad (9)$$

where $U_\mu^{pure}(x) = g_C(x) g_C^{-1}(x + a\hat{\mu})$. Then the glue spin operator \vec{S}_G reads,

$$\vec{S}_g = \int d^3x \text{Tr}(g_C \vec{E} g_C^{-1} \times \vec{A}^C) = \int d^3x \text{Tr}(\vec{E}^C \times \vec{A}^C) \quad (10)$$

where the trace Tr is taken over color indices and \vec{E}^C is the electric field in the Coulomb gauge.

In order to extract the glue spin contributions to the nucleon, we compute the ratio of the disconnected three-point function to the two-point function with the source and sink of the nucleon propagator located at t_0 and t , respectively. The glue spin operator is inserted at the time slice t' between t_0 and t . Then the ratio of the disconnected insertion three-point function to two-point function in the moving frame along the z direction is,

$$R(t, t', t_0) = \frac{\langle 0 | \Gamma_3^m \int d^3y e^{-ip_3 y_3} \chi(\vec{y}, t) S_g^3(t) \bar{\chi}(\vec{0}, t_0) | 0 \rangle}{\langle 0 | \Gamma^e \int d^3y e^{-ip_3 y_3} \chi(\vec{y}, t) \bar{\chi}(\vec{0}, t_0) | 0 \rangle} \quad (11)$$

where χ is the nucleon interpolation field and Γ_3^m/Γ^e is the polarized/unpolarized projection matrix of the proton respectively. The signal to noise ratio (SNR) can be improved by using the summed ratio method [15] and we denote the summed ratio by $SR(t, t_0) = \sum_{t'=t_0+1}^{t-1} R(t, t', t_0)$. $SR(t, t_0)$ has a linear behavior in the region where t is large enough that excited-state contamination is negligible,

$$SR(t - t_0) \equiv SR(t, t_0) \xrightarrow{t \gg 1} C_0 + (t - t_0) S_G + \mathcal{O}(e^{-\Delta E(t-t_0)}), \quad (12)$$

where S_G is the matrix element of the longitudinal glue spin operator in the proton.

A preliminary attempt [13] to calculate S_G on the lattice following the above prescription has been carried out on 2 + 1 flavor dynamical domain-wall configurations on a $24^3 \times 64$ lattice (24I) with the sea pion mass at 330 MeV. In this proceeding, we improve the statistics on the ensemble

Table 1: The parameters for the RBC/UKQCD configurations. $m_s^{(s)}$ and $m_\pi^{(s)}$ are the strange quark mass and the pion mass of the light sea quarks on the 2+1 flavor configurations, and N_{cfg} is the number of configurations used in the simulation.

Symbol	$L^3 \times T$	a (fm)	$m_s^{(s)}$ (MeV)	$m_\pi^{(s)}$ (MeV)	N_{cfg}
24I	$24^3 \times 64$	0.112(3)	120	330	203
32I	$32^3 \times 64$	0.084(2)	110	300	305

mentioned above and carry out the calculation on another ensemble with smaller lattice spacing, i.e. the $32^3 \times 64$ lattice, to check the $\mathcal{O}(a^2)$ correction to the glue spin. The parameters of the ensembles used in this proceeding are listed in Table 1.

To obtain S_G in a relatively large momentum frame, we calculate S_G for five different lattice momenta ($p = 0, 1, \sqrt{2}, \sqrt{3}, 2$) on the 24I ensemble and seven different lattice momenta ($p = 0, 1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}$) on the 32I ensemble. All the momenta are in unit of $2\pi/L$. On each ensemble, five quark masses with corresponding pion mass between 250 MeV and 400 MeV are used and a linear chiral extrapolation is applied to obtain the quantities at the physical point.

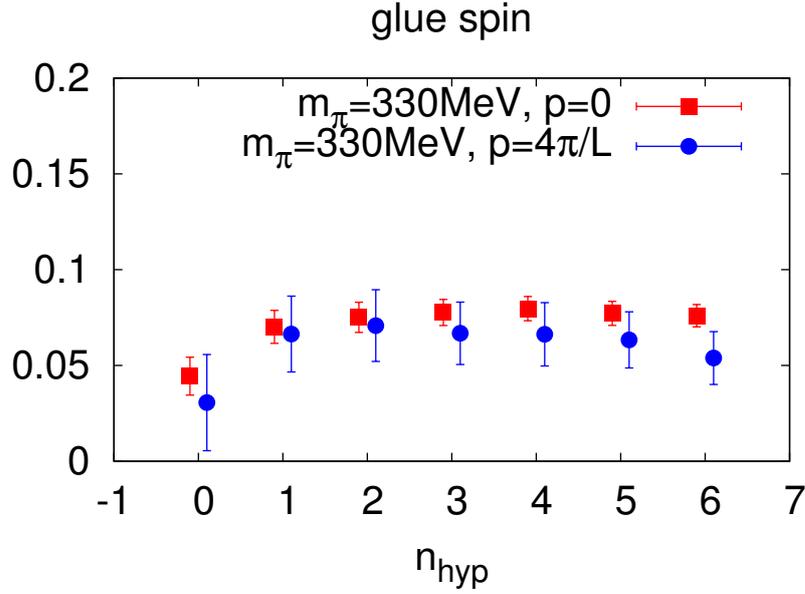


Figure 1: The HYP smearing dependence of the glue spin S_G , for $p = 0$ (red squares) and $p = 4\pi/L$ (blue dots). Both cases converge after one HYP smearing step.

To increase SNR, we generated the nucleon two point correlators with the source located on all the time slices on two ensembles and calculated the electric field with the clover definition,

$$F_{\mu\nu}^C = \frac{i}{8a^2 g_0} (\mathcal{P}_{\mu,\nu} - \mathcal{P}_{\nu,\mu} + \mathcal{P}_{\nu,-\mu} - \mathcal{P}_{-\mu,\nu} + \mathcal{P}_{-\mu,-\nu} - \mathcal{P}_{-\nu,-\mu} + \mathcal{P}_{-\nu,\mu} - \mathcal{P}_{\mu,-\nu}) \quad (13)$$

with HYP smeared gauge links [16] fixed in the Coulomb gauge, where $\mathcal{P}_{\mu,\nu} = U_\mu^C(x)U_\nu^C(x +$

$a\hat{\mu})U_{\mu}^{C\dagger}(x+a\hat{\nu})U_{\nu}^{C\dagger}(x)$ with $U_{\mu}^C(x)$ being the Coulomb gauge fixed Wilson link from $x+a\hat{\mu}$ to x . The Coulomb gauge fixing condition used here is enforced by requiring the spatial sum of the backward difference of the HYP smeared links to be zero,

$$\sum_{\mu=x,y,z} \left[U_{\mu}^C(x) - U_{\mu}^C(x-a\hat{\mu}) \right] = 0 \quad (14)$$

and the gauge fixed potential A_C is defined by $\left[\frac{U_{\mu}^C(x) - U_{\mu}^{C\dagger}(x)}{2ia_{g_0}} \right]_{\text{traceless}}$. It has been observed that the central values of the glue spin matrix elements as a function of HYP smearing steps are unchanged after one step of smearing, as shown in Fig. 1, while the SNR can be improved when more HYP smearing steps are applied. In this work, 6 and 11 steps HYP smearing are used for the 24I/32I ensembles respectively.

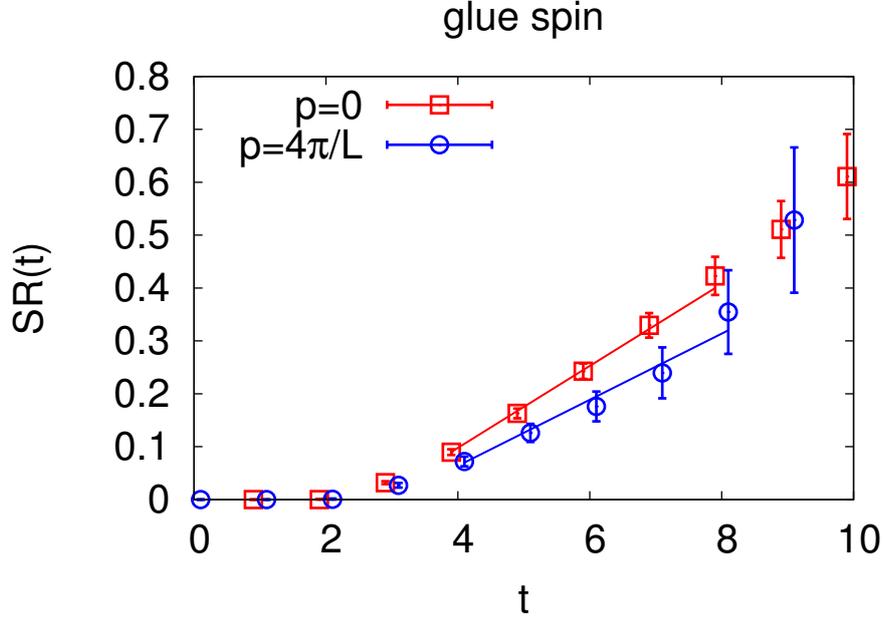


Figure 2: The summed ratio $SR(t)$ of the glue spin matrix elements for $p=0$ (red squares) and $p = 4\pi/L$ (blue dots) cases, at the unitary point of the 24I ensemble. Linear behaviors are observed, starting from $t=4$.

Upon incorporating these improvements, we see a clear linear behavior of the ratio $SR(t)$ as a function of time, which indicates that excited state contamination is negligible, thus allowing a clean and reliable extraction of the matrix element. Fig. 2 shows the case of the unitary point of the 24I ensemble, as an example.

The valence quark mass dependence is mild regardless the momentum involved, as in Fig. 3, which shows the case for the 24I ensemble. The sea quark mass dependence is not explored in this proceeding, and will be investigated in the future.

The glue helicity in proton spin, ΔG , corresponds to the glue longitudinal spin component S_G in the infinite momentum frame. An equivalent approach [17] is matching the glue spin under a regularization in a finite momentum frame (such as the lattice regularization) at a relatively large

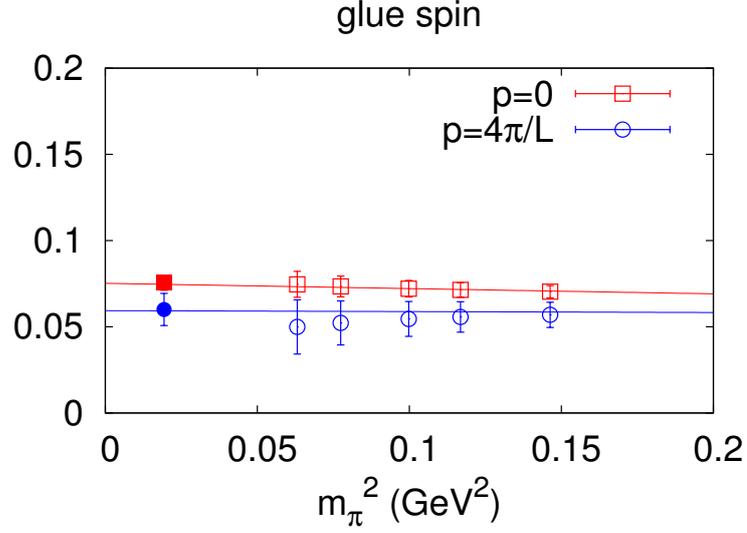


Figure 3: The quark mass dependence of the quark spin contribution in proton spin. The red squares are for zero momentum and the blue dots are for the $p = 4\pi/L$ (~ 800 MeV) case. These dependences are fairly mild and can be well described with a linear fit.

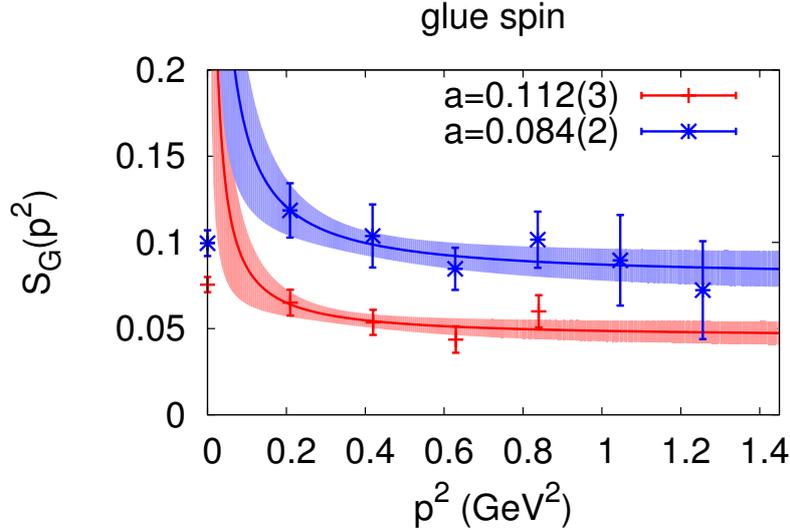


Figure 4: The result extrapolated to the physical pion mass, on two ensembles. The curves show the momentum (frame) dependence of the results.

scale (e.g. 2 GeV or larger) to the corresponding matrix element regularized in IMF, with the large-momentum effective field theory,

$$\tilde{\mathcal{O}}(P^z) = Z(P^z, 1/a)\mathcal{O}(1/a) + \frac{c_2}{(P^z)^2} + \frac{c_3}{(P^z)^4} + \dots \quad (15)$$

and including its mixing from the quark spin. This part of the calculation is in progress.

Setting $Z(P^z, 1/a) = 1$ for now, S_G can be extrapolated to $P_z=2$ GeV with Eq. (15). The

Table 2: Summary of the results. The values at 2 GeV are obtained by the fit with a polynomial series of $\frac{1}{(p^c)^2}$.

a (fm)	$S_G(p=0)$	$S_G(p=2 \text{ GeV})$
0.112(3)	0.075(4)	0.047(7)
0.084(2)	0.100(7)	0.084(10)

extrapolated results at the physical point of the pion mass, for different momenta on two ensembles, are listed in Table 2 and plotted in Fig. 4. An extrapolation to continuum with a linear $O(a^2)$ correction based on 24I and 32I lattices gives the unrenormalized $S_G(p=2 \text{ GeV}) = 0.13(3)$, which is consistent with $\langle \Delta g(Q^2) \rangle^{[0.05, 0.2]} = 0.17(6)$ in [3], if the Q^2 dependence is mild and the contribution from the other regions is negligible.

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