

Topological feature and phase diagram of QCD at complex chemical potential

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We propose new determination of the confinement-deconfinement (pseudo) critical temperature by using the analogy of the topological order discussed in the condensed matter physics. The imaginary chemical potential is interpreted as the Aharonov-Bohm phase, then the Roberge-Weiss endpoint would define the confinement-deconfinement (pseudo) critical temperature. Expected QCD phase diagram diagrams are presented at finite complex chemical potential based on the perturbative calculation.

The 33rd International Symposium on Lattice Field Theory

14 -18 July 2015

*Kobe International Conference Center, Kobe, Japan**

*Speaker.

1. Introduction

Understanding of QCD phase diagram at finite temperature (T) and real quark chemical potential (μ_R) is one of the interesting and important subjects in the nuclear and elementary particle physics. If we can obtain the QCD phase diagram from the first-principle calculation, there is no unclearness. However, such first principle calculation which is the lattice QCD simulation has the sign problem at finite μ_R . Even if several methods to circumvent the sign problem are used, we can not reach the $\mu_R/T > 1$ region. Therefore, several effective model calculations are performed so far. Figure 1 shows our current expectation of the QCD phase diagram.

In the study of the QCD phase diagram, the confinement-deconfinement transition plays a crucial role. However, our current understanding of the deconfinement transition is very limited. At least in the heavy quark mass limit, the Polyakov-loop which is the gauge invariant holonomy for the imaginary time circle becomes the exact order parameter of the deconfinement transition because the center (\mathbb{Z}_3) symmetry is directly related with the deconfinement transition in this case. Although, the Polyakov-loop is no longer the exact order parameter in the finite quark mass regime, the Polyakov-loop is usually treated as the approximate order parameter of the deconfinement transition, and the QCD phase diagram is drawn.

In this talk, we discuss the new determination of the deconfinement (pseudo) critical temperature by using the analogy of the topological order which is explained in Sec. 2. Also, we shows the expected QCD phase diagrams at complex chemical potential because we must extend the chemical potential to complex from real when we consider finite μ_R region in our present determination of the deconfinement (pseudo) critical temperature.

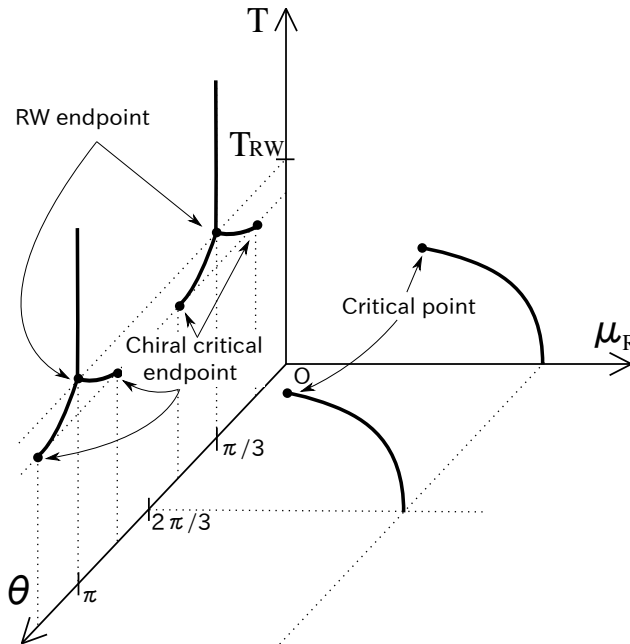


Figure 1: Schematic figure of our current expectation of the QCD phase diagram at finite μ_R and μ_I , respectively. Solid lines represent the first order transition line.

2. Free energy degeneracy and deconfinement transition

In the imaginary-time formalism, the $U(1)$ flux can be inserted to the closed loop of the imaginary-time direction and then the vector potential is induced by the $U(1)$ flux. Since the appearance form of the vector potential and the imaginary chemical potential is similar in the action, the imaginary chemical potential can be interpreted as the Aharonov-Bohm phase [1]. In this interpretation, we may use the discussion of the topological order discussed in the condensed matter physics [2]. It should be noted that actual applications to zero temperature QCD was already discussed in Ref. [3].

In Ref. [3], the author consider the torus T^3 at zero temperature. There are introduce three adiabatic operations: (a) Insert the $U(1)$ flux to spatial closed loops, (b) exchange i -th and $i+1$ -th quarks and (c) move a quark along loops. Commutation relations of the operation (b) and (c) follow the Braid group, and the Aharonov-Bohm effect determines the commutation relations of those operations with operation (a). If quarks are deconfined, above operations become non-commutable because quarks have the fractional charge. On the other hand, commutation relation is commutable if quarks are confined because physical states are characterized by hadron degrees of freedom with integer charges. Therefore, if there is only one vacuum in the deconfined phase, it is inconsistent with the non-commutability of the operations and thus vacuum degeneracy should exist.

At finite T , the topological order cannot be well defined, because thermal states are constructed by a mixture of pure states with the Boltzmann factor, and then we cannot operate (a), (b) and (c), adiabatically. Nevertheless, the RW periodicity shows significantly different behaviors in the confined and deconfined phases¹. It is induced by the nontrivial appearance of the RW periodicity in the deconfined phase. This fact suggests that we can distinguish the confined and deconfined phases at $\mu_I = 0$ from the non-trivial free energy degeneracy of the effective potential at $\theta = \pi/3$. Actually, we can have degenerated free energy minima at $\theta = \pi/3$ above T_{RW} , but not below T_{RW} . This degeneracy seems similar to the vacuum degeneracy in zero T systems used in the topological order and the analogy can be found; the response of hidden local minima by the flux insertion via μ_I tells us the non-trivial degeneracy of the free energy minima. Therefore, we propose that T_{RW} is the pseudo-critical temperature of the confinement-deconfinement transition.

It should be noted that the present definition and the standard definition determined by using the Polyakov-loop are consistent in the infinite quark mass limit where the Polyakov-loop is the exact order-parameter of the confinement-deconfinement transition. Therefore, we can find the relation $T_D = T_\Phi = T_{RW}$ where T_Φ is the critical temperature determined by the susceptibility of the Polyakov-loop and T_D is the deconfinement critical temperature. When the dynamical quarks are taken into account, T_Φ changed to the pseudo-critical temperature. The upper bound of the pseudo-critical temperature may be determined by the appearance of local minima of the effective potential at $\mu_I = 0$ in the $\text{Re } \Phi - \text{Im } \Phi$ plane. We call it $T_{\mathbb{Z}_3}$. From the lattice QCD and effective model calculations, we can have the relation $T_\Phi \leq T_{RW} \leq T_{\mathbb{Z}_3}$. The Polyakov-loop is no longer the exact order parameter and thus the determination of T_D is not unique, but T_{RW} is uniquely determined. Thus, T_{RW} which agrees with T_D in the infinite quark mass limit is unambiguously determined in the lattice QCD and effective models and provides a reasonable value as T_D in the

¹For example, this fact can be seen from the comparison between the perturbative one-loop calculation [4, 5] and the strong coupling limit of QCD with the mean field approximation [6, 7] at finite imaginary chemical potential.

case with the dynamical quark. If we adopt T_{RW} as T_D , we can say that the deconfinement transition is the topological phase transition.

3. QCD phase diagram at complex chemical potential

We now discuss the μ_R -dependence of the deconfinement pseudo-critical temperature defined by T_{RW} . We here give an model independent argument based on the perturbative expansion of the effective potential at finite μ_R as a first step to investigate the μ_R -dependence of the RW endpoint. It should be noted that non-perturbative model approaches have several difficulties. One of the promising effective models is the Polyakov-loop extended Nambu–Jona-Lasinio (PNJL) model [8], but the PNJL model also has the model sign problem at finite μ_R [9]. There are some proposals to circumvent the model sign problem, for example the complex integral path contour method [10, 11, 12] based on the Lefschetz thimble [13, 14, 15] and the complex Langevin method [16, 17]. Unfortunately those approaches cannot be directly used at finite complex chemical potential at the present because we cannot maintain the RW periodicity and some other desirable properties of QCD.

The effective potential with small μ_R can be expanded up to μ_R^2 order as

$$\begin{aligned} \mathcal{V}(T, \mu_R, \mu_I) &= \mathcal{V}(T, 0, \mu_I) - \left(\frac{\mu_R}{T}\right) (Tn_q(\mu_R, \mu_I)|_{\mu_R=0}) \\ &\quad - \frac{1}{2} \left(\frac{\mu_R}{T}\right)^2 \frac{d[Tn_q(\mu_R, \mu_I)]}{d\mu_R/T} \Big|_{\mu_R=0} + \mathcal{O}\left((\mu_R/T)^3\right), \end{aligned} \quad (3.1)$$

where

$$T \frac{dn_q}{d\mu_R/T} \Big|_{\mu_R=0} = T \frac{dn_q}{d(i\mu_I/T)} \Big|_{\mu_R=0} = -iT \frac{dn_q}{d\theta} \Big|_{\mu_R=0}. \quad (3.2)$$

Equation (3.2) is real. The second term in r.h.s. of Eq. (3.1) is pure imaginary.

Here, we consider the confinement and deconfinement configurations. We call the configuration at $(T_{RW} - \varepsilon)$ *confinement configuration* which is labeled as $\mathcal{C}_{-\varepsilon}$ where ε is a infinitesimal real positive value. Also, we call the configuration at $(T_{RW} + \varepsilon)$ *deconfinement configuration* which is labeled as $\mathcal{C}_{+\varepsilon}$. By comparing $\text{Re } \mathcal{V}$ with $\mathcal{C}_{-\varepsilon}$ to that with $\mathcal{C}_{+\varepsilon}$ in the $\varepsilon \rightarrow 0$ limit, we can distinguish whether T_{RW} decreases or increases if we can estimate the third term of r.h.s. of Eq. (3.1). From the lattice QCD and effective model calculations [18, 19], we can estimate that Eq. (3.2) is negative below T_{RW} and it becomes moderate above T_{RW} . The μ_R^2 correction term makes $\text{Re } \mathcal{V}$ with $\mathcal{C}_{-\varepsilon}$ higher than that with $\mathcal{C}_{+\varepsilon}$ because the μ_R^2 correction term is then positive with $\mathcal{C}_{+\varepsilon}$. This fact means that the first-order T_{RW} decreases with increasing μ_R at least in the small μ_R region. This behavior is consistent with the decreasing behavior of the pseudo-critical temperature of the deconfinement transition defined by using usual determinations which consider the Polyakov-loop; for example, see Refs [8, 20, 21].

By taking into account our perturbative result and the symmetry argument, we can sketch expected QCD phase diagrams at finite complex chemical potential. QCD phase diagrams expected from the symmetry argument and our present perturbative calculations are summarized in Fig. 2. Because of the RW periodicity, the phase structure should be periodic along the θ axis. The RW transition line on the (T, θ) plane ($\mu_R = 0$) may be connected with the first order phase transition boundary on the (T, μ_R) plane. Two of the first order transition lines starting from the RW endpoint

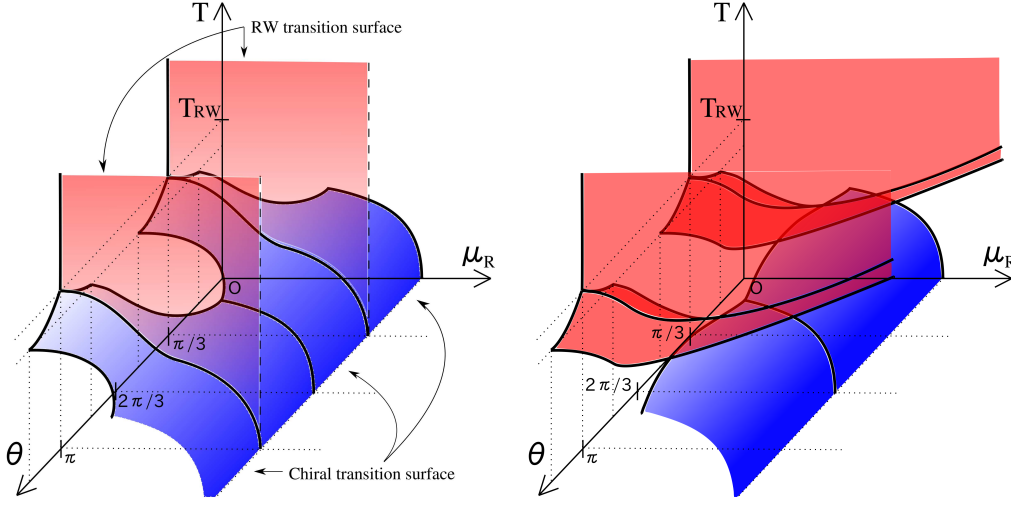


Figure 2: Two possible schematic QCD phase diagrams at finite complex chemical potential. The left (right) panel represents the correlated (uncorrelated) case between the chiral and RW transition surfaces.

have chiral transition natures and are referred to as the *chiral transition lines* [22]. Then, it is not unreasonable to expect that the endpoint of the chiral critical line on the (T, θ) plane is connected with the critical point on the (T, μ_R) plane. In this case, the first order phase boundary on the (T, μ_R) plane forms a chiral transition surface in the (T, μ_R, θ) space, and connects the (T, μ_R) plane and the (T, θ) plane. The RW transition line extends in the finite μ_R region and forms an RW transition surface in the (T, μ_R, θ) space. The RW endpoint may reach $T = 0$ as shown in the top panel of Fig. 2 or it may deviate from the chiral transition surface at some temperature. There is the possibility that T_{RW} line becomes smaller than the chiral critical surface at moderate μ_R and finally becomes larger than the chiral transition surface.. It is deeply related with a strength of the correlation between the chiral transition surface and the RW transition surface.

Another possibility is that the first order transition lines on the (T, θ) plane are separated from the first order phase boundary on the (T, μ_R) plane, as shown in the bottom panel of Fig. 2. T_{RW} first decreases at small μ_R , but does not go across the chiral transition surface. In this case, the deconfinement transition represented by the RW endpoint on the (T, θ) plane has less relevance to the first order phase boundary, which would be the chiral transition, on the (T, μ_R) plane. Therefore, we can call this possibility *uncorrelated case* and the former possibility *correlated case*.

The QCD phase diagram at finite complex chemical potential may be related with the following subjects. (I) In Ref. [23, 24], the authors use experimental data to construct the canonical partition function. Then, T_{RW} at $\mu_R = 0$ is used to clarify realized temperatures in experiments through the Lee-Yang zero analysis [25, 26]. In the analysis, T_{RW} at finite μ_R should be related with zeros inside the unit circle on the complex quark fugacity plane. If so, there is the possibility that we can strictly determine realized temperatures in experiments if we can systematically understand the behavior of zeros. (II) The analytic continuation in QCD from the imaginary to the real chemical potential is usually performed on the μ^2 plane. In the continuation, we may miss some information such as an inhomogeneous chiral condensate [27]. The analytic continuation on the

complex chemical potential plane may restore such information missing.

4. Summary

We have proposed that the Roberge-Weiss endpoint provides a reasonable deconfinement temperature. The imaginary chemical potential can be interpreted as the Aharonov-Bohm phase induced by $U(1)$ flux insertions and then the analogy of the topological order can be found. In the deconfined phase, we can find the degeneracy free energy degeneracy, but we cannot find such non-trivial structure of the free energy in the confined phase. This suggests that we can distinguish the confined and deconfined phases at $\theta = \mu_I/T = 0$ from the appearance of the non-trivial degeneracy of the effective potential. Then, T_{RW} can be considered as the pseudo-critical temperature at $\mu = 0$.

In this study, we have used the analogy of the topological order at finite T . The topological transition does not have the usual order parameter, but the relation with an entanglement and topological entropies has been discussed in the condensed matter physics; for example, see Ref. [28]. In QCD, such a relation is not clear, but it is an interesting direction to investigate the topological nature of the transition.

Using the perturbative expansion in terms of μ_R of the effective potential, we have investigated the behavior of T_{RW} at small μ_R and then the decreasing behavior of T_{RW} is obtained. Based on these results, we presented two scenarios of the QCD phase diagram at finite complex chemical potential. First scenario has the strong correlation between the chiral and deconfinement transitions. The RW endpoint at finite μ_R finally reaches the $T = 0$ point. Then, the critical point can become more complex than the usual expectation since two more first order transition lines are connected at the critical point. The second scenario is the uncorrelated case and then the RW endpoint is separated from the chiral transition surface.

Since the complex chemical potential is related with the Lee-Yang zero analysis and also the analytic continuation to the finite μ_R region, understanding the QCD phase diagram at finite complex chemical potential may be important to the beam energy scan program in heavy ion collider experiments, investigation of neutron star structures and so on.

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