Lattice simulation of QC_2D with $N_f = 2$ at non-zero baryon density

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The lattice simulations of QC_2D with two flavors of staggered fermions and non-zero quark chemical potential μ_q have been performed. Dependencies of the Polyakov loop, chiral condensate and baryon number density on μ_q were studied. We found that an increase of the baryon chemical potential leads to chiral symmetry restoration. At small values of μ_q, our results for the baryon number density agree with ChPT predictions.

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1. Introduction

The study of the phase diagram of QCD is very important for astrophysics, cosmology, and particle physics. However, the phase diagram of QCD is still incompletely studied. The reason is that quarks and gluons constitute a strongly interacting system, and this fact complicates the use of analytical methods. One of the possible approaches to the study of the properties of such a system is the LQCD, where functional integrals are numerically calculated using the Monte Carlo method. Calculations in lattice QCD allow ab initio study of the properties of a quark-gluon plasma [1].

However, calculations in LQCD with finite chemical potential are now impossible due to the sign problem of the fermionic determinant. Therefore, we shall study a simpler theory: QFT with $SU(2)$ gauge group and two degenerate flavors, where no sign problem arises [2, 3, 4], in order to better understand the qualitative features of the QCD phase diagram and to analyze the effect of a non-zero chemical potential on the properties of QGP. The point is that QC_2D has specific relation for the Dirac operator [2, 3]:

$$det\Big[M(\mu_q)\Big] = det\Big[(\tau_2 C\gamma_5)^{-1} M(\mu_q)(\tau_2 C\gamma_5)\Big] = det\Big[M(\mu_q^*)\Big]^*, \tag{1.1}$$

where $M(\mu_q) = \gamma_\mu D_\mu + m_q - \mu_q\gamma_4$ is the Dirac operator in continuum R^4, $\mu_q = \mu_B/2$ is the quark chemical potential, $C = \gamma_2\gamma_4$, and τ_2 is a generator of the $SU(2)$ group. Relation 1.1 guarantees, that $det\Big[M(\mu_q)\Big]$ is real for the real μ_q. One can also prove [4], that the spectrum of $M(\mu_q)^\dagger M(\mu_q)$ at finite real μ_q is strictly positive, both in the continuum and for the lattice formulation 2.3, if the quark mass is non-zero.

Our aim is to study the influence of the chemical potential on the Polyakov loop, chiral condensate and baryon number density. Especially interesting is to understand the effect of a non-zero baryon density on the breaking/recovery of the chiral symmetry. Similar investigations were performed in [3, 5, 6] for $N_f = 2$ with Wilson fermions and in [7][8] with $N_f = 4$ and 8 flavors of staggered fermions respectively. However, Wilson fermions explicitly violate the chiral symmetry [9], thus they may not reveal all the phase transition lines in the QC_2D phase diagram. In this paper we consider $N_f = 2$ flavors of staggered fermions, taking the fourth root of the $det\Big[M(\mu_q)^\dagger M(\mu_q)\Big]$ via the R-algorithm [10].

2. Lattice formulation

The partition function of the system under study has the form:

$$Z = \int DU\, det\Big[M^\dagger(\mu_q) M(\mu_q)\Big]^{\frac{1}{4}} e^{-S_G[U]}, \tag{2.1}$$

where the functional integration is performed over the $SU(2)$ group manifold, $M(\mu_q)$ is the lattice Dirac operator for Kogut-Susskind fermions with the baryon chemical potential, and $S_G[U]$ is the Wilson gauge action [11]:

$$S_G = \beta \sum_x \sum_{\mu<\nu=1}^{4} \Big(1 - \frac{1}{2}\mathrm{Tr}\, U_{x,\mu\nu}\Big). \tag{2.2}$$

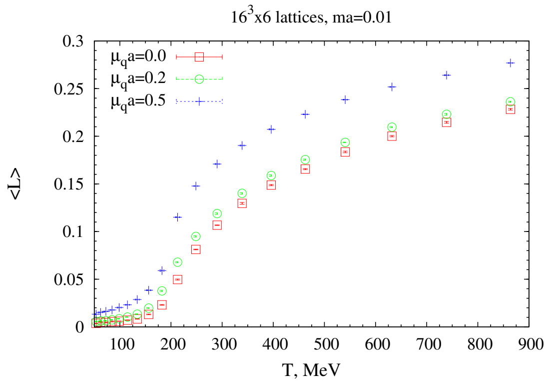

Figure 1: Polyakov loop as a function of T for three values of the baryon chemical potential.

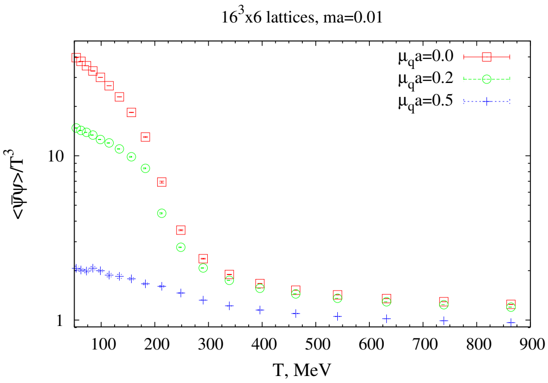

Figure 2: Chiral condensate as a function of T for three values of the baryon chemical potential. The ordinate axis is given on a logarithmic scale.

Here $\beta = \frac{4}{g^2}$, and $U_{x,\mu\nu} = U_{x,\mu}U_{x+\hat{\mu},\nu}U^\dagger_{x+\hat{\nu},\mu}U^\dagger_{x,\nu}$. The lattice Dirac operator $M(\mu_q)$ in 2.1 has the form:

$$M_{xy} = ma\delta_{xy} + \frac{1}{2}\sum_{\mu=1}^{4}\eta_\mu(x)\left[U_{x,\mu}\delta_{x+\hat{\mu},y}e^{\mu_q a\delta_{\mu,4}} - U^\dagger_{x-\hat{\mu},\mu}\delta_{x-\hat{\mu},y}e^{-\mu_q a\delta_{\mu,4}}\right], \tag{2.3}$$

where a is the lattice spacing, m is the mass of the quark, and the functions $\eta_1(x) = 1$, $\eta_2(x) = (-1)^{x_1}$, $\eta_3(x) = (-1)^{x_1+x_2}$, $\eta_4(x) = (-1)^{x_1+x_2+x_3}$ are the γ-matrices after the Kogut-Susskind transformation. The chemical potential μ_q is introduced in 2.3 by means of the multiplication of time links by the exponential factor $e^{\pm\mu_q a}$. This way of the introduction of the chemical potential makes it possible to avoid additional divergences and to reproduce the known result for free fermions in the continuum limit [12].

For partition function 2.1 in the continuum limit to correspond to two dynamic quark flavors, we extract the fourth order root of the fermionic determinant using the rational approximation with an accuracy of $O(10^{-15})$ [13]. Configurations were generated by means of the hybrid Monte Carlo method, Φ-algorithm [9] was employed. We considered a $16^3 \times 6$ lattice with the bare fermion mass $ma = 0.01$, $\beta = 1.6 \ldots 2.7$, and $\mu_q a = 0.0 \ldots 0.6$ (for each set of parameters, 400 independent configurations were generated). The program code was written with the use of CUDA. The calculations were performed at the ITEP supercomputer and IHEP cluster.

3. Numerical results and discussion

To study the physical properties of the system, we considered the following observables (triangular brackets mean thermodynamic averaging):

- Polyakov loop:

$$\langle L\rangle = \frac{1}{N_s^3}\sum_{x_1,x_2,x_3=0}^{N_s-1}\frac{1}{2}\left\langle \mathrm{Tr}\prod_{x_4=0}^{N_\tau-1}U_{x,4}\right\rangle; \tag{3.1}$$

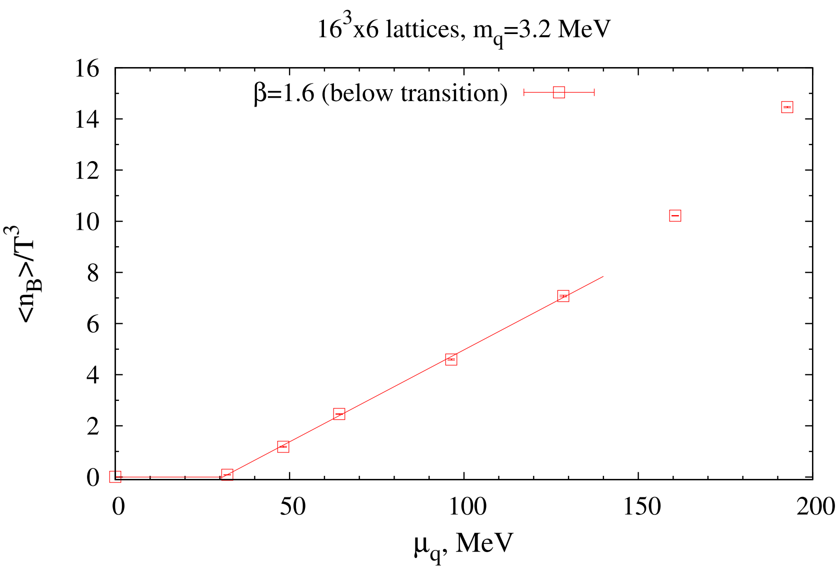

Figure 3: Baryon number density as a function of μ_q in the low-temperature phase ($\beta = 1.6$). Linear fit is shown.

- chiral condensate:

$$a^3 \langle \overline{\psi}\psi \rangle = -\frac{1}{N_s^3 N_\tau} \frac{1}{8} \frac{\partial}{\partial(ma)} log Z = \frac{1}{N_s^3 N_\tau} \frac{1}{8} \left\langle \mathrm{Tr} M^{-1} + \mathrm{Tr}\,(M^\dagger)^{-1} \right\rangle; \qquad (3.2)$$

- baryon number density:

$$a^3 \langle n_B \rangle = \frac{1}{N_s^3 N_\tau} \frac{1}{16} \frac{\partial(log Z)}{\partial(\mu_q a)} = \frac{1}{N_s^3 N_\tau} \frac{1}{8} \left\langle Re\mathrm{Tr} \left(\frac{\partial M}{\partial(\mu_q a)} M^{-1} \right) \right\rangle. \qquad (3.3)$$

In order to fix the scale we employed Sommer parameter r_0 = 0.468(4) fm [1] and performed the measurements on $16^3 \times 32$ lattices with $m_q a = 0.01$. Lattice spacings and pion masses are listed in the Table 1.

Figure 1 shows the dependences of the Polyakov loop on temperature for three μ_q values, a crossover phase transition is observed. It may also be seen in the figure, that an increase in the baryon chemical potential results in an slight increase in $\langle L \rangle$ for the same temperature. However,

β	a, fm	M_π, MeV
1.7	0.45(1)	109(3)
1.9	0.20(1)	216(6)
2.1	0.135(2)	430(13)
2.2	0.097(1)	551(16)

Table 1: Lattice spacings and pion masses.

the susceptibilities of $\langle L \rangle$ can not be measured with the existing statistics and the influence of the baryon chemical potential on T_cs cannot be determined.

Figure 2 shows the dependences of the chiral condensate on T for various μ_q values. It can be seen that an increase in the baryon chemical potential leads to a significant decrease in $\langle \overline{\psi}\psi \rangle$, i.e., to the recovery of the chiral symmetry. These results are in agreement with the previous results obtained for Wilson fermions [5] and with the outcomes of [14]. On the figure 3 the dependence of the baryon number density on μ_q in the confinement phase is shown. One can see, that at $\mu_q \approx m_\pi/2$ it begins to rise linearly, and then the increment becomes non-linear. Such a behaviour agrees well with the ChPT predictions [14, 15].

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