Determination of $U_A(1)$ restoration from meson screening masses by using the entanglement PNJL model: Toward chiral regime

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We determine temperature ($T$) dependence of $U_A(1)$ restoration from meson screening masses calculated with 2+1 flavor lattice QCD, using Polyakov-loop extended Nambu–Jona-Lasinio (PNJL) model with entanglement vertex. The entanglement PNJL (EPNJL) model exhibits the $U_A(1)$ anomaly through the Kobayashi–Maskawa–’t Hooft (KMT) interaction. $T$ dependence of KMT interaction strength is determined from the difference between pion and $a_0$ meson screening masses. The strength is strongly suppressed around the pseudocritical temperature of chiral transition. Using this $T$-dependent KMT interaction, we draw the Columbia plot near the physical point. In the light-quark chiral-limit with the strange quark mass fixed at the physical value, the chiral transition becomes the second order. This indicates that there exists a tricritical point. Hence the location is estimated.

The 33rd International Symposium on Lattice Field Theory
14 -18 July 2015
Kobe International Conference Center, Kobe, Japan

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1. Introduction

In the Quantum Chromodynamics (QCD) vacuum, $U_A(1)$ symmetry is explicitly broken by the $U_A(1)$ anomaly through the topologically nontrivial gauge configurations. For high temperature ($T$), the instanton density $dn_{\text{inst}}(T)$ is suppressed by the Debye-type screening \[ (1.1) \]

\[ dn_{\text{inst}}(T) \sim dn_{\text{inst}}(0) \exp \left[ - \frac{\pi^2 \rho^2 T^2}{3 N_c + \frac{1}{3} N_f} \right], \]

where $N_c(N_f)$ means number of colors (flavors) and $\rho$ means instanton radius. The suppression suggests that $U_A(1)$ symmetry is effectively restored at high temperature. The restoration of $U_A(1)$ symmetry is related with the order of the chiral phase transition in 2-flavor QCD at zero light-quark mass. In Ref. \[ (2) \], it is suggested that the order may be second order with 3d Izing $O(4)$ universality class if the effective restoration is not completed at $T = T_c$, where $T_c$ is transition temperature for chiral phase transition. When $U_A(1)$ symmetry is restored completely at $T = T_c$, the chiral transition becomes the first order \[ (2) \]. Recently, however, it was pointed out in Ref. \[ (3) \] that the second order is still possible. In this case, the universality class is not $O(4)$ but $U(2)_L \times U(2)_R$. There are many lattice QCD (LQCD) simulations and effective model analyses made so far to clarify the order and its universality class in the two-flavor chiral limit and the light-quark chiral limit where light-quark mass vanishes with strange-quark mass fixed at the physical value, but these are still controversial.

In this talk, we incorporate the effective restoration of $U_A(1)$ symmetry in entanglement Polyakov-loop extended Nambu–Jona-Lasinio (EPNJL) model by introducing a temperature-dependent strength $K(T)$ to the Kobayashi-Maskawa-'t Hooft (KMT) determinant interaction. $T$ dependence of $K(T)$ is well determined from the results of state-of-the-art 2+1-flavor lattice QCD simulations on pion and $a_0$-meson screening masses. Using the EPNJL model, we draw the Columbia plot near the physical point and determine the order of chiral transition in the light-quark chiral limit with $m_s$ fixed at the physical value.

2. Model setting

2.1 EPNJL model

We start with the 2+1 flavor EPNJL model \[ (4) \]. The Lagrangian density is

\[ \mathcal{L} = \bar{\psi}(i \gamma_\mu D^\mu - \hat{m}_0) \psi + G_s(\Phi) \sum_{a=0}^{8} \left[ (\bar{\psi} \lambda_a \psi)^2 + (\bar{\psi} i \gamma_5 \lambda_a \psi)^2 \right] - K(T) \left[ \det \psi_f (1 + \gamma_5) \psi_f + \det \bar{\psi}_f (1 - \gamma_5) \bar{\psi}_f \right] - \mathcal{W} (\Phi[A], \Phi[A], T) \]

\[ (2.1) \]

with quark fields $\psi = (\psi_u, \psi_d, \psi_s)^T$ and $D^\mu = \partial^\mu + i A^\mu$ with $A^\mu = \delta^{\mu}_{0} g(A^3)_{a} t_{a}/2 = -\delta^{\mu}_{0} ig(A_{4})_{a} t_{a}/2$ for the gauge coupling $g$, where the $\lambda_a (t_{a})$ are the Gell-Mann matrices in flavor (color) space and $\lambda_{0} = \sqrt{2/3} \, I$ for the unit matrix $I$ in flavor space. The determinant in \[ (2.1) \] is taken in flavor space. For the 2+1 flavor system, the current quark masses $\hat{m}_0 = \text{diag}(m_u, m_d, m_s)$ satisfy a relation $m_u > m_d \equiv m_u = m_d$. In the EPNJL model, the coupling strength $G_s(\Phi)$ of the scalar-type four-quark interaction depends on the Polyakov loop $\Phi$ and its Hermitian conjugate $\bar{\Phi}$ as

\[ G_s(\Phi) = G_s(0) \times \left[ 1 - \alpha_l \Phi \bar{\Phi} \right]. \]

\[ (2.2) \]
This entanglement coupling is charge-conjugation and $Z_3$ symmetric.

For $T$ dependence of $K(T)$, we assume the following form phenomenologically:

$$
K(T) = \begin{cases} 
K(0) & (T < T_1) \\
K(0)e^{-(T-T_1)c/b^2} & (T \geq T_1)
\end{cases} 
$$

(2.3)

For high $T$ satisfying $T \gg T_1$, the form (2.3) is reduced to (2.4).

In the EPNJL model, the time component of $A_\mu$ is treated as a homogeneous and static background field, which is governed by the Polyakov-loop potential $\mathcal{U}$. In the Polyakov gauge, $\Phi$ and $\bar{\Phi}$ are obtained by

$$
\Phi = \frac{1}{3} \text{tr}_c(L), \quad \bar{\Phi} = \frac{1}{3} \text{tr}_c(L^*)
$$

(2.4)

with $L = \exp[iA_4/T] = \exp[i\text{diag}(A_{41}^{11}, A_{41}^{22}, A_{43}^{33})/T]$ for real variables $A_{41}^{ij}$ satisfying $A_{41}^{11} + A_{41}^{22} + A_{43}^{33} = 0$. For zero chemical potential where $\Phi = \bar{\Phi}$, one can set $A_{43}^{33} = 0$ and determine the others as $A_{41}^{22} = -A_{41}^{11} = \cos^{-1}((3\Phi - 1)/2)$.

We use the logarithm-type Polyakov-loop potential of Ref. [5] as $\mathcal{U}$, but we refit the parameter $T_0$ to 180 MeV in order to reproduce the chiral transition temperature $T_c = 154 \pm 9$ MeV [2, 3, 5] and deconfinement transition temperature $T_{\text{deconf}} = 170 \pm 7$ MeV [3].

Making the mean field approximation (MFA) to (2.4) and the path integral over quark fields, one can get the thermodynamic potential (per unit volume) as

$$
\Omega = U_M + \mathcal{U} - 2 \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left[ 3E_{p,f} + \frac{1}{\beta} \ln \left[ 1 + 3(\Phi + \bar{\Phi} e^{-\beta E_{p,f}}) e^{-\beta E_{p,f}} + e^{-3\beta E_{p,f}} \right] \\
+ \frac{1}{\beta} \ln \left[ 1 + 3(\Phi + \bar{\Phi} e^{-\beta E_{p,f}}) e^{-\beta E_{p,f}} + e^{-3\beta E_{p,f}} \right] \right]
$$

(2.5)

with $\beta = 1/T$ and $E_{p,f} = \sqrt{p^2 + M_f^2}$. The effective quark mass $M_f$ is $M_f = m_f - 4G_u(\Phi)\sigma_f + 2K(T)\sigma_f \bar{\sigma}_{f'}$ with $f \neq f' \neq f''$. The mesonic potential $U_M$ is $U_M = 2G_u(\Phi)(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 4K(T)\sigma_1 \bar{\sigma}_1 \bar{\sigma}_1$. Here, $\sigma_f$ means the chiral condensate $\langle \bar{\psi}_f \psi_f \rangle$ for flavor $f$. We determine the mean-field variables $(X = \sigma_1, \sigma_2, \Phi, \bar{\Phi})$ from the stationary conditions:

$$
\frac{\partial \Omega}{\partial X} = 0,
$$

(2.6)

where isospin symmetry is assumed for the light-quark sector, i.e., $\sigma_x = \sigma_u = \sigma_d$.

On the right-hand side of (2.5), the first term (vacuum term) in the momentum integral diverges. We then use the PV regularization [11]. In the scheme, the integral $I(M_f)$ is regularized as

$$
I^{\text{reg}}(M_f) = \sum_{\alpha=0}^2 C_{\alpha} I(M_{f;\alpha}),
$$

(2.7)

where $M_{f;0} = M_f$ and the $M_{f;\alpha}$ ($\alpha \geq 1$) mean masses of auxiliary particles. The parameters $M_{f;\alpha}$ and $C_{\alpha}$ should satisfy the condition $\sum_{\alpha=0}^2 C_{\alpha} = \sum_{\alpha=0}^2 C_{\alpha} M_{f;\alpha}^2 = 0$. We then assume $(C_0, C_1, C_2) = (1, 1, -2)$ and $(M_{f;1}^2, M_{f;2}^2) = (M_f^2 + 2\Lambda^2, M_f^2 + \Lambda^2)$. We keep the parameter $\Lambda$ finite even after the
subtraction (\[\mathcal{M}\]), since the present model is non-renormalizable. The parameters are taken from Ref. [12] and they are \(m_t = 6.2\) MeV, \(m_q = 175.0\) MeV, \(G_s(0)\Lambda^2 = 2.35\) and \(K(0)\Lambda^5 = 27.8\) for \(\Lambda = 795\) MeV. This parameter set reproduces mesonic observables at vacuum, i.e., the pion and kaon decay constants \((f_\pi = 92\) MeV and \(f_K = 105\) MeV\)) and their masses \((M_\pi = 141\) MeV and \(M_K = 512\) MeV\)) and the \(\eta^\prime\)-meson mass \((M_{\eta'} = 920\) MeV\)). In the present work, we analyze LQCD results of Ref. [12] for pion and \(a_0\)-meson screening masses. In the LQCD simulation, the pion mass \(M_\pi(0)\) at vacuum \((T = 0)\) is 175 MeV and a bit heavier than the experimental value 138 MeV. We then change \(m_t\) to 9.9 MeV in the EPNJL model in order to reproduce \(M_\pi(0) = 175\) MeV.

2.2 Meson screening mass

We derive the equations for pion and \(a_0\)-meson screening masses, following Refs. [13, 14]. The current corresponding to a meson of type \(\xi\) is

\[
J_{\xi}(x) = \bar{\psi}(x)\Gamma_{\xi}\psi(x) - \langle \psi(x)\Gamma_{\xi}\bar{\psi}(x) \rangle,
\]

where \(\Gamma_{\pi} = i\gamma_5\lambda_3\) for \(\pi\) meson and \(\Gamma_{a_0} = \lambda_3\) for \(a_0\) meson. We denote the Fourier transform of the mesonic correlation function \(\eta_{\xi\xi}(q) \equiv \langle 0|T\{J_{\xi}(x)\bar{J}_{\xi}(0)\}|0\rangle\) by \(\chi_{\xi\xi}(q^2, \bar{q}^2)\) as

\[
\chi_{\xi\xi}(q^2, \bar{q}^2) = i\int d^4x e^{i\bar{q}x}\eta_{\xi\xi}(x),
\]

where \(\bar{q} = \pm|q|\) for \(q = (q_0, \mathbf{q})\) and \(T\) stands for the time-ordered product. Using the random-phase (ring) approximation, one can obtain the Schwinger-Dyson equations for \(\xi = \pi, a_0\) channels

\[
\chi_{\xi\xi} = \frac{\Pi_{\xi}}{1 - 2G_{\xi}\Pi_{\xi}}
\]

with the effective couplings \(G_{\pi}\) and \(G_{a_0}\) defined by

\[
G_{a_0} = G_s(\Phi) + \frac{1}{2}K(T)\sigma_3, \quad G_{\pi} = G_s(\Phi) - \frac{1}{2}K(T)\sigma_3,
\]

and the one-loop polarization function \(\Pi_{\xi}\) defined by

\[
\Pi_{a_0} = 4i[I_1 + I_2 - (q^2 - 4M^2)I_3], \quad \Pi_{\pi} = 4i[I_1 + I_2 - q^2I_3].
\]

\(M\) is the effective light-quark mass and \(M_a = M_d\). At \(T = 0\), three integrals \(I_1, I_2, I_3\) are obtained by

\[
I_1 = \int \frac{d^4p}{(2\pi)^4} \mathfrak{tr} \left[ \frac{1}{p^2 - M^2} \right], \quad I_2 = \int \frac{d^4p}{(2\pi)^4} \mathfrak{tr} \left[ \frac{1}{(p' + q)^2 - M^2} \right],
\]

\[
I_3 = \int \frac{d^4p}{(2\pi)^4} \mathfrak{tr} \left[ \frac{1}{(p' + q)^2 - M^2} \right] \frac{1}{(p^2 - M^2)},
\]

with \(p' = (p_0 + iA_4, \mathbf{p})\). \(\mathfrak{tr}\) means the trace in color space. For finite \(T\), the corresponding equations are obtained by the replacement

\[
p_0 \rightarrow i\omega_n = i(2n + 1)\pi T, \quad \int \frac{d^4p}{(2\pi)^4} \rightarrow iT \sum_{n = -\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4},
\]

\[
(2.14)
\]
Meson screening mass is defined by the exponential damping of the meson propagator \( \eta_{\xi \xi}(r) \) in the long distance limit \( (r \rightarrow \infty) \):

\[
M_{\xi,scr} = -\lim_{r \rightarrow \infty} \frac{d \ln \eta_{\xi \xi}(r)}{dr},
\]

(2.15)

where \( \eta_{\xi \xi}(r) \) is obtained by the Fourier transform of \( \chi_{\xi \xi}(0, q^2) \) from the momentum \( \vec{q} \) space to the coordinate space \( r \):

\[
\eta_{\xi \xi}(r) = \frac{1}{4\pi^2 i r} \int_{-\infty}^{\infty} d\vec{q} \, \chi_{\xi \xi}(0, \vec{q}^2) e^{i \vec{q} \cdot \vec{r}}.
\]

(2.16)

It is not easy to make this Fourier transformation particularly at large \( r \) due to the highly oscillating function \( e^{i \vec{q} \cdot \vec{r}} \). In order to avoid this problem, one can consider the Fourier transformation as a contour integral in the complex \( \vec{q} \) plane by using the Cauchy’s integral theorem. However, it is reported in Ref. [13] that \( \chi_{\xi \xi}(0, q^2) \) has logarithmic cuts in the vicinity of the real \( \vec{q} \) axis and heavy numerical calculations are necessary for evaluating the cut effects. In our previous work [14], we showed that these logarithmic cuts are unphysical and removable. If we make the \( p \) integral before taking the Matsubara summation \( \sum_n \) in (2.14), we can express \( I_3^{\text{reg}}(0, q^2) \) as an infinite series of analytic functions:

\[
I_3^{\text{reg}}(0, q^2) = \frac{iT}{4\pi \vec{q}} \sum_{j,n,\alpha} C_\alpha \sin^{-1} \left( \frac{\vec{q}}{\sqrt{\frac{q^2}{4} + M_{j,n,\alpha}^2}} \right)
\]

(2.17)

with

\[
M_{j,n,\alpha}(T) = \sqrt{M_{\alpha}^2 + ((2n + 1)\pi T + A_{\alpha}^j)^2},
\]

(2.18)

where \( M_{\alpha} = M_{\pi,\alpha} = M_{\eta,\alpha} \). Each term of \( I_3^{\text{reg}}(0, q^2) \) has two physical cuts on the imaginary axis, one is an upward vertical line starting from \( \vec{q} = 2iM_{j,n,\alpha} \) and the other is a downward vertical line from \( \vec{q} = -2iM_{j,n,\alpha} \). There are two lowest branch points \( \vec{q} = 2iM_{j=1,n=0,\alpha=0} = 2iM_{j=2,n=-1,\alpha=0} \). We call them “threshold mass” in the sense that they come from the quark-antiquark continuum state.

We can obtain the meson screening mass \( M_{\xi,scr} \) as a pole of \( \chi_{\xi \xi}(0, q^2) \),

\[
\big| [1 - 2G_\xi \Pi_\xi(0, q^2)] \big|_{\vec{q} = iM_{\xi,scr}} = 0.
\]

(2.19)

If the pole at \( \vec{q} = iM_{\xi,scr} \) is well isolated from the cut, i.e., \( M_{\xi,scr} < 2M_{j=1,n=0,\alpha=0} \), one can determine the screening mass from the pole location without making the \( \vec{q} \) integral.

3. Numerical Results

The EPNJL model has three adjustable parameters, \( \alpha_t \) in the entanglement coupling \( G_s(\Phi) \) and \( b \) and \( T_1 \) in the KMT interaction \( K(T) \). These parameters can be clearly determined from 2+1 flavor LQCD data [12] for pion and \( a_0 \)-meson screening masses, \( M_{\pi,scr} \) and \( M_{a_0,scr} \) as shown below.

Figure 1 shows \( T \) dependence of \( M_{\pi,scr}, M_{a_0,scr} \). The EPNJL results for \( M_{\pi,scr} \) (\( M_{a_0,scr} \)) are represented by solid (dotted) line and LQCD ones are plotted with closed squares (open circles). Best fitting is obtained, when \( \alpha_t = 1.0, \, T_1 = 0.79T_c = 121 \text{ MeV} \) and \( b = 0.23T_c = 36 \text{ MeV} \). The
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Fig. 1: $T$ dependence of $M_{\pi,\text{scr}}$ and $M_{a_0,\text{scr}}$. The solid (dotted) line denotes $M_{\pi,\text{scr}}$ ($M_{a_0,\text{scr}}$) calculated by the EPNJL model. LQCD data are taken from Ref. [12]; closed squares (open circles) correspond to the 2+1 flavor data for $M_{\pi,\text{scr}}$ ($M_{a_0,\text{scr}}$). LQCD data are rescaled by the factor 154/196.

Fig. 2: Order of chiral transition near physical point in the $m_l-m_s$ plane. The value of $\log[\chi_{ll}(T_c)]$ is shown by a change in hue. Simulation point, physical point, light-quark chiral-limit point and tricritical point are denoted by S, P, $C_l$ and TCP. The solid lines stand for second-order chiral transitions.

parameters thus obtained lead to $K(T_c)/K(0) = 0.434$ and it indicates the rapid suppression of $K(T)$ in the vicinity of $T_c$, but $U_A(1)$ symmetry is not completely restored at $T_c$. Here, we rescale the LQCD results of [12] with multiplying them by the factor 154/196 to reproduce $T_c = 154 \pm 9$ MeV. This is because $T_c$ in the simulations [12] is about 196 MeV, although it becomes $T_c = 154 \pm 9$ MeV in finer LQCD simulations [6, 7, 8] close to the continuum limit.

Using the $T$-dependent KMT interaction, we draw the Columbia plot near the physical point in $m_l-m_s$ plane, as shown in Fig. 2. The S point represents the location of $(m_l,m_s)$ for the LQCD simulation [12] and it is located at $(m_l,m_s) = (9.9$ [MeV], $175$ [MeV]). S-point is close to light-quark chiral limit point ($C_l$), therefore, we can extrapolate the LQCD results [12] from S-point to $C_l$-point by using the EPNJL model. Varying both $m_l$ and $m_s$, we determine the order of chiral transition from the divergence of $T$ dependence of chiral susceptibility $\chi_{ll}(T)$ and the discontinuity of chiral condensate. The value of $\log[\chi_{ll}(T_c)]$ is denoted by a change in hue. In the light-quark chiral-limit with the strange quark mass ($m_s$) fixed at the physical value $m_s = m_s^{\text{phys}} = 175$ MeV, the chiral transition becomes the second order in the mean field approximation. The second-order chiral transitions (solid lines) meet at $(m_l^{\text{tric}}, m_s^{\text{tric}}) \approx (0, 0.726 m_s^{\text{phys}}) = (0$[MeV], $127$[MeV]). This is a tricritical point (TCP) of chiral phase transition.

4. Summary

In summary, we incorporated the effective restoration of $U_A(1)$ symmetry in the 2+1 flavor EPNJL model by introducing a $T$-dependent coupling strength $K(T)$ to the KMT interaction. The $T$ dependence was well determined from state-of-the-art 2+1 flavor LQCD data on pion and $a_0$-meson screening masses. The strength $K(T)$ thus obtained is suppressed in the vicinity of the
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The pseudocritical temperature of chiral transition. However, $U_A(1)$ symmetry breaking still remains at $T_c$. Using the EPNJL model with the present parameter set, we showed that, at least in the mean field level, the order of chiral transition is second order at the light-quark chiral-limit point of $m_l = 0$ and $m_s = 175$ MeV (the physical value). This result indicates that there exists a tricritical point near the light-quark chiral-limit point in the $m_l$--$m_s$ plane. We then estimated the location of the tricritical point as $(m_l^{\text{tric}}, m_s^{\text{tric}}) \approx (0, 0.726 m_s^{\text{phys}}) = (0, 127\text{MeV})$.

Acknowledgments

M. I, J. T., H. K., and M. Y. are supported by Grant-in-Aid for Scientific Research (No. 27-3944, No. 25-3944, No. 26400279 and No. 26400278) from the Japan Society for the Promotion of Science (JSPS).

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