SU(3) gauge theory with four degenerate fundamental fermions on the lattice

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The LatKMI Collaboration has been studying SU(3) gauge theories with a large number of fermion flavors, N_f. Here, we report results from lattice simulations of SU(3) gauge theory with four fundamental fermions. We first show the fermion mass dependence of F_π, $\langle\bar{\psi}\psi\rangle$ and their chiral extrapolations, showing evidence of chiral symmetry breaking. Then we report the mass spectrum of a vector meson and nucleon, showing that their behavior is very close to that of real-world QCD. We also show preliminary results of the measurement of the mass of the flavor-singlet scalar bound state.

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1. Introduction

As a part of the project studying large N_f QCD, the LatKMI Collaboration has been investigating the SU(3) gauge theory with four fundamental fermions (four-flavor QCD). The main purpose of studying four-flavor QCD is to provide a qualitative comparison to $N_f = 8$, 12, 16 QCD; however, a quantitative comparison to real-world QCD is also interesting. To make such comparisons more meaningful, it is desirable to use the same kind of lattice action consistently, so that qualitative difference of different theories is less affected by artifacts of lattice discretization. Here, we adopt the highly-improved staggered quark action with the tree-level Symanzik gauge action (HISQ/tree), which is exactly the same as the setup for our simulations for $SU(3)$ gauge theories with $N_f = 8$, 12 and 16 fundamental fermions [1–3]. In the next section, we show the fermion mass dependence of F_π, $\langle \bar{\psi}\psi \rangle$, M_π, M_ρ, M_N and their chiral extrapolations. In section 3, preliminary results of the measurement of the mass of the flavor-singlet scalar bound state will be reported.

2. Chiral properties and masses of the vector meson and the nucleon

For our study of four-flavor QCD, we generated gauge configurations at $\beta = 3.7$ with three volumes, $(L, T) = (12, 18)$, $(16, 24)$, $(20, 30)$, and four or five bare mass parameter, m_f, in the range $0.01 \leq m_f \leq 0.05$ depending on the volume.

In the left panel of Fig. 1, we show the decay constant of the Nambu–Goldstone (NG) boson, F_π, as a function of bare mass parameter m_f, and in the right panel, the chiral condensation, $\langle \bar{\psi}\psi \rangle$, is shown again as a functions of m_f. Quadratic fits of F_π and $\langle \bar{\psi}\psi \rangle$ show a non-zero value in the chiral limit, which can be considered as evidence for four-flavor QCD being in the chirally broken phase. As a consistency check, we also calculated the chiral condensate by using the GMOR relation, which is shown in the plot of $\langle \bar{\psi}\psi \rangle$. From the figure, we can see that the value of the chiral condensate obtained from the GMOR relation agrees with that of $\langle \bar{\psi}\psi \rangle$ in the chiral limit.
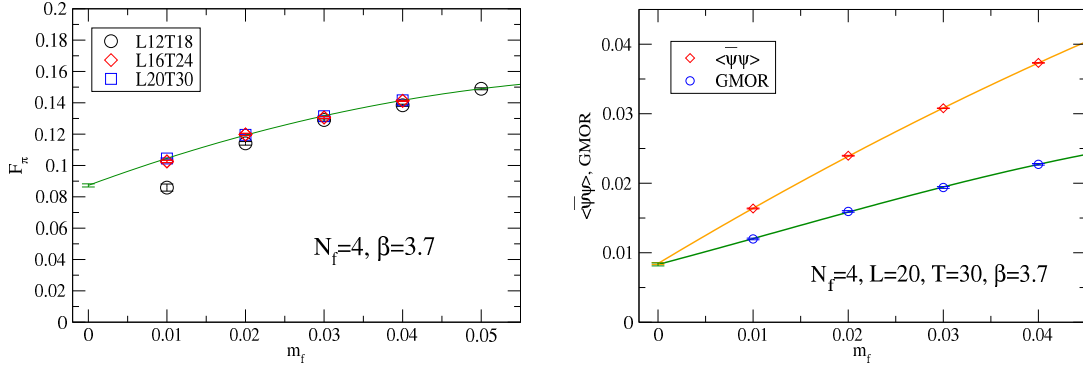

Figure 1: Left: The decay constant of the NG boson, F_π, for various values of m_f for $N_f = 4$, $\beta = 3.7$. The curve appearing in the figure is obtained by fitting the quadratic function to the largest volume data. **Right:** $\langle \bar{\psi}\psi \rangle$ for various values of m_f for $N_f = 4$, $\beta = 3.7$. In the figure only the largest volume data is plotted. The chiral condensate estimated by using the GMOR relation is also plotted in the figure. The curves appearing in both figures are obtained by fitting the quadratic function to the largest volume data.

In Fig. 2, we show the plot of the mass-squared (M_π^2) of the NG boson obtained from our simulations as a function of m_f. The largest volume data of M_π^2 can be fitted well by a liner ansatz; if the quadratic term is allowed, its coefficient is small. This is consistent with leading order predictions from chiral perturbation theory. In Fig. 3, the mass-squared of the pseudoscalar mesons for different tastes are plotted as functions of m_f. One can see the almost constant shift in mass-squared for different tastes, which is consistent with the picture of the staggered χPT. (See also Ref. [4]) From these results, we conclude that the SU(3) gauge theory with four fundamental fermions is in the chiral symmetry breaking phase. We also show the finite-size hyperscaling
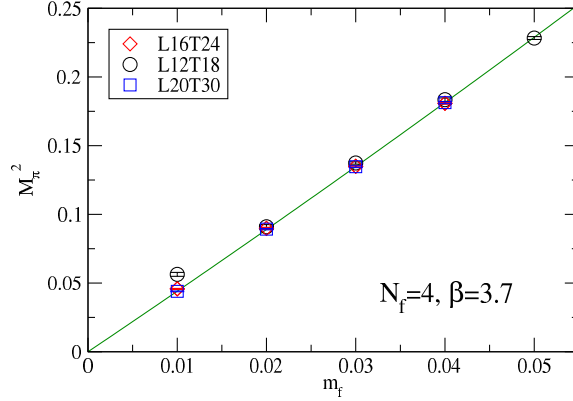

Figure 2: Mass-squared of the NG boson as a function of m_f. Quadratic fit to the largest volume data is also shown.
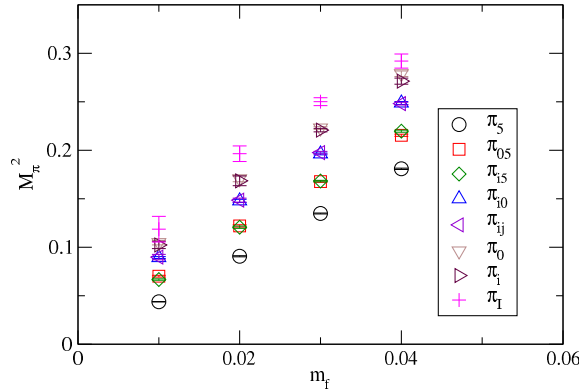

Figure 3: Mass-squared of the NG boson for different tastes as functions of m_f.

test [5] for $N_f = 4$ QCD by using the data of F_π obtained here. In Fig. 4, we show the finite-size hyperscaling plot for input values of $\gamma = 0.0, 1.0$ and 2.0. As we expect, the data show no alignment in the range of $0 \leq \gamma \leq 2$. This should be regarded as a typical property of QCD-like theory, and contrasted to the case of $N_f = 8, 12$.

Fig. 5 shows the ratio of the mass of the vector meson, M_ρ, to F_π for various values of m_f. The curve in the figure is the quadratic fit to the data of the largest volume, from which we can see the value of the ratio in the chiral limit being $M_\rho/F_\pi \simeq 6$. This value is very close to the ratio in the
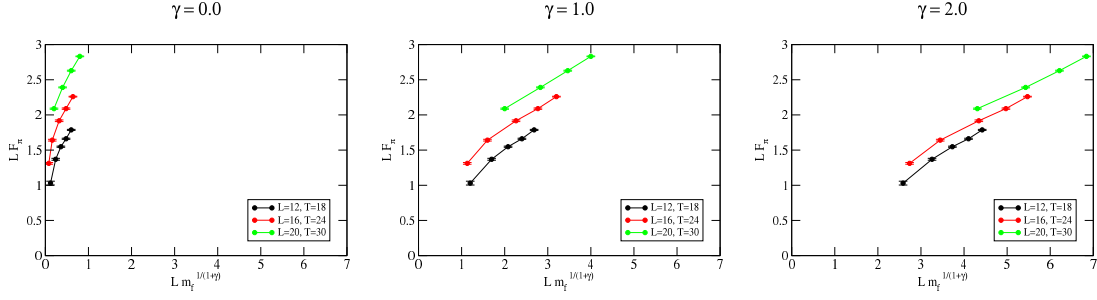
Figure 4: Finite size hyperscaling test of F_π in $N_f = 4$ QCD. Input values of γ are, from left to right panels, $\gamma = 0.0, 1.0$ and 2.0, respectively.
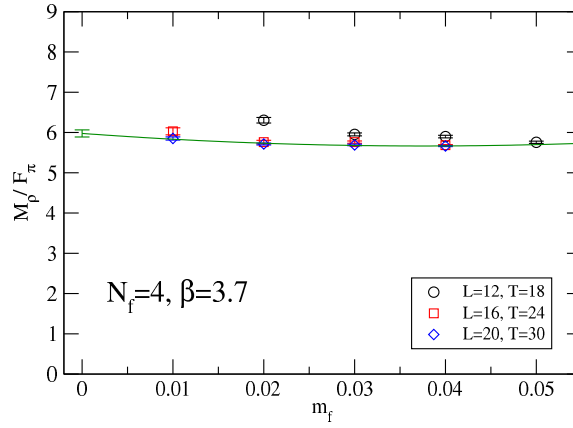

Figure 5: The ratio of the mass of the vector meson, M_ρ, to F_π for various values of m_f. Quadratic fit to the largest volume data is also shown.

real-world QCD, $775\,\mathrm{MeV}/130\,\mathrm{MeV} \simeq 6$. In Fig. 6, we show the ratio of the mass of the nucleon, M_N, to M_ρ for various values of m_f which are obtained from the largest volume data. The curve in the figure is quadratic fit to the data, from which we can see that the value of the ratio is, again, very close to that of the real-world QCD.

In Fig. 7, M_N/M_ρ for various values of M_π/M_ρ are plotted. The corresponding value of real-world QCD is indicated by a diamond. For comparison, data of $N_f = 8$ is plotted in the figure as filled circle symbols. From top right to bottom left of the $N_f = 8$ data, the input bare mass is $m_f = 0.08, 0.06, 0.04, 0.03, 0.02, 0.015$, and 0.012. In Fig. 8, plot of M_N/F_π for various values of M_ρ/F_π are plotted. The corresponding value in the chiral limit is indicated as magenta diamond and that of real-world QCD is indicated as red diamond. For comparison, corresponding data of $N_f = 8$ is plotted in the figure as triangle symbols. From top right to bottom left of the $N_f = 8$ data, the input bare mass is $m_f = 0.08, 0.06, 0.04, 0.03, 0.015, 0.012, 0.02$. From these figures, one can see that the data of $N_f = 4$ is approaching to the real-world QCD in the chiral limit, while $N_f = 8$ data stays in the area of the parameter space which is far from the real-world QCD value.
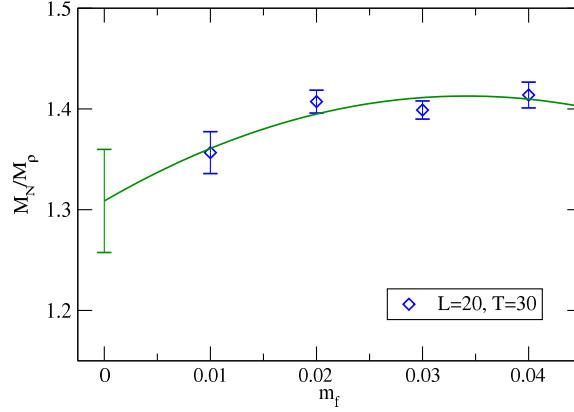
Figure 6: The ratio of the mass of the nucleon, M_N, to M_ρ for various values of m_f. Quadratic fit to the largest volume data is also shown.
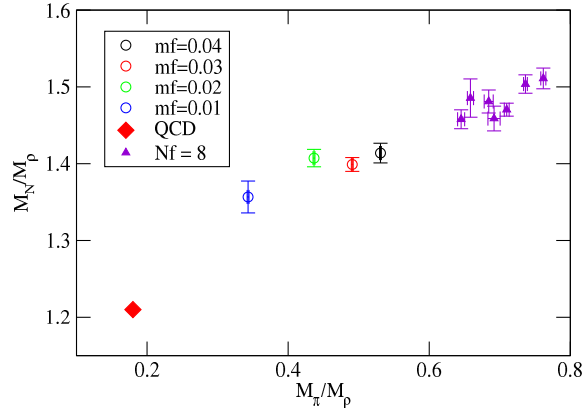

Figure 7: Plot of M_N/M_ρ versus M_π/M_ρ. The corresponding value of real-world QCD is indicated as diamond. For comparison, data of $N_f = 8$ is plotted in the figure as filled triangle symbols. From top right to bottom left, the input bare mass is $m_f = 0.08, 0.06, 0.04, 0.03, 0.02, 0.015$, and 0.012.

3. Flavor-singlet scalar bound state

The flavor-singlet scalar meson is a very interesting object both in large N_f QCD and in real-world QCD. In the former case, it could be related to the 126 GeV Higgs boson discovered at the LHC, while for the latter case, it is related to the σ meson. We calculate the flavor-singlet scalar correlator to extract the bound state mass by using the same technique we adopted for $N_f = 8$ and 12 QCD. (See Refs. [6, 7] for details. See also Ref. [8] for earlier lattice study on σ meson.) The preliminary effective mass plot for $L = 20$, $T = 30$, $m_f = 0.01$ is shown in Fig. 9. The plateau is observed in the disconnected channel, and the mass extracted from it is about 0.3, which is heavier than the pseudo NG boson ($M_\pi \sim 0.2$) in the same parameter. This is quite different from $N_f = 8$ and 12 QCD, in which it was shown that the mass of the flavor-singlet scalar bound state is comparable to or smaller than the pseudo NG boson [6, 7].
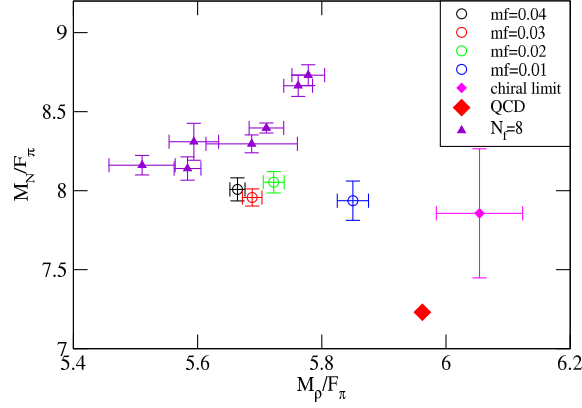
Figure 8: Plot of M_N/F_π versus M_ρ/F_π. The corresponding value in the chiral limit is indicated as magenta diamond and that of real-world QCD is indicated as red diamond. For comparison, the corresponding data of $N_f = 8$ is plotted in the figure as triangle symbols. From top right to bottom left, the input bare mass is $m_f = 0.08, 0.06, 0.04, 0.03, 0.015, 0.012, 0.02$.

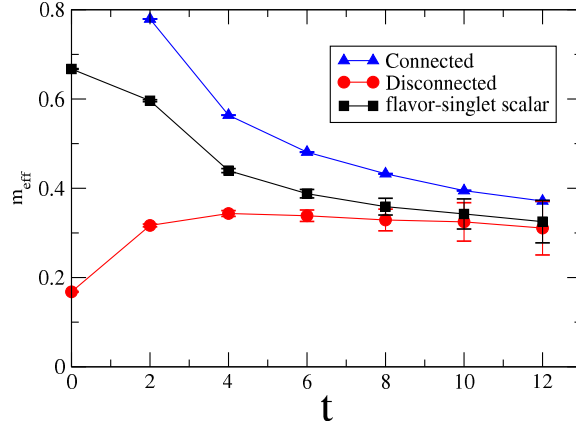$$N_f{=}4,\ L{=}20,\ T{=}30,\ \beta{=}3.7,\ m_f = 0.01$$

Figure 9: The effective mass plot of the flavor-singlet scalar for $L = 20$, $T = 30$, $m_f = 0.01$. The (black) square symbols represent effective mass calculated from the full scalar correlator, while (blue) triangle and (red) circle symbols represent those obtained from the connected and disconnected correlators, respectively.

4. Summary

In this proceedings, we showed our results of four-flavor lattice QCD simulation, which was performed by using the HISQ action with the tree-level Symanzik gauge action. It was shown that the theory is in the chiral symmetry breaking phase, and the ratio of M_ρ to F_π, as well as M_N to M_ρ in the chiral limit are quite close to that of real-world QCD. Preliminary results for flavor-singlet bound state mass were also shown, and the qualitative difference between four-flavor QCD and (near-)conformal theories such as $N_f = 8$ and 12 QCD was pointed out. More detailed analysis with higher statistics and similar investigation in multiple mass parameter would help to understand

the physical interpretation of the result, which will be pursued in the future.

Acknowledgments

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