Gradient flow and IR fixed point in SU(2) with Nf=8 flavors

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We study the running of the coupling in SU(2) gauge theory with 8 massless fundamental representation fermion flavours, using the gradient flow method with Schrödinger functional boundary conditions. Gradient flow allows us to measure robust continuum limit for the step scaling function. The results show a clear indication of infrared fixed point consistent with perturbation theory.

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1. Introduction

There exists a class of asymptotically free gauge theories where the running coupling approaches a non-trivial infrared fixed point (IRFP) and long-distance physics is conformal. These theories have applications in model building beyond the standard model, particularly in techni-color theories, where the electroweak symmetry is broken by the dynamical formation of the chiral condensate analogously to QCD. In addition to direct applications in particle phenomenology, the theories with IRFP are interesting from purely theoretical viewpoint of understanding the structure of gauge field theories. In particular, as a function of number of colours N, flavours N_f and fermion representations, one is interested in determining the boundary where the the IRFP of the theory is lost and the behaviour becomes generically QCD-like. Where this happens signifies the location of the edge of the so called conformal window in the gauge theory phase diagram. Typically the fixed point coupling near this boundary is expected to be large and non-perturbative methods are needed.

In this work we study the running of the coupling in SU(2) gauge field theory with 8 fermions in the fundamental representation. Analytic estimates suggest that this theory can be expected to lie within the conformal window, i.e has an IRFP. It has been studied before in [1], but the results were inconclusive. The phase structure of the model with staggered fermions was studied in [2].

The SU(2) gauge theory with 4–10 fundamental representation fermion flavours has been studied also earlier [3–6]. In these works, the coupling was measured via the Schrödinger functional method [7], but the results were inconclusive regarding the location of the edge of the conformal window. Since the running is slow close to the IRFP, very high accuracy is needed to discern the continuum behaviour, hindering the progress towards large enough volumes.

In this work we use the gradient flow method [8] with Schrödinger functional boundary conditions [9]. This allows us to control the leading order discretization errors and hence improve the continuum limit. Preliminary results of this study were published in [10]. Schrödinger functional boundaries also make it possible to measure the mass anomalous dimension as described in more detail in [11].

2. Methods and Results

We study the model using a HEX smeared [12], clover improved Wilson fermion action and a partially smeared plaquette gauge action. The full action can be written as

$$S = (1 - c_g)S_G(U) + c_g S_G(V) + S_F(V) + c_{SW}\delta S_{SW}(V),$$

where V is the smeared gauge field and U is the unsmeared one. As a result of the smearing, the action is non-perturbatively improved to order a with the Sheikholeslami-Wohlert coefficient $c_{SW} \approx 1$ and we simply choose $c_{SW} = 1$. The gauge action smearing, tuned by the coefficient c_g, removes the unphysical bulk phase transition from the region of interest in the parameter space. In our case it is sufficient to choose $c_g = 0.5$.

We also use Schrödinger Functional boundary conditions, which enables us to measure the mass anomalous dimension of the model from the same dataset. The gauge fields are set to unity

and the fermion fields are set to zero at time slices $x_0 = 0, L$ on a lattice of size L^4:

$$U_k(x) = V_k(x) = 1, \qquad \psi(x) = 0 \quad \text{when} \quad x_0 = 0, L$$
$$U_\mu(x + L\hat{k}) = U_\mu(x), \quad V_\mu(x + L\hat{k}) = V_\mu(x)$$
$$\psi(x + L\hat{k}) = \psi(x).$$

Here $k = 1, 2, 3$ labels one of the spatial directions.

We use the gradient flow method to measure the running coupling. The gradient flow is defined by introducing a fictitious flow time t and studying how the fields evolve according to flow equation:

$$\partial_t B_{t,\mu} = D_{t,\mu} B_{t,\mu\nu},$$
$$B_{0,\mu} = A_\mu$$
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}].$$

Here $B_{t,\mu}$ is the flow field parametrized by the flow time t, and A_μ is the original gauge field. In this study we evolve the flow fields using the Symanzik gauge action. The flow smears the gauge field over a radius $\sqrt{8t}$, removing the UV divergences and automatically renormalizing gauge invariant observables [8].

The coupling is measured from the evolution of the field strength

$$\langle E(t) \rangle = \frac{1}{4} \langle G_{\mu\nu}(t) G_{\mu\nu}(t) \rangle.$$

To the leading order in perturbation theory in $\overline{\text{MS}}$ scheme, it has the form [13] $\langle E(t) \rangle = \mathcal{N} g^2 / t^2 + \mathcal{O}(g^4)$. Since the translation symmetry is broken by the boundary conditions, we measure the field strength only at the middle time slice. The dimensionless renormalized quantity $t^2 < E(t) >$ can then be used to define the renormalized coupling [14]:

$$g_{GF}^2(\mu) = \mathcal{N}^{-1} t^2 \langle E(t) \rangle |_{x_0 = L/2, t = 1/8\mu^2}, \tag{2.1}$$

where the normalization factor $\mathcal{N}$ for these boundary conditions has been calculated in [9]. To be free of both lattice artifacts and finite volume effects the scale μ must be within $1/L \ll \mu \ll 1/a$. To ensure this we relate the lattice scale with the renormalization scale as $\mu^{-1} = cL$. Unless otherwise indicated we use $c = 0.4$ in our analysis below.

We measure the coupling with several physical lattice sizes and bare couplings in order to quantify the running using the step scaling function [15]:

$$\Sigma(u, a/L) = g_{GF}^2(g_0, 2L/a) \big|_{g_{GF}^2(g_0, L/a) = u} \tag{2.2}$$
$$\sigma(u) = \lim_{a \to 0} \Sigma(u, a/L).$$

The step scaling function describes how the coupling evolves when the linear size of the system is increased.

As was observed in [16], the gradient flow coupling suffers from large order $(a/L)^2$ errors. These can be alleviated with a correction τ_0 to the lattice value of the flow time [17]

$$g_{GF}^2 = \mathcal{N}^{-1} t^2 \langle E(t + \tau_0 a^2) \rangle = \mathcal{N}^{-1} t^2 \langle E(t) \rangle + \mathcal{N}^{-1} t^2 \left\langle \frac{\partial E(t)}{\partial t} \right\rangle \tau_0 a^2. \tag{2.3}$$

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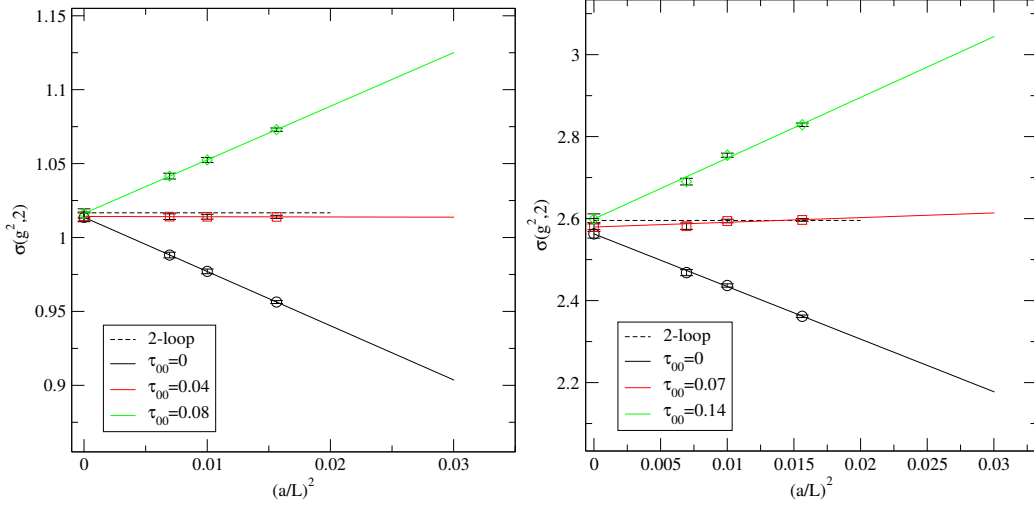
Figure 1: The continuum limit (2.6) with several values of the flow time correction τ_0 with $u = 1$ (left) and $u = 2.5$ (right).
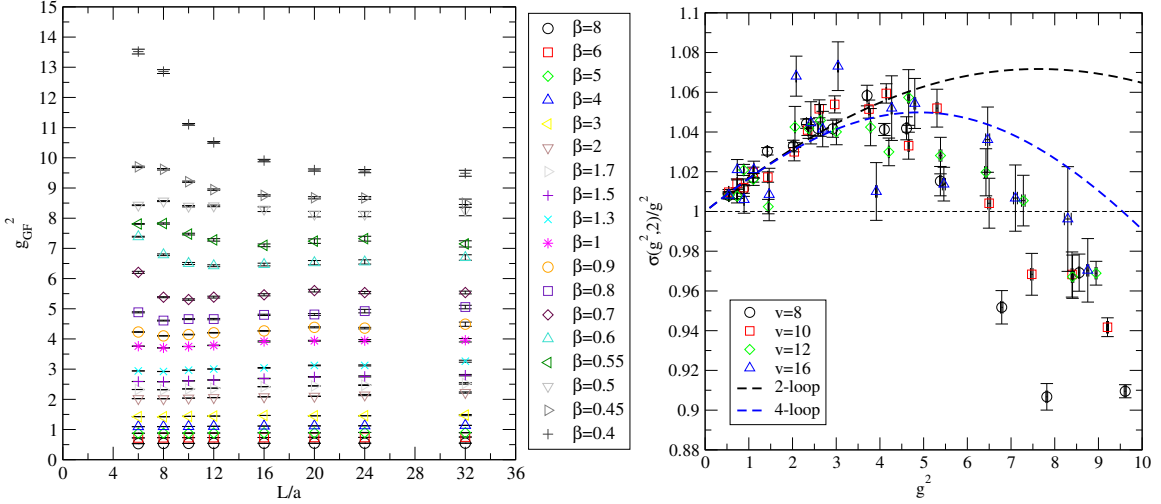

Figure 2: The plot on the left shows the gradient flow coupling (2.1) measured at each β and L/a at $c = 0.4$. The plot on the right shows the lattice step scaling function (2.2) for these couplings. For both pictures τ_0 correction (2.4) was used.

The effect of the correction on the step scaling function can be seen in figure 1. As can be seen, τ_0 does change the slope but not the continuum limit of the step scaling function. In the following results we choose a functional form that approximately removes the $\mathcal{O}(a^2)$ effects. For the $c = 0.3 - 0.5$ we use

$$\tau_0 = 0.06 \log(1 + g^2). \tag{2.4}$$

The measured couplings g^2_{GF} and lattice step scaling function Σ with this τ_0 correction are shown in figure 2. The continuum limit of the step scaling function has to be evaluated at constant coupling g^2. We use two different methods for this. First method uses the traditional polynomial

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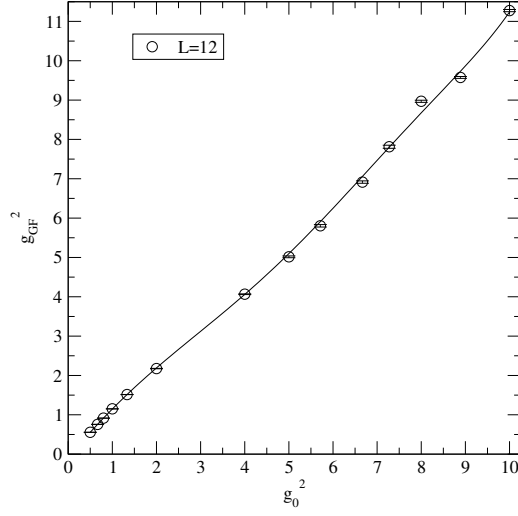
Figure 3: The gradient flow coupling and the interpolating function (2.5).
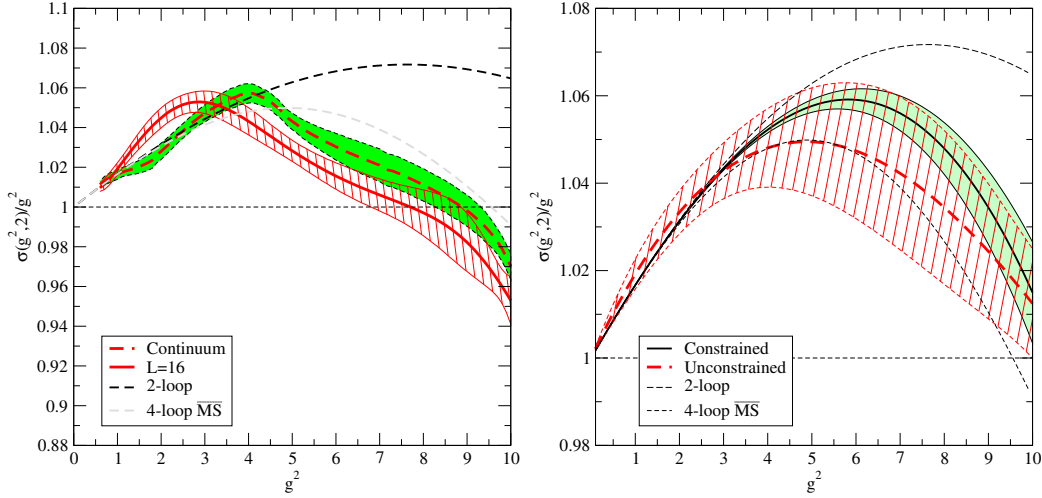
Figure 4: The step scaling function with $c = 0.4$, $\tau_0 = 0.06\log(1+u)$ calculated with two continuum extrapolation methods. On the left: traditional coupling interpolation (2.5) for the biggest lattice and continuum extrapolation (2.6). on the right: power series (2.7) for $(a/L)^2 + (a/L)^3$. The solid line is constrained to perturbative results, while the hashed line is not. 4-loop $\overline{\text{MS}}$ results are show as a guide for the eye.

interpolation:

$$g_{GF}^2(g_0, a/L) = \sum_{i=1}^{m} a_i g_0^{2i}, \ \ m = 6 \tag{2.5}$$

shown in figure 3. The continuum limit $\sigma(u)$ can then be approximated by fitting to a function of a form:

$$\Sigma(u, a/L) = \sigma(u) + c_2(u)(a/L)^2. \tag{2.6}$$

This method does not utilize the fact that the step scaling σ must follow the universal perturbative 2-loop prediction. As a second method, we carry out a continuum extrapolation, where we present

$\sigma(u)$ as truncated power series, parametrize the discretization errors as series in u and do a single fit to all measured data [18]. This second method has the form:

$$\Sigma(u, a/L) = \sigma(u) + \sum_{k=2}^{n_a} f_k(u) \frac{a^k}{L^k} \qquad (2.7)$$

$$\sigma(u) = 1 + \sum_{i=1}^{n} s_i u^i$$

$$f_k(u) = \sum_{l=0}^{n_f^k} c_{k,l} u^l,$$

where c_i and $c_{k,l}$ are fit parameters, s_i are determined from universal 2-loop β-function i.e. $s_1 = 2b_0 \ln(2)$. We set $n = 3$, $n_a = 3$ and $n_f^2 = n_f^3 = 4$ in order to minimize $\chi^2/\text{d.o.f}$ of the fit. These two methods are compared in 4. They both follow the perturbative value closely up to $g^2 = 4$ and then diverge towards a fixed point. While the 4-loop $\overline{\text{MS}}$ result is scheme dependent, the nonperturbative results obtained with either of the two methods behave similar to it. The power series method seems to give IRFP at a bit higher value than the traditional approach.

3. Conclusions

We have studied the running coupling in the SU(2) lattice gauge theory with 8 fermions in the fundamental representation. Gradient flow algorithm with Schrödinger functional boundaries and flow time correction (2.3) seems to work well even on relatively high couplings. The calculated step scaling function shows a clear indication of a fixed point. We have also measured the mass anomalous dimension using the mass step scaling and spectral density methods. The results are reported in [11].

4. Acknowledgments

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