

S-parameter and vector decay constant in QCD with eight fundamental fermions

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SU(3) gauge theory with eight massless fundamental fermions seems to be near the conformal boundary, and is a candidate theory of walking technicolor. Along the series of study by LatKMI collaboration using HISQ fermions, S -parameter and vector decay constant, which provide important constraints in the model of electroweak symmetry breaking, are calculated for this theory. Use of various volumes allows a systematic investigation of finite volume effects. A strong sensitivity of the S -parameter to the volume is found.

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1. Introduction

Many-flavor QCD provides a possibility for the strong dynamics realization of the Higgs sector in the Standard Model. Among the tested models, SU(3) gauge theory with eight mass-less fundamental fermions has drawn much attention recently¹, especially as it could be a successful candidate of walking technicolor theory with a flavor-singlet scalar possibly light enough to realize 125 GeV Higgs [2]. In addition to the tests of walking dynamics and light Higgs realization, the Peskin-Takeuchi S parameter [3] provides an important constraint. As increasing number of fermions would naturally enhance S , there is a concern that many-flavor models will critically face the limit set by electroweak precision measurements.

The S parameter in the many flavor theories has been studied first by using Bethe-Salpeter equation [4, 5]. Lattice computation of the S parameter is first performed using the overlap fermions [6] and domain-wall fermions [7] on the conventional QCD configurations, mainly aiming to test a methodology. Chiral symmetry at finite lattice spacing is essential for the computation, which is the reason why the chiral fermion formulation is used in these studies.

In the context of strong dynamics theory for the electroweak symmetry breaking, LSD collaboration is the first to systematically study the S parameter on the dependence to the number of flavors N_f adopting the domain-wall-fermion formulation in lattice gauge computation. The results are reported for SU(3) gauge theory with $N_f = 2, 6$ [8] and 8 fundamental fermions [9]. LSD results indicate a decreasing trend of S towards lighter fermion mass m_f for $N_f = 6$ and 8, which would ease the model building with these theories. The trend is also backed up by the tendency of the observed spectrum towards parity doubling. One caveat, if any, that these studies have is that only one spatial volume is used thus the finite volume effects might have been overlooked. This is particularly important in this context as the finite volume effect tends to reduce the degree of chiral symmetry breaking, which could drive the system towards parity doubling, hence reduce S .

We have been studying $N_f = 8$ SU(3) gauge theory with hadron spectrum as a main probe, where the indication of the theory being close to the conformal phase boundary and a parametrically light flavor singlet scalar are found [10, 2, 11]. Therefore, we regard $N_f = 8$ SU(3) gauge theory as a candidate of walking technicolor theory, which has a potential to coop with the experiments. We use the highly-improved staggered quarks (HISQ) throughout our many flavor studies for $N_f = 4, 8, 12, 16$. To fully utilize the gauge configuration ensembles so far accumulated to a good statistics, staggered-fermion calculation for the S parameter is required. We developed the formulation and reported initial results in the past lattice conferences². Here we report on our computation of the S parameter for $N_f = 8$ and provide an update.

The calculation method, which makes use of the exact chiral symmetry on staggered fermions with multiple staggered field, is briefly described and the preliminary results will be reported in the next section. As a byproduct of the measurements of S parameter, the vector meson decay constant is also measured and reported.

¹see for example reviews by Hasenfratz (these proceedings) and Kuti [1].

²Lattice 2013 talk by Y. Aoki and Lattice 2014 talk by H. Ohki.

2. Vacuum Polarization Functions and Spectrum

Peskin-Takeuchi *S* parameter is calculated from the slope of the transverse $V - A$ vacuum polarization function $\Pi(q^2)$ with respect to the infinitesimal squared momentum transfer q^2 ,

$$S = 4\pi\Pi'(q^2 = 0), \quad (2.1)$$

$$\Pi_{\mu\nu}(q) = \delta_{\mu\nu}\Pi(q^2) - \frac{q_\mu q_\nu}{q^2} \tilde{\Pi}(q^2), \quad (2.2)$$

$$\Pi_{\mu\nu}(q) \equiv \int dx^4 e^{iqx} \left\{ \langle V_\mu^{(1,2)}(x) V_\nu^{(2,1)}(0) \rangle - \langle A_\mu^{(1,2)}(x) A_\nu^{(2,1)}(0) \rangle \right\}, \quad (2.3)$$

where $V_\mu^{(i,j)}$ and $A_\mu^{(i,j)}$ are the flavor non-singlet vector and axial vector current respectively, made of anti-quark field with i -flavor and quark field with j -flavor. This defines the contribution of *S* parameter from one electro-weak doublet.

On the lattice a naive definition of *S* parameter may easily suffer from ultra-violet power-divergent contribution due to a lack of continuum symmetry[6]. An exact symmetry which transforms V_μ into A_μ and vice versa can be used to cancel the UV divergence in the difference. In the literature overlap fermions or domain-wall fermions are used for the computation of *S* parameter [6, 7, 8, 9] due to this reason. However such calculations are computationally expensive.

As we use staggered fermions, namely HISQ, it is useful to have a formulation to calculate *S* parameter within HISQ. Staggered fermions have a exact flavor and chiral symmetry which is part of the full flavor and chiral symmetry in the continuum. This flavor symmetry is of the baryon number. The associated current is the flavor singlet vector current in four-flavor (one-species) theory, thus is not suitable to compute the *S* parameter. Fortunately our primary interest is for the eight flavors, where the exact symmetry enhances to contain $SU(2)_L \times SU(2)_R$. Therefore the flavor non-singlet non-renormalized vector and axialvector currents can be constructed as the Noether currents of the symmetry.

The conserved currents of HISQ involve length-one and three point-split bilinears. All have the same spin-taste structures

$$(\gamma_\mu \otimes 1)\tau^a, (\gamma_\mu \gamma_5 \otimes \xi_5)\tau^a \quad (2.4)$$

for vector and axial vector respectively. τ^a , being $SU(2)$ generators for the eight-flavor theory, bind two different staggered fields. The length-three operators are from the Naik term in the action. Apart from the spin-taste structure, the vector and the axial vector currents are exactly same. The exact chiral symmetry with the generator $(\gamma_5 \otimes \xi_5)\tau^a$ transforms one into another. It is this transformation property that ensures the cancellation of the UV power divergence when the $V - A$ difference of the vacuum polarization function is composed. Therefore, the use of the conserved current is not necessary in general. For example, the operator with just the length-one part from the conserved current will do the job, though renormalization of the operator is then necessary.

In practice we chose to use the one-link point-split currents at the source position of the two-point function and the full conserved currents at the sink. This makes both the numerical effort and coding complexity minimal while keeping useful Ward-Takahashi identities intact. The necessary renormalization factor for the one-link operator is calculated by taking the ratio:

$$Z_A(t) = \frac{\langle \mathcal{A}_4(t)P(0) \rangle}{\langle A_4(t)P(0) \rangle}, \quad (2.5)$$

where $\mathcal{A}_4(t)$ and $A_4(t)$ are the temporal component of the conserved and one-link axial vector current respectively with zero-momentum projection at time t . $P(0)$ is the pseudoscalar operator with the structure $(\gamma_5 \otimes \xi_5)$ at the origin. The axial current renormalization Z_A for $\mathcal{A}_\mu = Z_A A_\mu$ is determined at plateau at large t . The left panel of Figure 1 shows the ratio $Z_A(t)$ for the so-

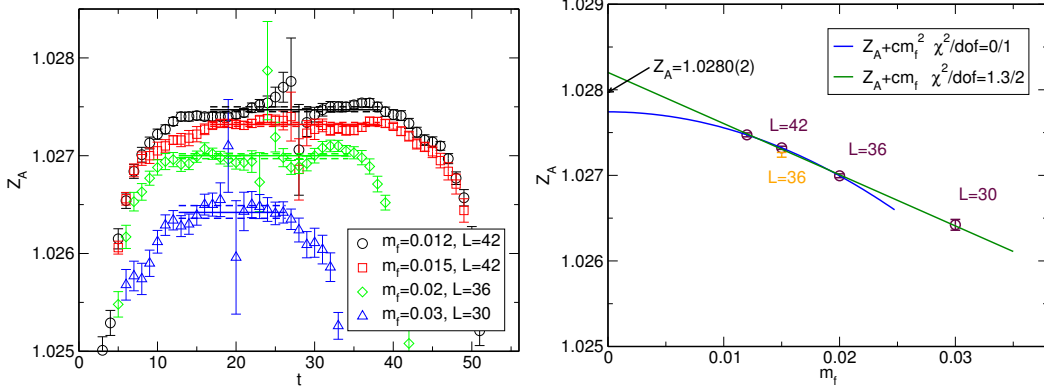


Figure 1: Ratio and plateau fitting for Z_A (left) and chiral extrapolation with linear or quadratic in m_f (right).

far smallest masses. The right panel shows the extrapolation to the chiral limit using these four points, where we use the largest volume for each mass $L = 30$ ($m_f = 0.03$), 36 ($m_f = 0.02$) and 42 ($m_f = 0.015$ and 0.0125) for $L^3 \times T$ ($T = 4/3 * L$) lattice. Both linear extrapolation with all four points and quadratic with lightest three are good. We adopt the average for our Z_A and the systematic error of the extrapolation is estimated from the difference from the average to linear or quadratic, which is small (0.02%). The statistical error is less than 0.005%, thus negligible.

For the S -parameter, vacuum polarization from the vector current two-point function is needed, too. The one-link vector current is renormalized with Z_A so far obtained by taking advantage of the exact chiral symmetry, which leads to $Z_V = Z_A$.

The results of $V - A$ transverse vacuum polarization function $\Pi(q^2)$ are shown in Fig. 2. These results are from the bare two point functions. The one-link operator at the source position needs to be renormalized, which will be taken care of by multiplying one factor of Z_A to the results. The Solid lines show fits with the Padé approximation $f(q^2) = (b_0 + b_1 q^2)/(1 + c_1 q^2 + c_2 q^4)$ [8].

The strong-dynamics contribution to the S parameter per electroweak doublet, Eq. (2.1) is calculated through the Padé fit and shown in Fig. 3 as a function of quark mass m_f (left panel). Multiplication of Z_A has been done to get renormalized S parameter. Following the same color toward smaller m_f , S bends down. This movement is of a finite volume effect as changing to larger volume will move the point back upward. The size of the effect is quite large. For example, the change of S at $m_f = 0.015$ from $L = 36$ to 42 is 8%, while the pion mass shift at the same point is only 0.04% and statistically consistent with zero. Note that for the test against experiment, subtraction of the standard Higgs sector from this result is needed [3, 12]. The finite volume effect discussed here, however, is independent from that.

The right panel drawn against $m_\pi L$, a dimension-less product of techni-pion mass m_π and the linear spatial size L , shows a same structure for different L . The left-most points of $L = 18$ and 36 suffer from the finite volume effect, which is apparent from the left panel. The bending-down

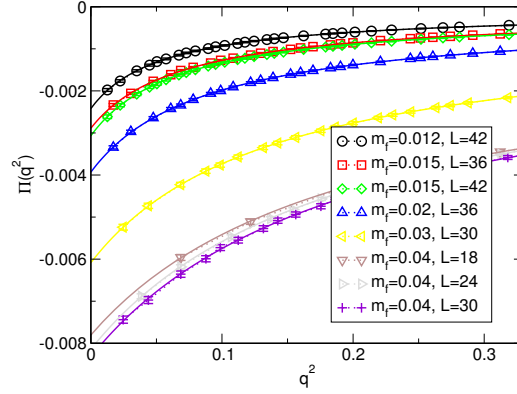


Figure 2: Bare $V - A$ transverse vacuum pluralization function $\Pi(q^2)$. Solid lines show fits with the Padé approximation $f(q^2) = (b_0 + b_1q^2)/(1 + c_1q^2 + c_2q^4)$.

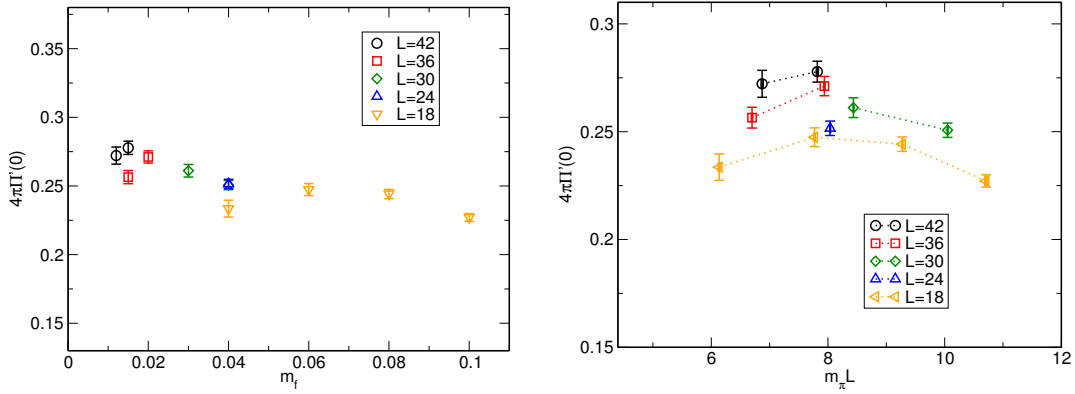


Figure 3: Strong-dynamics contribution to the S parameter per electroweak doublet as a function of quark mass m_f (left) or of a dimension-less product of techni-pion mass m_π and the linear spatial size L (right).

seems to occur around $m_\pi L \simeq 7$. As the smallest mass point $m_f = 0.012$ with $L = 42$ is in the range of $m_\pi L < 7$, the finite volume effect is suspected.

The left panel of Fig. 4 shows the vector and axialvector meson masses measured with the PV channel ($\gamma_k \gamma_4 \otimes \xi_k \xi_4$). Although this channel is different from the one that is used to calculate S , we use this as the signal is much better than that due to the locality of the operator in the staggered hypercube. We expect that good taste symmetry, which has been confirmed in the pion sector, also holds for the these sectors.

A remarkable finite volume dependence is observed for the vector meson mass, which drives the meson heavier as the volume is reduced. Due to the larger statistical noise the situation is not very clear for the axialvector. However for $m_f = 0.02$ and 0.03 , the effect appear opposite to the vector meson. The movement to the small volume is towards the parity doubling, thus seems consistent with the volume effect on S through the dispersive analysis using spectral decomposition [3]. We have to note, however, the correspondence between the parity doubling and the decrease of S is not entirely clear with the current result. For example, as already noted S decreased 8% from

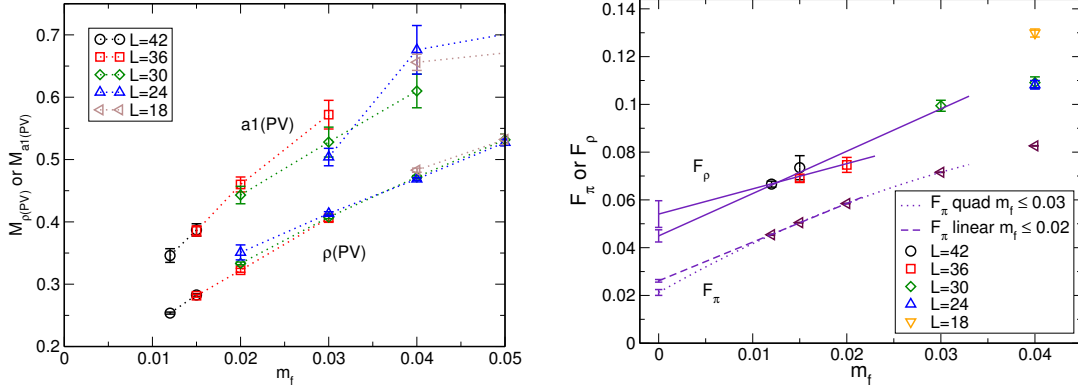


Figure 4: Vector (ρ) and axialvector (a_1) meson masses measured with *PV* channel as a function of quark mass (left). Vector meson decay constant through the smeared one-link current is plotted (colored symbols) in comparison with the pion decay constant (black symbols) (right). Linear extrapolation for the chiral limit F_ρ is performed with two different mass ranges.

$L = 42$ to 36 at $m_f = 0.015$, while the meson masses at the same parameter are stable.

In the spectral decomposition of the *S* parameter [3] not only the mass but also the decay constant enters. The right panel of Fig. 4 shows the vector meson decay constant computed through the symmetric two-point function with the one-link operator in both source and sink position. There, significant volume dependence is not observed except for the smallest volume, $L = 18$.

One also needs to study the axialvector decay constant, which is not shown in this report and remains to be a future task. With our data the vector and axialvector decay constant can in principle be determined. However the axial vector channel is an excited state in the two point correlation function, therefore quite noisy. This is, unfortunately, a drawback of the use of staggered fermions.

Let us get back to the vector decay constant and try to infer the value in the chiral limit. A rough analysis leads to the chiral limit value of the decay constant which appear approximately twice as the pion decay constant $f_\rho \simeq 2f_\pi$. This is consistent with the results obtained by LSD collaboration [9] with domain-wall fermions.

3. Summary and Outlook

In this report we briefly described how the Peskin-Takeuchi *S* parameter is computed using Highly Improved Staggered Quarks (HISQ) and preliminary results obtained by applying the method to the eight-flavor $SU(3)$ gauge theory were shown. A remarkable strong sensitivity of the *S* parameter to the system volume was found, which indicates a careful volume study is indispensable to obtain the correct chiral behavior of the *S* parameter. The spectrum closely related with *S* parameter includes vector and axialvector meson masses and decay constants. Noise reduction, where a brute-force method might suffice, especially on the axialvector sector will be useful for an in-depth understanding of the finite volume effect on *S*. $V - A$ vacuum polarization can be used as a probe of (near) conformality through the mass scaling [13], which will be of interest when the finite volume issues are settled.

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