

Analysis of short-distance current correlators using OPE

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We investigate the correlators of flavor non-singlet bilinear operators calculated on the lattice at short distances. In the continuum theory, non-perturbative effects are encoded in the form of the operator product expansion (OPE). We test the prediction of OPE by comparing lattice results with those in the continuum theory. We also determine the renormalization factors of quark currents.

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1. Introduction

Current correlators provide a rich source of information on the QCD vacuum. Correlators in the sufficiently short-distance region (< 0.1 fm) are well predicted by perturbation theory, which is currently available to four-loop order [1], while the non-perturbative dynamics dominates in the region of long distance (> 1 fm), as systematically understood by chiral perturbation theory. Dynamics in the middle range is, on the other hand, quite nontrivial, and lattice calculation is needed to obtain quantitative predictions.

The operator product expansion (OPE) [2] accommodates some non-perturbative effects and is used to analyze experimental data such as the hadronic τ decays. Since the number of operators to be included rapidly increases as one considers longer distance physics the range of distances suitable for OPE is limited. In this work, we test the applicability of OPE for current correlators in the region $0.1 \text{ fm} < x < 0.5 \text{ fm}$ by comparing current correlators in the continuum theory and lattice calculations.

As a by-product, we can determine the renormalization constants of quark bilinear operators following the analysis of [3–5], in which the renormalization condition is imposed on the correlator at a certain distance in the coordinate space. Unlike the RI/MOM scheme [6], this method enables us to renormalize composite operators in a fully gauge invariant manner and to use the perturbative matching factor available to the four-loop level. Like RI/MOM, the window problem remains, *i.e.* we need to use the correlators in the region satisfying $a \ll x \ll \Lambda_{\text{QCD}}^{-1}$ to avoid discretization effects on the lattice and non-perturbative effects on the continuum side. We investigate these effects and introduce various techniques to reduce them.

In this report, we present the status of these analyses. We employ $2 + 1$ flavor Möbius domain wall fermions with stout link smearing and the Symanzik improved gauge action. We work on $32^3 \times 64$ lattices at $a^{-1} = 2.45$ GeV, $48^3 \times 96$ lattices at $a^{-1} = 3.61$ GeV and a $64^3 \times 128$ lattice at $a^{-1} = 4.50$ GeV, all of which have matched physical volume and pion masses of $M_\pi = 300 \sim 500$ MeV. Calculation of masses and decay constants for the light mesons and the heavy-light mesons on these ensembles is reported in [7].

2. Current correlators

We calculate correlation functions of light quark bilinear operators in the coordinate space,

$$\begin{aligned} \Pi_S(x) &= \langle S(x)S(0)^\dagger \rangle, & \Pi_P(x) &= \langle P(x)P(0)^\dagger \rangle, \\ \Pi_{V,\mu\nu}(x) &= \langle V_\mu(x)V_\nu(0)^\dagger \rangle, & \Pi_{A,\mu\nu}(x) &= \langle A_\mu(x)A_\nu(0)^\dagger \rangle, \end{aligned} \quad (2.1)$$

where S and P are the scalar and pseudoscalar densities, while V_μ and A_μ are the vector and axial-vector currents. The flavor indices are omitted for simplicity, but they are understood as isospin triplet operators of light quarks. We also use the vector and axial-vector correlators after taking a trace of the Lorentz diagonal components,

$$\Pi_{V/A}(x) = \sum_{\mu} \Pi_{V/A,\mu\mu}(x). \quad (2.2)$$

Figure 1 shows the pseudoscalar correlator calculated non-perturbatively on the lattice (circle) and that in the free system both in the continuum (dashed curve) and lattice theory (diamond). Due

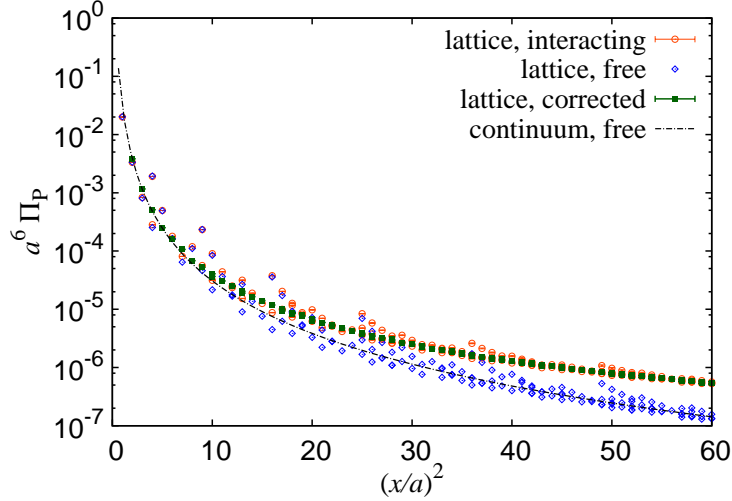


Figure 1: The pseudoscalar correlators measured on the lattice before (circle) and after (square) subtracting discretization effects in the tree level. The correlators in the free system calculated in the continuum theory (dashed curve) and lattice (diamond) are also plotted. This data are calculated on the $48^3 \times 96$ lattice at $\beta = 4.35$ and input mass $am_q = 0.0042, am_s = 0.0180$.

to the discretization effect, the non-perturbative data are not on a smooth line. However, the similar effect is already seen in the free correlator, which implies that the difference of the free theory between the lattice and the continuum describes the discretization effect in the interacting system to a good approximation. In fact, by applying a subtraction,

$$\Pi_{\Gamma}^{\text{lat}} \rightarrow \Pi_{\Gamma}^{\text{lat}} - (\Pi_{\Gamma}^{\text{lat, free}} - \Pi_{\Gamma}^{\text{free, cont}}), \quad (2.3)$$

we obtain smoother correlator as shown in Fig. 1 by squares.

Next, we examine the convergence of the continuum perturbation theory to identify the valid region of x . It is sufficient to investigate the scalar and vector channels because these channels coincide with the pseudoscalar and axial-vector channels, respectively, in the massless limit. Perturbative coefficients of correlators are calculated to four-loop in [1]. The beta function [8] and the mass anomalous dimension [9, 10] are also known up to four-loop level. However, if we naïvely use these results, the ratio shows poor convergence as shown in the left panel of Fig. 2. In this plot, reasonable convergence is found only in the region $x \lesssim 0.1$ fm, which is in the same order as our lattice spacing $a = 0.04 \sim 0.08$ fm. We can improve the convergence of the perturbative expansion by choosing an appropriate renormalization scale μ^* instead of using $\tilde{\mu} \simeq 1.12/x$ suggested in [1]. After some investigation of the convergence property and the size of systematic errors, we found that $\mu^* = 2.86/x$ for the scalar and $\mu^* = 5.47/x$ for the vector are optimal choices. As shown in the right panel of Fig. 2, the convergence is much better and the difference among 2-, 3- and 4-loop is hardly visible for $x < 0.5$ fm.

The lattice correlators of the scalar and pseudoscalar channels are also plotted on the right panel of Fig. 2. These correlators are multiplied by $Z_S^{\text{MS}}(2 \text{ GeV})^2$, the renormalization factor of the scalar density determined in Sec. 4. Lattice result and the continuum perturbation theory agree very well in $x \lesssim 0.25$ fm. Significant difference found in $x > 0.25$ fm is due to non-perturbative effects as discussed in the following sections.

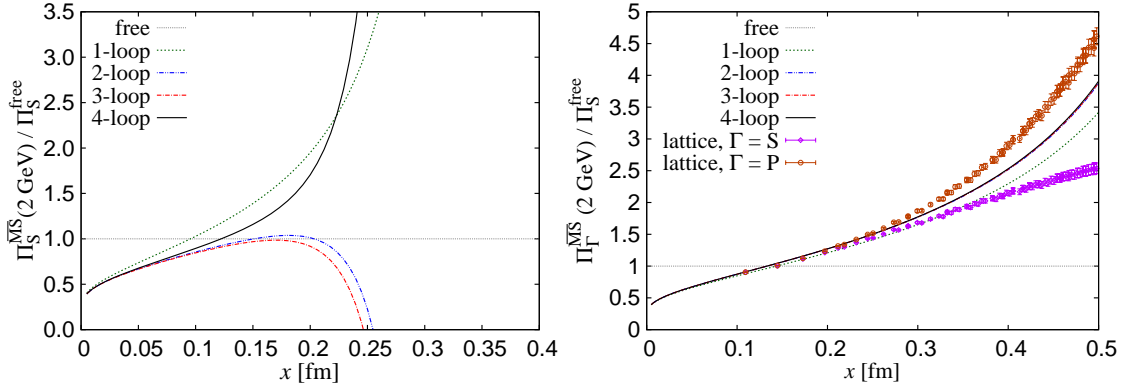


Figure 2: The scalar correlator in the continuum perturbation theory calculated with the coupling constant $\alpha_s(\mu^*)/\pi$ renormalized at $\mu^* = 1/x$ (left) and $2.86/x$ (right) in the $\overline{\text{MS}}$ scheme. The correlators are renormalized at 2 GeV in the $\overline{\text{MS}}$ scheme with $n_f = 3$. In the right panel, we also plot lattice correlators of the scalar and pseudoscalar channels measured on the same ensemble as Fig. 1.

3. Non-perturbative effect on current correlators

We discuss non-perturbative effects on the vector and axial-vector correlators. Using the PCAC relation, one can relate V/A correlators to the chiral condensate [11, 12]. When the valence masses are degenerate, the relation is written as

$$\Sigma_{m_q}(x) \equiv -\frac{\pi^2 (Z_V^{\overline{\text{MS}}})^2}{2m_q} x^2 x_\nu \partial_\mu (\Pi_{A,\mu\nu}(x) - \Pi_{V,\mu\nu}(x)) = \langle \bar{q}q \rangle + O(m_q). \quad (3.1)$$

Here, $Z_V^{\overline{\text{MS}}}$ is to renormalize the vector and axial-vector currents constructed by the local operators on the lattice. The subtraction of $\partial_\mu \Pi_{V,\mu\nu}$, which vanishes in the continuum theory, is to cancel the discretization effects. Figure 3 shows the lattice results of $\Sigma_{m_q}(x)$ renormalized by $Z_S^{\overline{\text{MS}}}(2 \text{ GeV})$ for three input masses, corresponding to $M_\pi \sim 300, 400, \text{ and } 500 \text{ MeV}$. The lattice data would be flat in the chiral limit and coincide with the gray band, which is the FLAG average [13] for $n_f = 2 + 1$, $(\langle \bar{q}q \rangle^{\overline{\text{MS}}}(2 \text{ GeV}))^{1/3} = -271(15) \text{ MeV}$.

In contrast, the prediction for the scalar and pseudoscalar correlators from OPE in literature fails to reproduce the lattice data. It becomes prominent when one compares the difference of these channels, *i.e.* the lattice result shows larger difference than the OPE prediction. Such inconsistency has been known [14–17], and a possible description by the instanton-induced 't Hooft interaction is suggested.

4. Renormalization of quark currents

In this section, we report the renormalization factors of quark currents using correlators. On each ensemble, the renormalization condition is applied by

$$\left(\tilde{Z}_\Gamma^{\overline{\text{MS}}} \right)^2 \Pi_\Gamma^{\text{lat}}(x) = \Pi_\Gamma^{\overline{\text{MS}}}(x), \quad (4.1)$$

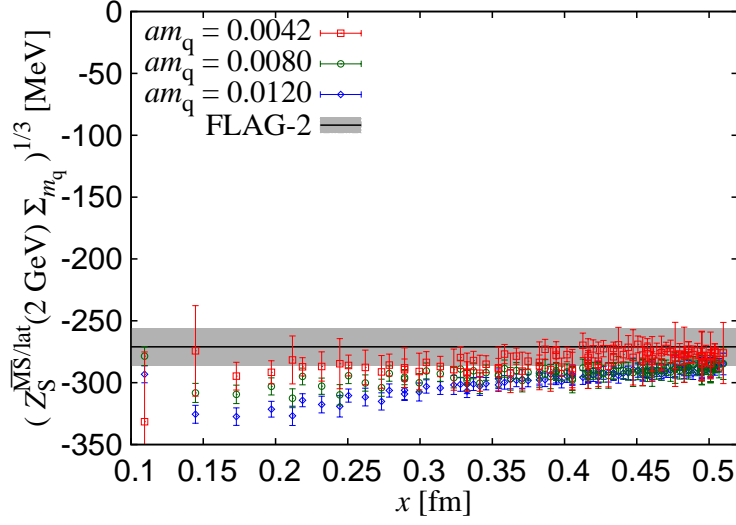


Figure 3: The cubic root of Σ_{m_q} renormalized by $Z_S^{\overline{\text{MS}}}(2 \text{ GeV})$. Input volume, β , and m_s are same as Fig. 1 but result of three valence masses are shown. The solid line and the gray band stand for the FLAG average of the chiral condensate at 2 GeV in the $\overline{\text{MS}}$ scheme.

at a certain distance x . The data of $\widetilde{Z}_T^{\overline{\text{MS}}}$ may show some x -dependence reflecting the discretization effects and non-perturbative effects contained in Π_T^{lat} . As Fig. 4 shows, both the x -dependence and mass-dependence of $\widetilde{Z}_{V/A}^{\overline{\text{MS}}}$ are found to be significant. We take account of the non-perturbative effects using the OPE. The vector and axial-vector correlators are expressed as a linear combination of vacuum expectation values of local operators,

$$\Pi_{V/A}(x) = \frac{c_0}{x^6} + \frac{c_{4,\bar{q}q}^{V/A} m_q \langle \bar{q}q \rangle + c_{4,G}^{V/A} \langle GG \rangle}{x^2} + \dots \quad (4.2)$$

Because of the relation $c_{4,\bar{q}q}^V/c_{4,\bar{q}q}^A = -3/5$ [2, 12], the combination $\frac{1}{8}(5\Pi_V + 3\Pi_A)$ cancels the bulk of the contribution from the chiral condensate $\langle \bar{q}q \rangle$. This combination $\widetilde{Z}_{5V+3A}^{\overline{\text{MS}}}$ is shown by crosses, triangles, and pentagons for different quark masses in Fig. 4, where the dependences on x and on valence masses are dramatically reduced. Although there is no mass dependence remains for the operators of dimension four, the mass dependence of the data is still seen at $x > 0.4 \text{ fm}$, which may be attributed to the contributions from higher dimensional operators, including the four-quark condensate $\langle \bar{q}q q q \rangle$ and $m_q^2 \langle GG \rangle$.

Figure 5 shows $\widetilde{Z}_{(5V+3A)/8}^{\overline{\text{MS}}}(x)$ at each β . The position where $\widetilde{Z}_{(5V+3A)/8}^{\overline{\text{MS}}}$ starts deviating from a constant toward short-distance moves as the lattice spacing, indicating the discretization effect. The most significant discretization effect is of $O(a^2)$ which appears as $(a/x)^2$. Since we have already subtracted the discretization effects at the tree level, the remaining effect is $\alpha_s(a/x)^2$.

Taking account of these non-perturbative effects and discretization effects, we determine the renormalization factor $Z_V^{\overline{\text{MS}}}$ in the massless limit by a simultaneous fit of all ensembles using the fit function

$$\widetilde{Z}_{(5V+3A)/8}^{\overline{\text{MS}}}(a; x) = Z_V^{\overline{\text{MS}}}(\beta) + C_{-2} \alpha_s(a) (a/x)^2 + C_{4,G} x^4 + (C_{6,q} + C_{6,G} m_q^2) x^6. \quad (4.3)$$

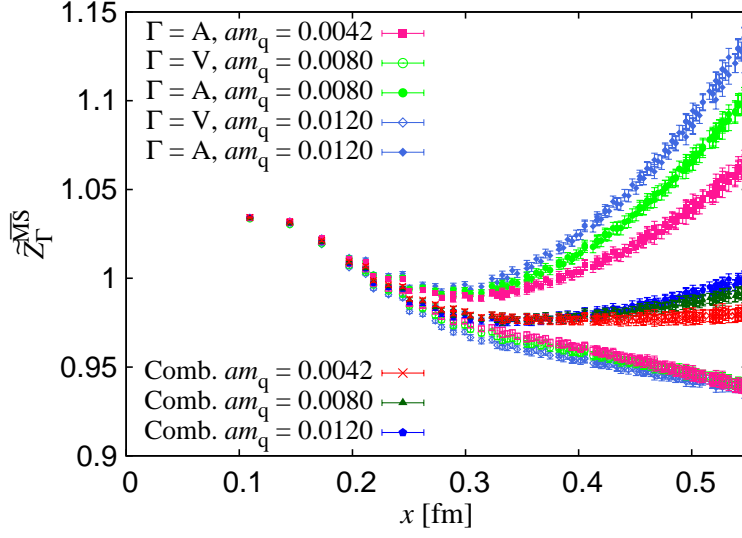


Figure 4: $\tilde{Z}_{V/A}^{\overline{\text{MS}}}$ with the dependence on x , calculated by (4.1) (square, circle, diamond) and by taking the combination $\frac{1}{8}(5\Pi_V + 3\Pi_A)$ (cross, triangle, pentagon). The results of same ensembles as Fig. 3 are shown.

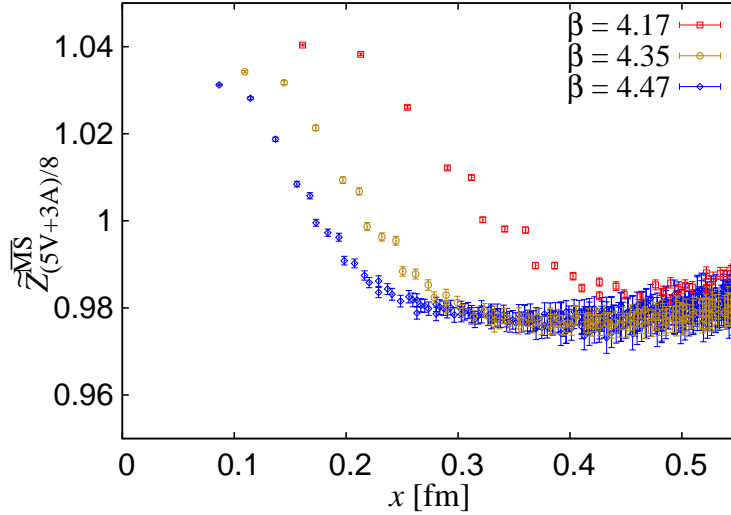


Figure 5: $\tilde{Z}_{(5V+3A)/8}^{\overline{\text{MS}}}(x)$ for each β at similar pion masses $M_\pi \simeq 300$ MeV.

Our results are $Z_V^{\overline{\text{MS}}} = 0.951(4), 0.956(3), 0.961(3)$ at $\beta = 4.17, 4.35, 4.47$, respectively.

Determination of Z_S is more complicated due to the possible instanton-induced effect. Since instantons affect the scalar and pseudoscalar correlators to the opposite direction with the same magnitude, the naïve average $\frac{1}{2}(\Pi_S + \Pi_P)$ is independent of such an effect. According to the OPE, the average contains the contribution of the chiral condensate, which may be cancelled by the difference between the vector and axial-vector correlators. Taking a combination of $\Pi_S + \Pi_P$ and $\Pi_V - \Pi_A$ to cancel these known x -dependence, we are able to fit the Z -factor using the fit form similar to (4.3). We obtain $Z_S^{\overline{\text{MS}}}(2 \text{ GeV}) = 1.024(15), 0.922(11), 0.880(7)$ at $\beta = 4.17, 4.35, 4.47$, respectively.

5. Summary

The main purpose of this work is to understand how precisely the continuum theory predicts the short-distance behavior of current correlators by comparing lattice results with the continuum theories. The vector and axial-vector correlators agree with the OPE in the region $x \lesssim 0.5$ fm, while the scalar and pseudoscalar channels need more study to understand.

Controlling non-perturbative effects and discretization effects, we determine renormalization factors of quark currents using correlators. This procedure enables us to renormalize in a gauge invariant manner and to perform the perturbative matching to the four-loop level.

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