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Mass and Axial current renormalization in the Schrödinger functional scheme for the RG-improved gauge and the stout smeared $O(a)$-improved Wilson quark actions

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We present the quark mass and axial current renormalization factors for the RG-improved Iwasaki gauge action and three flavors of the stout smeared $O(a)$-improved Wilson quark action. We employ $\alpha = 0.1$ and $n_{\rm step} = 6$ for the stout link smearing parameters and all links in the quark action are replaced with the smeared links. Using the Schrödinger functional scheme we evaluate the renormalization factors at $\beta = 1.82$ where large scale simulations are being carried out.

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1. Introduction

We are accumulating the configurations near the physical masses of up-down (degenerated) and strange quarks ($N_f = 2+1$) on a 96^4 lattice with the lattice cutoff $a^{-1} \sim 2.3$ [GeV] under the project of HPCI (High Performance Computing Infrastructure) Strategic Programs for Innovative Research (SPIRE) Field 5, "The origin of matter and the universe" [1]. The properties of these ensembles have to be determined via the detailed analysis on the physical quantities such as the light hadron spectrum, quark masses, and hadronic observables, extracted from the ensembles.

In this poster, we report on the mass and axial current renormalization factors determined with the Schrödinger functional (SF) scheme for the RG-improved Iwasaki gluon action and three flavors of the stout smeared $O(a)$-improved Wilson quark action. We employ $\alpha = 0.1$ and $n_{\mathrm{step}} = 6$ for the stout link smearing [2] parameters, and all link variables contained in the $O(a)$-improved Wilson quark action are replaced with the stout smeared ones. The $O(a)$-improvement parameter c_{SW} has been determined nonperturbatively in Ref. [3] using the SF method. We determine the renormalization factors at $\beta = 1.82$ ($a^{-1} \sim 2.3$ [GeV]) where the simulations on the 96^4 lattice with this lattice action are being carried out.

The renormalization constants for the axial and pseudo-scalar operators (and vector operator as a byproduct) with the SF scheme are determined using the standard method described in Refs. [4, 5, 6, 7, 8]. The temporal and spatial lattice sizes are finite at $T = aN_T$ and $L = aN_S$, and a Dirichlet boundary condition in the temporal direction is imposed to define the SF scheme. The boundary gauge fields at $n_4 = 0$ and N_T are kept fixed by the Dirichlet boundary condition and the same boundary condition is applied on the smeared gauge field during the stout smearing steps. The up, down and strange quark masses are degenerate and tuned to vanish in determining the renormalization constants. The HMC (two-flavor part) and RHMC (single-flavor part) algorithms with the SF boundary condition are employed to generate the gauge configuration. The action parameters are set to be $\beta = 1.82$ and $c_{\mathrm{SW}} = 1.11$ [3], and the boundary parameters are to be $c_t^P = 1$ ($c_t^R = 3/2$) for the plaquette (rectangular) term for the gauge action and $\tilde{c}_t = 1$ for the quark action.

2. Axial, vector, and pseudo-scalar operator renormalization constants

The operators to be renormalized are defined by

$$A_4^a(x) = \overline{q}(x)\gamma_4\gamma_5 T^a q(x), \qquad V_4^a(x) = \overline{q}(x)\gamma_4 T^a q(x), \qquad P^a(x) = \overline{q}(x)\gamma_5 T^a q(x). \tag{2.1}$$

As it was observed that the nonperturbative $O(a)$-mixing correction to the axial current was consistent with zero [3], we assume a negligible $O(a)$-mixing and employ the unimproved current operators in determining the renormalization factors. The correlation functions used for setting the renormalization conditions are

$$f_{XY}(t,s) = -\frac{2}{N_f^2(N_f^2-1)}\sum_{\vec{x},\vec{y}} f^{abc}f^{cde}\langle O'^d X^a(\vec{x},t)Y^b(\vec{y},s)O^e\rangle, \tag{2.2}$$

$$f_X(t) = -\frac{1}{N_f^2-1}\sum_{\vec{x}}\langle X^a(\vec{x},t)O^a\rangle, \tag{2.3}$$

$$f_1 = -\frac{1}{N_f^2-1}\langle O'^a O^a\rangle, \tag{2.4}$$

$$f_V(t) = \frac{1}{N_f(N_f^2-1)}\sum_{\vec{x}} i f^{abc}\langle O'^a V_4^b(\vec{x},t)O^c\rangle, \tag{2.5}$$

where f^{abc} is the structure constant of $SU(N_f)$. The operator X (Y) can be A_4 or P. O^a and O'^a are boundary operators defined by

$$O^a = \frac{1}{(L/a)^3}\sum_{\vec{y},\vec{z}}\overline{\zeta}(\vec{y})\gamma_5 T^a\zeta(\vec{z}), \qquad O'^a = \frac{1}{(L/a)^3}\sum_{\vec{y},\vec{z}}\overline{\zeta}'(\vec{y})\gamma_5 T^a\zeta'(\vec{z}), \tag{2.6}$$

where ζ and ζ' are boundary quark fields located at $n_4 = 0$ and $n_4 = N_T$ respectively.

Using the correlation functions (2.2)-(2.5), the renormalization factors are determined with

$$Z_A = \left.\sqrt{\tilde{Z}_A(2T/3)}\right|_{m_q\to 0}, \qquad \tilde{Z}_A(t) = \frac{f_1}{n_A}\left[f_{AA}(t,T/3) - 2m_{\mathrm{PCAC}}f_{PA}(t,T/3)\right]^{-1}, \tag{2.7}$$

$$Z_V = \left.\tilde{Z}_V(T/2)\right|_{m_q\to 0}, \qquad \tilde{Z}_V(t) = \frac{f_1}{n_V f_V(t)}, \tag{2.8}$$

$$Z_P = \left.\tilde{Z}_P(T/2)\right|_{m_q\to 0}, \qquad \tilde{Z}_P(t) = \frac{\sqrt{3f_1}}{n_P f_P(t)}, \tag{2.9}$$

where n_A, n_V, and n_P are normalization constants evaluated at the tree-level so as to be $Z_A = 1$, $Z_V = 1$, and $Z_P = 1$. To take the mass-less limit we employ an averaged PCAC mass for Z_A and Z_V;

$$am_q = am^{\mathrm{PCAC}} = \frac{1}{3}\sum_{t=T/2-a}^{T/2+a}\frac{af_A(t+a) - af_A(t-a)}{4f_P(t)}, \tag{2.10}$$

while a non-averaged mass for Z_P;

$$am_q = am^{\mathrm{PCAC}} = \frac{af_A(T/2+a) - af_A(T/2-a)}{4f_P(T/2)}. \tag{2.11}$$

The simulation parameters are shown in Table 1. The phase angle θ is the parameter of the generalized periodic boundary condition in each spatial direction for the quark field. The boundary gauge fields are fixed to the identity matrix. The data are measured at every trajectory and the statistical errors are estimated with the jackknife method after blocking data with the size of 100 trajectories. The simulations (A1S) and (A1L) are dedicated for Z_A (and Z_V as a byproduct), while (P4a) and (P4b) are for Z_P. The hopping parameter $\kappa = 0.126110$ corresponds to the critical value κ_c determined in Ref. [3]. Almost vanishing masses are realized in the (A1S) and (A1L) runs.

The time dependence of $\tilde{Z}_A(t)$ and $\tilde{Z}_V(t)$ from (A1L) is shown in Figure 1. $\tilde{Z}_V(t)$ is almost time-independent, and a short plateau around $t = 17-22$ ($\sim 2T/3$) is observed for $\tilde{Z}_A(t)$ when

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Run	$L/a, T/a$	θ	κ	traj.	HMC Acc.
(A1S)	$8, 18$	1/2	0.126110	20000	0.8811(80)
(A1L)	$12, 30$	1/2	0.126110	34700	0.9120(53)
(P4a)	$4, 4$	1/2	0.126110	100000	0.8792(11)
(P4b)	$4, 4$	1/2	0.125120	80000	0.8787(15)

Table 1: Parameters and statistics for the renormalization factors.

Run	am^{PCAC}	Z_V	Z_A	Z_P
(A1S)	0.00041(61)	0.9664(20)	0.9745(48)	-
(A1L)	-0.00080(33)	0.95153(76)	0.9650(68)	-
(P4a)	-0.021859(94)	-	-	1.01317(43)
(P4b)	0.013241(99)	-	-	1.00670(45)

Table 2: PCAC masses and renormalization factors Z_V, Z_A, and Z_P.

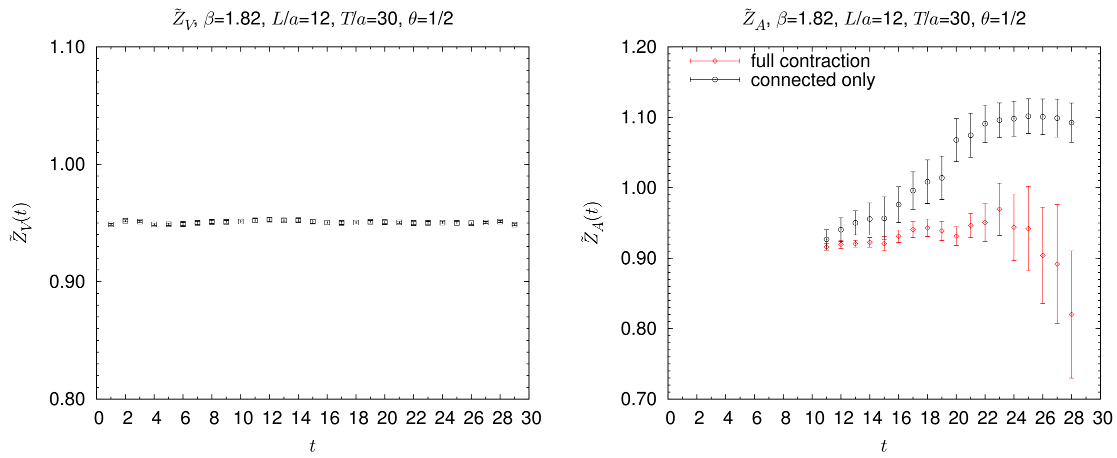

Figure 1: Time dependence of $\tilde{Z}_V(t)$ (left) and $\tilde{Z}_A(t)$ (right) on a $12^3 \times 30$ lattice (A1L).

disconnected diagrams are included properly. A similar behavior is observed in (A1S). The renormalization factors extracted with the definitions (2.7)-(2.9) are tabulated in Table 2. We assign the discrepancy of Z_A and Z_V between two runs, (A1S) and (A1L), to the systematic error. We observe that $Z_A \simeq Z_V$ and $Z_A \simeq 1$. This could indicate a better chiral property of the stout smeared quark action we employed.

The renormalization factor for the pseudo-scalar operator depends on the renormalization scale. The renormalization scale corresponds to the physical box size L and the scale is implicitly defined by the value of the renormalized coupling ($g^2_{\mathrm{SF}}(L)$) in the SF scheme. To convert $Z_P^{\mathrm{SF}}(1/L)$ to the mass renormalization factor $Z_m^{\overline{\mathrm{MS}}}(\mu)$ at a reference scale μ in the $\overline{\mathrm{MS}}$ scheme, we need the RG evolution of Z_P together with the running of the renormalized coupling constant $g^2(\mu)$ in both the SF and $\overline{\mathrm{MS}}$ schemes. The details of the mass renormalization factor are described in the next section.

3. Scale setting and mass renormalization

The renormalization factor of the pseudo-scalar operator Z_P is evaluated at $T = L = 4a$ and $\beta = 1.82$. The simulation parameters and results are shown in Tables 1 and 2. We have two simulations, (P4a) and (P4b). The scale $L = 4a$ is chosen so that the RG evolution by the step scaling of the coupling is available from the scale $L_{\max} = 4a$. The renormalized coupling in the SF scheme at $L = 4a$ and $\beta = 1.82$ is shown in Table 3, where the boundary condition defining the SF scheme coupling is imposed on the gauge field. The PCAC mass in Table 3 is defined by Eq. (2.11). The step scaling evolution for the SF scheme coupling from $g^2_{\mathrm{SF}}(1/L_{\max}) \sim 3.7-3.8$ is available in the continuum limit [9]. The PCAC masses at $\kappa_c = 0.126110$ is slightly off the vanishing point as seen in (P4a) and (G4a) runs. The discrepancy between $\kappa = 0.12512$ and κ_c are assigned to the systematic error of Z_m.

Combining the RG evolutions for Z_P and the coupling in both schemes, we can extract the mass renormalization constant $Z_m^{\overline{\mathrm{MS}}}(g_0,\mu)$ in the $\overline{\mathrm{MS}}$ scheme [8, 9, 10, 11] by

$$Z_m^{\overline{\mathrm{MS}}}(g_0,\mu) = \left(\frac{m^{\overline{\mathrm{MS}}}(\mu)}{M^{\mathrm{RGI}}}\right)\left(\frac{M^{\mathrm{RGI}}}{m^{\mathrm{SF}}(1/L_{\max})}\right)\left(\frac{Z_A^{\mathrm{SF}}(g_0,a/L)}{Z_P^{\mathrm{SF}}(g_0,a/L_{\max})}\right), \tag{3.1}$$

where $L_{\max} = 4a$ will be applied. The mass ratios between the renormalization group invariant (RGI) mass M^{RGI} and the renormalized masses $m^{\overline{\mathrm{MS}}}$ (or m^{SF}) are defined by

$$\left(\frac{M^{\mathrm{RGI}}}{m^{\overline{\mathrm{MS}}}(\mu)}\right) = \left(2b_0\bar{g}^2(\mu)\right)^{-d_0/(2b_0)} \exp\left[-\int_0^{\bar{g}(\mu)} dg\left(\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right)\right]\Bigg|_{\overline{\mathrm{MS}}}, \tag{3.2}$$

$$\left(\frac{M^{\mathrm{RGI}}}{m^{\mathrm{SF}}(1/L_{\max})}\right) = \left[\prod_{j=1}^{n}\sigma_P(u_j)\right]\left(2b_0\bar{g}^2(1/L_n)\right)^{-d_0/(2b_0)} \exp\left[-\int_0^{\bar{g}(1/L_n)} dg\left(\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right)\right]\Bigg|_{\mathrm{SF}}. \tag{3.3}$$

$\bar{g}^2(\mu)$ is the renormalized coupling constant in the $\overline{\mathrm{MS}}$ scheme for Eq. (3.2), and that in the SF scheme for Eq. (3.3). $\sigma_P(u)$ is the step scaling function for Z_P in the SF scheme. The argument u_j is the renormalized coupling defined by $u_j = g^2_{\mathrm{SF}}(2^j/L_{\max})$ which is evolved from $u_0 = g^2_{\mathrm{SF}}(1/L_{\max})$ using the step scaling function $\sigma(u)$ for the coupling via $u_{j+1} = \sigma(u_j)$.

In order to evaluate the mass renormalization constant $Z_m^{\overline{\mathrm{MS}}}(g_0,\mu)$, we employ $\sigma_P(u)$ from Ref. [8] and $\sigma(u)$ from Ref. [9]. The number of steps n is chosen to be 5 from which we can evaluate the exponent of Eq. (3.3) perturbatively. The two-loop mass anomalous dimension $\tau(g)$ [12] and the three-loop beta function $\beta(g)$ [13] are used in the SF scheme, while the four-loop estimates for the mass anomalous dimension and beta function are employed in the $\overline{\mathrm{MS}}$ scheme in evaluating the mass renormalization constant. In order to evaluate the mass renormalization constant at the reference scale $\mu = 2$ [GeV], we need the scale of a^{-1} in physical unit at $\beta = 1.82$.

4. Results

The axial and vector current renormalization factors at $\beta = 1.82$ are evaluated as

$$Z_A = 0.9650(68)(95), \tag{4.1}$$

$$Z_V = 0.95153(76)(1487), \tag{4.2}$$

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Run	θ	κ	traj.	HMC Acc.	am^{PCAC}	g^2_{SF}
(G4a)	$\pi/5$	0.126110	70000	0.8787(14)	-0.04550(26)	3.662(17)
(G4b)	$\pi/5$	0.125120	80000	0.8791(15)	0.00042(25)	3.776(16)

Table 3: Parameters, statistics and the renormalized coupling g^2_{SF} at $\beta = 1.82$ and $L/a = T/a = 4$.

where the central values are from (A1L). The first error is the statistical one and the second is the systematic one which is evaluated from the discrepancy between the two runs.

The mass renormalization constant $Z_m^{\overline{\mathrm{MS}}}(g_0, \mu)$ at $\mu = 2$ [GeV] and $\beta = 1.82$ is evaluated as

$$Z_m^{\overline{\mathrm{MS}}}(g_0, \mu = 2[\mathrm{GeV}]) = 0.9950(111)(89), \tag{4.3}$$

where the central value is extracted by combining the factors Z_A from (A1L), Z_P from (P4a), and u_0 from (G4a). For the scale a^{-1}, we use the preliminary value $a^{-1} = 2.332(18)$ [GeV] from Ref. [14]. The first error is the statistical error and the second is the systematic one estimated from the discrepancy to the mass renormalization constant evaluated using (P4b) and (G4b).

5. Summary

In this poster, we presented the determination of the renormalization constants for the axial and vector currents and the quark mass in the Schrödinger functional scheme. The values in Eqs. (4.1)-(4.3) were obtained for the RG-improved Iwasaki gluon and three flavors of the stout smeared quarks at $\beta = 1.82$. By applying these renormalization factors to the results from the simulation on the 96^4 lattice, we can obtain the quark masses and the decay constants precisely [14].

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