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Hadronic form factors for rare semileptonic B decays

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We discuss first results for the computation of short distance contributions to semileptonic form factors for the rare B decays $B \to K^* \ell^+ \ell^-$ and $B_s \to \phi \ell^+ \ell^-$. Our simulations are based on RBC/UKQCD's $N_f = 2+1$ ensembles with domain wall light quarks and the Iwasaki gauge action. For the valence b-quark we chose the relativistic heavy quark action.

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1. Introduction

Flavour physics plays a crucial role in the quest for new physics beyond the Standard Model (SM). Here we concentrate on flavour changing processes involving b-quarks. Its large mass may give rise to (virtual) new heavy particles or exhibit large enough couplings to e.g. charged Higgs bosons postulated by the 2-Higgs doublet model [1, 2]. In order to detect such signs of new physics, it is important to improve our understanding of flavour changing processes which occur at tree and loop level in the SM. Compared to tree level flavour changing processes in the SM, transitions proceeding via Flavour Changing Neutral Currents (FCNC) are highly suppressed. FCNC are therefore particularly attractive probes for discovering potential contributions from physics beyond the SM (BSM). So far however no discovery has been made. Both experiment and theory are working on reducing uncertainties and improving bounds. On the theoretical side predictions of hadronic uncertainties are typically dominating the error budget. Here we are presenting our efforts in using simulations of lattice QCD for making reliable and precise predictions for hadronic form factors that are crucial inputs to the analysis.

FCNC are conventionally described in the SM by a set of 20 operators which contribute to the effective Hamiltonian [3–8]. Weak $b \to q$ decays (with q a down or a strange quark) are parametrised by

$$\mathscr{H}_{\text{eff}}^{b\to q} = -\frac{4G_F}{\sqrt{2}} V_{tq}^* V_{tb} \sum_{i=1}^{10} C_i O_i + C_i' O_i', \tag{1.1}$$

where V_{tq}^* and V_{tb} are CKM matrix elements, $O_i^{(\prime)}$ are local operators and $C_i^{(\prime)}$ their corresponding Wilson coefficients determined in [9–11]. The leading short distance contributions are given by

$$O_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{q}\gamma^\mu P_{L(R)} b \bar{\ell}\gamma_\mu \ell, \qquad O_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{q}\gamma^\mu P_{L(R)} b \bar{\ell}\gamma_\mu \ell, \tag{1.2}$$

and the dileptonic operator

$$O_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{q}\sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu}. \tag{1.3}$$

In Equations (1.2) and (1.3) the lepton is denoted by ℓ, the mass of the b-quark by m_b and $P_{L(R)} = \frac{1}{2}(1 \mp \gamma^5)$, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Further, long distance effects contribute which are commonly estimated perturbatively [12, 13]. These theoretical estimates are however questioned because charm resonances may not be captured well enough [14]. More research is needed to understand the implications and to also explore current tensions between SM predictions and experimental results. Of particular interest is e.g. the observed tension for the observable P_5' in $B_s \to K\ell^+\ell^-$ decays [15, 16] which can also be explained by different models of new physics, see e.g. [17–19].

In the following we focus on the computation of the dominant short distance operators for which a lattice calculation is suitable. Moreover, in these proceedings we restrict ourselves to the computation of decays with a pseudoscalar $B_{(s)}$ meson in the initial and a vector meson in the final state. As in all other contemporary computations of these decays the final state vector meson will be approximated as a stable particle. Our computations provide important, independent checks to existing calculations by the Cambridge group [20–22], HPQCD [23], and the Fermilab/MILC [24, 25] collaborations which are all based on overlapping sets of MILC's staggered gauge field

$L^3 \times T$	a^{-1} [GeV]	am_l	am_h	am'_s	M_π [MeV]	total # of configs
$24^3 \times 64$	1.785(5)	0.005	0.040	0.0343	338	1636
$24^3 \times 64$	1.785(5)	0.010	0.040	0.0343	434	1419

Table 1: Parameters of the calculation. We use the coarse ensembles generated by the RBC and UKQCD collaboration [26, 27] with 2+1 flavor domain-wall fermions and Iwasaki gauge actions. The domain-wall fermions are created with fifth dimension $L_s = 16$ and domain-wall height $M_5 = 1.8$. Values for the inverse lattice spacing and the quark and meson masses are taken from the refined analysis [32]; am_l labels the mass of the light sea-quark, am_h the mass of the heavy sea quark, and am'_s the mass of the near physical strange quark mass. The physical value of the strange quark mass is $am_s = 0.03224(18)$ [32].

configurations. In the following Section we describe our computational setup to obtain these contributions and present in Sec. 3 our first numerical results before summarising in Sec. 4.

2. Computational setup

Our simulations are performed using 2+1 flavour domain-wall fermions and Iwasaki gauge-field ensembles generated by the RBC and UKQCD Collaborations [26, 27]. In the valence sector we use the domain-wall action [28, 29] for u/d and s-quarks and the relativistic heavy quark action [30] with nonperturbatively tuned parameters [31] for the b-quarks. Further details of our simulations are summarised in Tab. 1. Autocorrelations between our lattices are effectively reduced by shifting the gauge fields by a random 4-vector as first step of our simulations.

We parametrise the contributions of the short distance operators defined in Eqs. (1.2) and (1.3) by a set of seven form factors, f_V, f_{A_0}, f_{A_1}, f_{A_2}, f_{T_1}, f_{T_2}, and f_{T_3} assuming we have a $B_{(s)}$ meson in the initial and a light vector meson (V) in the final state. We determine the seven form factors by evaluating the following hadronic weak matrix elements [11]:

$$\langle V(k,\varepsilon)|\bar{q}\gamma^\mu b|B_{(s)}(p)\rangle = f_V(q^2)\frac{2i\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma}{M_{B_{(s)}}+M_V} \tag{2.1}$$

$$\begin{aligned}\langle V(k,\varepsilon)|\bar{q}\gamma^\mu\gamma_5 b|B_{(s)}(p)\rangle = {} & f_{A_0}(q^2)\frac{2M_V\varepsilon^*\cdot q}{q^2}q^\mu \\ & + f_{A_1}(q^2)(M_{B_{(s)}}+M_V)\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right] \\ & - f_{A_2}(q^2)\frac{\varepsilon^*\cdot q}{M_{B_{(s)}}+M_V}\left[k^\mu+p^\mu-\frac{M^2_{B_{(s)}}-M^2_V}{q^2}q^\mu\right]\end{aligned} \tag{2.2}$$

$$q_\nu\langle V(k,\varepsilon)|\bar{q}\sigma^{\nu\mu}b|B_{(s)}(p)\rangle = 2f_{T_1}(q^2)\epsilon^{\mu\rho\tau\sigma}\varepsilon^*_\rho k_\tau p_\sigma \tag{2.3}$$

$$\begin{aligned}q_\nu\langle V(k,\varepsilon)|\bar{q}\sigma^{\nu\mu}\gamma^5 b|B_{(s)}(p)\rangle = {} & if_{T_2}(q^2)\left[\varepsilon^{*\mu}(M^2_{B_{(s)}}-M^2_V)-(\varepsilon^*\cdot q)(p+k)^\mu\right] \\ & + if_{T_3}(q^2)(\varepsilon^*\cdot q)\left[q^\mu-\frac{q^2}{M^2_{B_{(s)}}-M^2_V}(p+k)^\mu\right]\end{aligned} \tag{2.4}$$

where p (k) is the momentum of the $B_{(s)}$ (vector) meson and $q = (M_{B_{(s)}} - E_V(|\vec{k}|), -\vec{k})$. The masses of the $B_{(s)}$ and the vector meson are given by $M_{B_{(s)}}$ and M_V, respectively, and ε denotes the polarisation vector of the final state meson.

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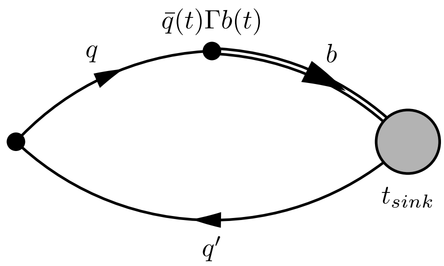

Figure 1: Three point correlator function used to obtain the $B \to V$ form factors. The single and double lines correspond to light and b-quark propagators, respectively.

The matrix elements Eqs. (2.1) – (2.4) are evaluated by computing ratios of 3-point over 2-point correlation functions and we choose to carry out the computation in the $B_{(s)}$-meson rest frame

$$R^{\alpha\Gamma}_{B_{(s)}\to V}(t, t_{\rm sink}, k) = \frac{C^{\alpha\Gamma}_{B_{(s)}\to V}(t, t_{\rm sink}, k)}{\sqrt{\frac{1}{3}\sum_i C^{ii}_V(t,k) C_{B_{(s)}}(t_{\rm sink} - t)}} \sqrt{\frac{4E_V M_{B_{(s)}} \sum_\lambda \varepsilon^j(k,\lambda)\varepsilon^{j*}(k,\lambda)}{e^{-E_V t} e^{-M_{B_{(s)}}(t_{\rm sink}-t)}}} \tag{2.5}$$

$$\xrightarrow{t, t_{\rm sink}\to\infty} \sum_\lambda \varepsilon^\alpha(k,\lambda) \langle V(k,\lambda)|\bar{q}\Gamma b|B_{(s)}(p)\rangle. \tag{2.6}$$

The 3-point functions

$$C^{\alpha\Gamma}_{B_{(s)}\to V}(t, t_{\rm sink}, \vec{k}) = \sum_{\vec{x},\vec{y}} e^{i\vec{k}\cdot\vec{y}} \langle \mathscr{O}^\alpha_V(0,\vec{0}) \bar{q}(t,\vec{y}) \Gamma b(t,\vec{y}) \mathscr{O}_{B_{(s)}}(t_{\rm sink}, \vec{x})\rangle, \tag{2.7}$$

are sketched in Fig. 1 and constructed in the following way: A light quark propagator with the appropriate Dirac-structure and Fourier phase at its sink is used as the source for a sequential b-quark inversion which is then contracted with another light quark propagator originating form the same location as the first one. The contractions are performed by inserting a vector or tensor current $\bar{q}\Gamma b$ with $\Gamma = \{\gamma^\mu, \gamma^5\gamma^\mu, \sigma^{\mu\nu}, \gamma^5\sigma^{\mu\nu}\}$ between the interpolating meson operators $\mathscr{O}_V = \bar{q}\gamma^\mu q$ and $\mathscr{O}_{B_{(s)}} = \bar{b}\gamma^5 q$. As usual we project the result onto states of discrete momenta $\vec{k}$. The 2-point functions in the denominator of Eq. (2.5) are given by

$$C_{B_{(s)}}(t) = \sum_{\vec{x}} \langle \mathscr{O}^\dagger_{B_{(s)}}(t,\vec{x}) \mathscr{O}_{B_{(s)}}(0,\vec{0})\rangle, \qquad \text{and} \qquad C_V(t,\vec{k}) = \sum_{\vec{x}} e^{i\vec{k}\cdot\vec{x}} \langle \mathscr{O}^\dagger_V(t,\vec{x}) \mathscr{O}_V(0,\vec{0})\rangle. \tag{2.8}$$

In addition we use the correlators (2.8) to extract the $B_{(s)}$ meson mass, $M_{B_{(s)}}$, and the energy of the vector meson V with momentum $\vec{k}$, $E_V(|\vec{k}|)$.

We reduce excited state contamination by generating the heavy b-quark propagators with a Gaussian smeared source using the parameters determined in [31], while light u/d and s-quark propagators are generated from a point source.

3. First results

Central to the numerical part of this project is the calculation of the 3-point functions defined in Eq. (2.7). We implement those as inline function in Chroma [33]. In order to optimise the signal, we study four different source-sink separations seeking the choice which results in the longest plateau

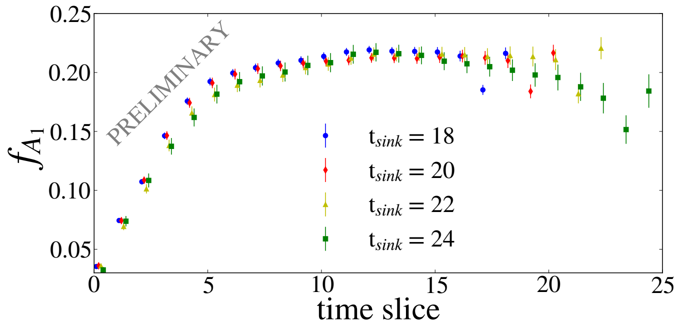

Figure 2: Bare form factor f_{A_1} for the $B_s \to \phi\ell^+\ell^-$ decay at zero momentum for four different source-sink separations t_{sink} on the coarse $a \approx 0.11$fm ensemble with $am_l = 0.005$.

and small statistical errors. The outcome is shown in Fig. 2 where we compare the form factor f_{A_1} for the $B_s \to \phi\ell^+\ell^-$ decay at zero momentum using $t_{\rm sink} = 18, 20, 22, 24$. Within statistical uncertainties, all choices for $t_{\rm sink}$ agree. The best signal is however obtained for $t_{\rm sink} = 20$. We extract the value of the 3-point functions by fitting the range of the plateau sufficiently far from the source and sink locations in order to minimise the influence of excited states. In this way we have obtained results for all form factors f_V, f_{A_0}, f_{A_1}, f_{A_2}, f_{T_1}, f_{T_2}, and f_{T_3}, some of which we show in Fig. 3. For all form factors we find good signals and can extract values from correlated fits to plateaus ranging over several time slices.

4. Summary and outlook

In this paper we present our first results computing the short distance contributions to $B_s \to \phi\ell^+\ell^-$ decays on the lattice. We have validated our code and calculated the form factors parametrising this rare FCNC decay using two ensembles at our coarse lattice spacing of $a^{-1} = 1.785$ GeV. In parallel we are computing renormalisation factors using a *mostly nonperturbative setup* [34] and are also analysing form factors for $B \to K^*\ell^+\ell^-$ decays.

The next steps are to include additional terms for the 1-loop $O(a)$ improvement of the operators and subsequently recalculate the form factors on the two coarse ensembles presented here and in addition on three finer ensembles with $a^{-1} = 2.38$ GeV. Moreover we are exploring to add ensembles with physical light quark masses or an even finer lattice spacing ($a^{-1} \approx 2.8$ GeV). With a set of measurements obtained for different sea- and valence-quark masses and at different values of the lattice spacing, we then intend to perform a combined chiral- and continuum extrapolation based on heavy meson chiral perturbation theory in order to facilitate a kinematical extrapolation (z-expansion) to zero q^2.

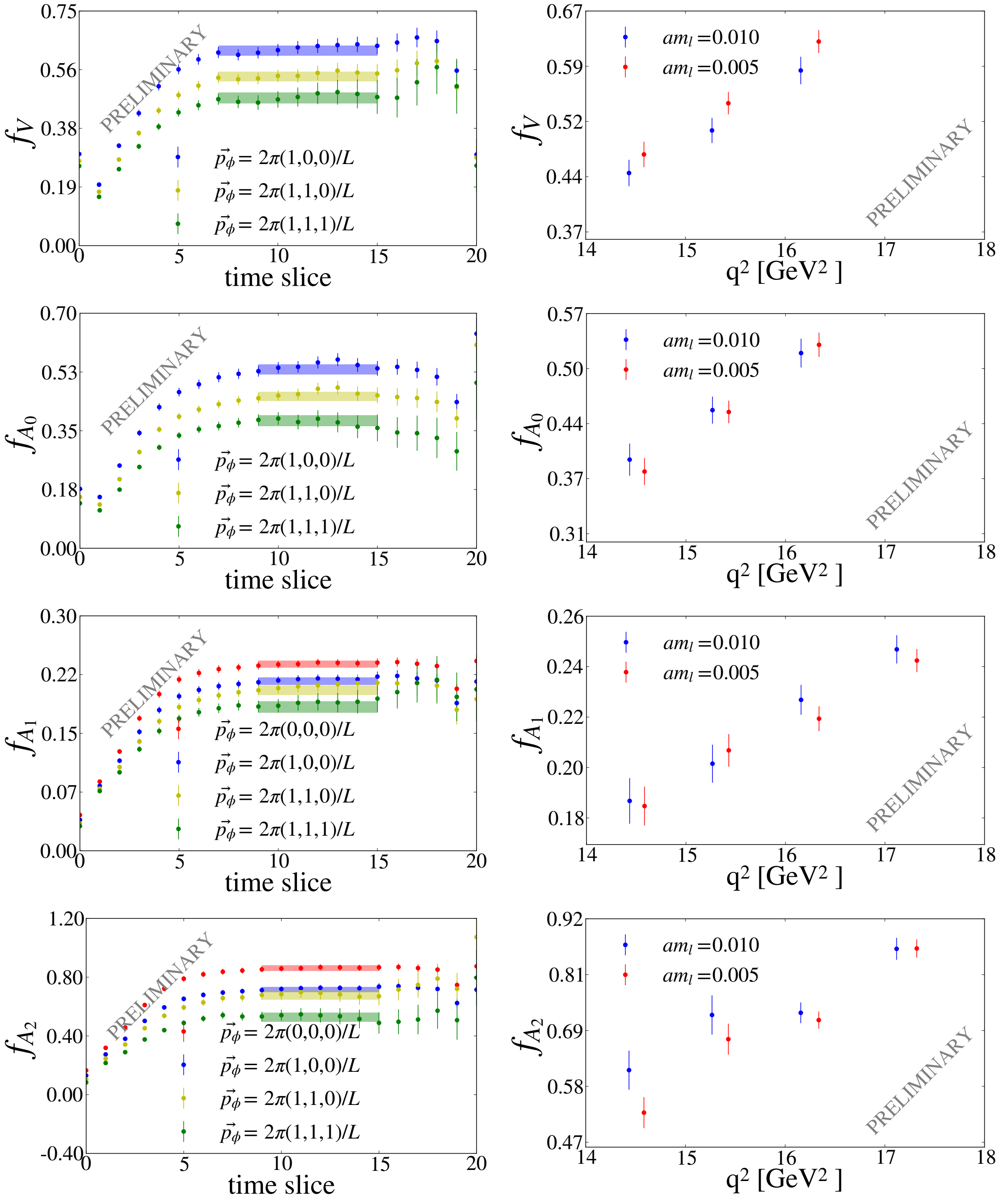

Figure 3: Bare form factors f_V, f_{A_0}, f_{A_1}, f_{A_2} for $B_s \to \phi\ell^+\ell^-$ decays. Left: plateau fits for the coarse ensemble with $am_l = 0.005$, right: extracted form factor values vs. q^2 in GeV obtained on both coarse ensembles, $am_l = 0.005$ and $am_l = 0.010$.

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