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Leading logarithms for mesons and nucleons

Johan Bijnens*
Department of Astronomy and Theoretical Physics, Lund University, Sölvegatan 14A, SE 22362 Lund, Sweden
E-mail: bijnens@thep.lu.se

Karol Kampf
Institute of Particle and Nuclear Physics, Charles University, Prague, Czech Republic
Email: karol.kampf@mff.cuni.cz

Alexey Vladimirov
Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany
E-mail: vladimirov.aleksey@gmail.com

This talk describes the work done in calculating leading logarithms in massive effective field theories. We discuss shortly leading logarithms in renormalizable theories and how they can be calculated using only one-loop calculations in effective field theories. The remainder of the talk discusses masses, decay constants, condensates and anomalous processes in mesonic effective field theories like Chiral Perturbation Theory and the expansion of the nucleon mass.

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*Speaker.

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1. Introduction

The main motivation for the work described in this talk is to obtain information on high orders in effective field theories. The work described in this talk is published in [1, 2, 3, 4, 5]. There are also previous talks containing some of these results, these include [6, 7, 8]

The main use of our formulas is within Chiral Perturbation Theory (ChPT) [9, 10, 11] but we envisage other applications as well. A review of ChPT at loop level is [12], a somewhat shorter more recent review is [13]. The present mesonic status is reviewed in the plenary talk by Gerhard Ecker [14].

In Sect. 2 we give a short introduction to leading logarithms (LLs) in renormalizable field theories. The underlying principle that allows to calculate LLs also in effective field theories, is discussed in Sect. 3. The remaining sections are devoted to the different results we obtained. Section 4 discusses a number of mesonic properties in the massive $O(N+1)/O(N)$ model including masses, decay constants and meson-meson scattering. The next section includes the anomaly and shows a number of results for $\pi^0 \to \gamma^*\gamma^*$ and the $\gamma\pi\pi\pi$ vertex. Sect. 6 discusses the LL for N-flavour equal mass ChPT. Sect. 7 discusses the extensions of the arguments needed for baryon properties and the nucleon mass with results up to seven loops. There are many more results in the papers, with similar conclusions, not discussed here due to lack of time.

2. Leading logarithms in renormalizable theories

The term leading logarithms (LLs) is used for many different things. In this talk it means the leading dependence on the subtraction scale μ. For a dimensionless observable F depending on a single scale M, Quantum field theory (QFT) tells us that the dependence on the subtraction scale is via $L \equiv \log(\mu/M)$ in the form

$$F = F_0 + \left(F_1^1 L + F_0^1\right) + \left(F_2^2 L^2 + F_1^2 L + F_0^2\right) + \left(F_3^3 L^3 + \cdots\right) + \cdots \tag{2.1}$$

Here F_m^ℓ means the L^m contribution at order ℓ in the expansion. The LLs are the terms $F_\ell^\ell L^\ell$. In QFT these terms can be more easily calculated than the remainder. The reason is that an observable cannot depend on the subtraction scale $\mu(dF/d\mu) \equiv 0$ and that ultra-violet divergences in QFT are always local. We rewrite (2.1) for the case of a renormalizable theory with an expansion in α

$$F = \alpha + \left(f_1^1 \alpha^2 L + f_0^1 \alpha^2\right) + \left(f_2^2 \alpha^3 L^2 + f_1^2 \alpha^3 L + f_0^2 \alpha^3\right) + \left(f_3^3 \alpha^4 L^3 + \cdots\right) + \cdots \tag{2.2}$$

Taking $\mu(d/d\mu)$ of (2.2), and setting it to zero using the beta-function $\mu(d\alpha/d\mu) = \beta_1\alpha^2 + \beta_2\alpha^3 + \cdots$, gives

$$\left(\beta_1 + f_1^1\right)\alpha^2 + \left(2\beta_1 f_1^1 + 2 f_2^2\right)\alpha^3 L + \left(\beta_2 + 2\beta_1 f_0^1 + f_1^2\right)\alpha^3 + \left(3\beta_1 f_2^2 + 3 f_3^3\right)\alpha^4 L^2 + \cdots = 0\,. \tag{2.3}$$

The terms with highest power in L at each order in α lead to

$$\begin{aligned} f_1^1 &= -\beta_1,\ f_2^2 = \beta_1^2,\ f_3^3 = -\beta_1^3, \ldots \qquad \Longrightarrow \\ F(M) &= \alpha\left(1 - \alpha\beta_1 L + (\alpha\beta_1 L)^2 - (\alpha\beta_1 L)^3 + \cdots\right) + \cdots \\ &= \frac{\alpha(\mu)}{1 + \alpha(\mu)\beta_1 \log(\mu/M)} + \cdots = \alpha(M) + \cdots \end{aligned}$$

In the last line we showed first explicitly $\alpha = \alpha(\mu)$ and how the LL in this case can be absorbed into a running coupling constant. The argument can be generalized to sub-leading logarithms and extended in general to the renormalization group.

An important part in this derivation is that the underlying theory is the same at all orders. α is the same in all terms. This reasoning is no longer true in effective field theories. In effective field theory we have a different Lagrangian at each order, with different and new coupling constants.

3. Weinberg's argument

However, even if the argument used in Sect. 2 no longer holds, some possibilities remain. Weinberg [9] pointed out that two-loop leading logarithms can be calculated using only one-loop calculations, the method is later called "Weinberg consistency conditions" and relates divergences from different types of diagrams. This method was used for $\pi\pi$-scattering at two loops [15] and the general mesonic two-loop LL structure [16]. The extension to all orders was done in [17] and later with a diagrammatic [2] and an operator method [5].

Let us give the argument as presented in [2]. We introduce μ, the subtraction scale, and a parameter $\hbar$ that keeps track of the order in the expansion. Dimensional regularization is used throughout with $d = 4 - w$. The bare Lagrangian is expanded

$$\mathscr{L}^{\text{bare}} = \sum_{n\geq 0} \hbar^n \mu^{-nw} \mathscr{L}^{(n)}, \quad \mathscr{L}^{(n)} = \sum_i c_i^{(n)} \mathscr{O}_i, \quad c_i^{(n)} = \sum_{k=0,n} \frac{c_{ki}^{(n)}}{w^k}. \tag{3.1}$$

The last shows how the coefficients of the Lagrangians are expanded in the divergences. From QFT it follows that divergences are always local and that only the $c_{0i}^{(n)}$ have a direct μ-dependence. The $c_{ki}^{(n)}$ $k \geq 1$ only depend on μ through their dependence on the lower order coupling constants $c_{0i}^{(m<n)}$. The ℓ-loop contribution at order $\hbar^n$ can be similarly expanded in the divergences coming from the loop integrations

$$L_\ell^n = \sum_{k=0,l} \frac{1}{w^k} L_{k\ell}^n. \tag{3.2}$$

The remaining parts of the argument basically use that all divergences must cancel including the non-local ones. At one-loop level we get a contribution

$$\frac{1}{w}\left(\mu^{-w} L_{00}^1(\{c\}_1^1) + L_{11}^1\right) + \mu^{-w} L_{00}^1(\{c\}_0^1) + L_{10}^1. \tag{3.3}$$

Expanding $\mu^{-w} = 1 - w\log\mu + \frac{1}{2}w^2\log^2\mu + \cdots$ to get the $\log\mu$ dependence we see that this is $-\log\mu\, L_{00}^1(\{c\}_1^1) \equiv \log\mu\, L_{11}^1$ obtainable by a one-loop calculation and by canceling the $1/w$ term we obtain the c_{1i}^1. At two loop-order we get more nontrivial results. The contribution is

$$\begin{aligned}&\frac{1}{w^2}\left(\mu^{-2w} L_{00}^2(\{c\}_2^2) + \mu^{-w} L_{11}^2(\{c\}_1^1) + L_{22}^2\right) + \frac{1}{w}\Big(\mu^{-2w} L_{00}^2(\{c\}_1^2) + \mu^{-w} L_{11}^2(\{c\}_0^1)\\ &+ \mu^{-w} L_{10}^2(\{c\}_1^1) + L_{21}^2\Big) + \left(\mu^{-2w} L_{00}^2(\{c\}_0^2) + \mu^{-w} L_{10}^2(\{c\}_0^1) + L_{20}^2\right).\end{aligned} \tag{3.4}$$

Canceling infinities leads to two equations

$$L_{00}^2(\{c\}_2^2) + L_{11}^2(\{c\}_1^1) + L_{22}^2 = 0, \quad 2L_{00}^2(\{c\}_2^2) + L_{11}^2(\{c\}_1^1) = 0. \tag{3.5}$$

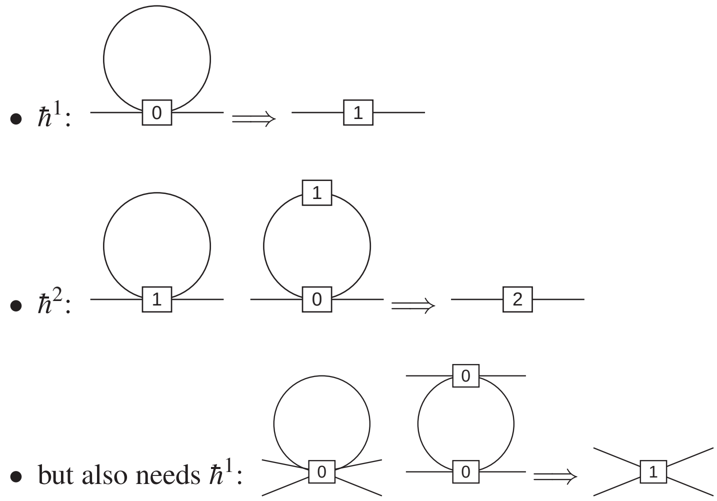

Figure 1:
The diagrams needed for the LL up to order $\hbar^2$ for the mass. Top line, one-loop order; Middle line: two-loop order; Bottom line: the extra diagrams needed for the divergences of the four meson vertex at one-loop order. This vertex is needed in the first diagram in the second line. $\boxed{\text{n}}$ indicates a vertex from $\mathscr{L}^{(n)}$.

These determine the c^2_{2i} and allow to fix the LL explicit μ-dependence as $-\frac{1}{2}L^2_{11}(\{c\}^1_1)\log^2\mu$. This was essentially Weinberg's argument in [9]. This reasoning works to all orders, the full argument in this form can be found in [2].

We can thus calculate LLs using only one-loop diagrams, but for each new order we need to take into account the new vertices. The main reason why it is difficult to push this to higher orders is that we also get more and more complicated diagrams at each order. This is illustrated at two-loop order in Fig. 1. At higher orders the number of diagrams increases fast. As an example we show the diagrams needed for the mass to six loops in Fig 2.

So in practice we calculate the divergence and rewrite it in terms of a local Lagrangian. Since we use dimensional regularization, we know that the results have all the symmetries present in the result, so we do not need to rewrite the Lagrangians in a nice form. This together with the fact that we do not need to rewrite the Lagrangian in a minimal form allows to automatize the process fully. The speed of FORM [18] played a major role in obtaining the results described below. A small technical comment, we have required all one-particle-irreducible diagrams to be finite, the extra counter-terms needed for this do not affect physical results.

4. $O(N+1)/O(N)$

In this section we discuss a few results from the massive $O(N)$ nonlinear sigma model. The Lagrangian is given by

$$\mathscr{L}_{n\sigma} = \frac{F^2}{2}\partial_\mu\Phi^T\partial^\mu\Phi + F^2\chi^T\Phi. \tag{4.1}$$

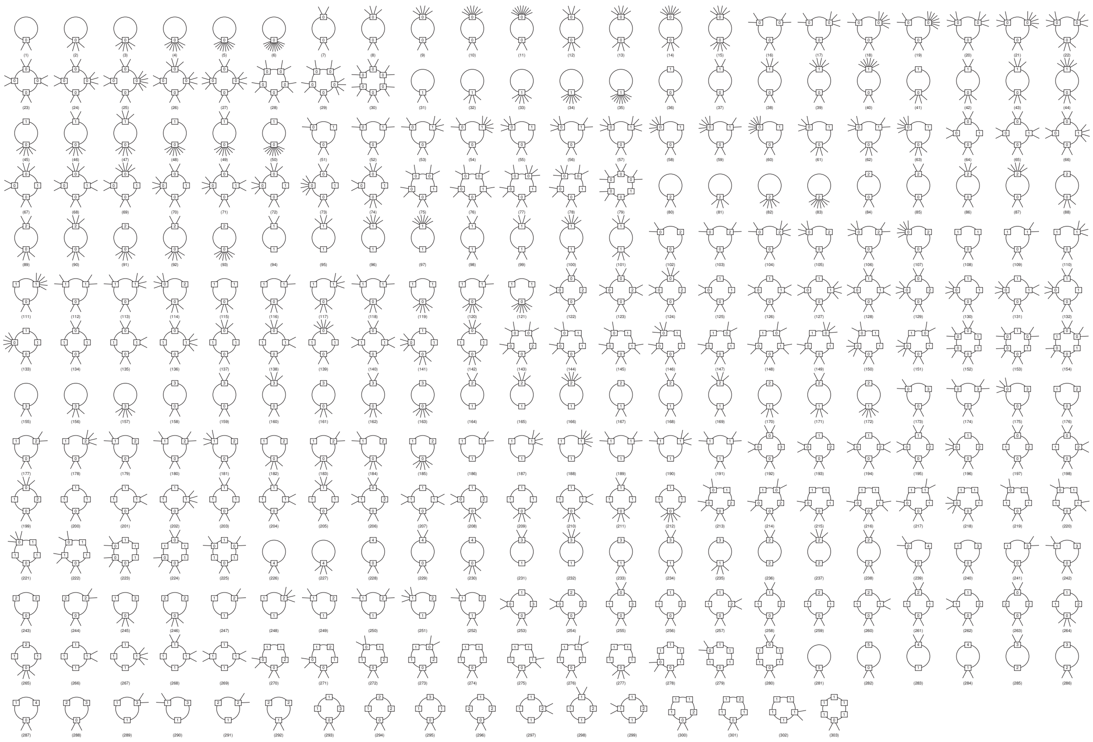

Figure 2: The number of diagrams needed for the LL for the meson mass at to six loops.

with Φ a real $N+1$ vector transforming as $\Phi \to O\Phi$ under $O(N+1)$ and $\Phi^T\Phi = 1$. We choose as vacuum expectation value $\langle \Phi^T \rangle = (1\ 0 \ldots 0)$ and the model includes explicit symmetry breaking via $\chi^T = \left(M^2\ 0 \ldots 0\right)$. It has thus both spontaneous and explicit symmetry breaking with a surviving $O(N)$ global symmetry. $N = 3$ is two-flavour Chiral Perturbation Theory. The N (pseudo-)Nambu-Goldstone bosons are described by an N-vector ϕ.

Calculations of this complexity need to be checked in as many ways as possible. One good check is to use different parametrizations of Φ in terms of ϕ. Contributions of different diagrams can be very different with different parametrizations, while physical quantities should be independent of this choice. The work in [1, 2, 3] used up to five different representations. In particular we used the Gasser-Leutwyler [10], Weinberg [19] and CCWZ [20, 21].

An example of results is the meson mass squared LLs to six loops [2, 3].

$$M^2_{phys} = M^2\left(1 + a_1 L_M + a_2 L_M^2 + \cdots\right), \quad L_M = \frac{M^2}{16\pi^2 F^2}\log\frac{\mu^2}{\mathscr{M}^2} \tag{4.2}$$

The usual choice for the physical scale in the logarithm is $\mathscr{M} = M$. The coefficients are shown in Tab. 1. One question is whether one can guess at an all-order function reproducing these coefficients. Unfortunately we did not succeed in that. Similar results for the decay constant and vacuum expectation value can be found in [1, 2, 3].

Using the LLs we can study how fast a series converges when rewritten in terms of different

i	a_i, $N=3$	a_i for general N
1	$-\frac{1}{2}$	$1-\frac{N}{2}$
2	$\frac{17}{8}$	$\frac{7}{4}-\frac{7N}{4}+\frac{5\ N^2}{8}$
3	$-\frac{103}{24}$	$\frac{37}{12}-\frac{113N}{24}+\frac{15\ N^2}{4}-N^3$
4	$\frac{24367}{1152}$	$\frac{839}{144}-\frac{1601\ N}{144}+\frac{695\ N^2}{48}-\frac{135\ N^3}{16}+\frac{231\ N^4}{128}$
5	$-\frac{8821}{144}$	$\frac{33661}{2400}-\frac{1151407\ N}{43200}+\frac{197587\ N^2}{4320}-\frac{12709\ N^3}{300}+\frac{6271\ N^4}{320}-\frac{7\ N^5}{2}$
6	$\frac{1922964667}{6220800}$	$158393809/3888000-182792131/2592000\ N$ $+1046805817/7776000\ N^2-17241967/103680\ N^3$ $+70046633/576000\ N^4-23775/512\ N^5+7293/1024\ N^6$

Table 1: The coefficients a_i defined in (4.2) for the physical mass in terms of the lowest order mass.

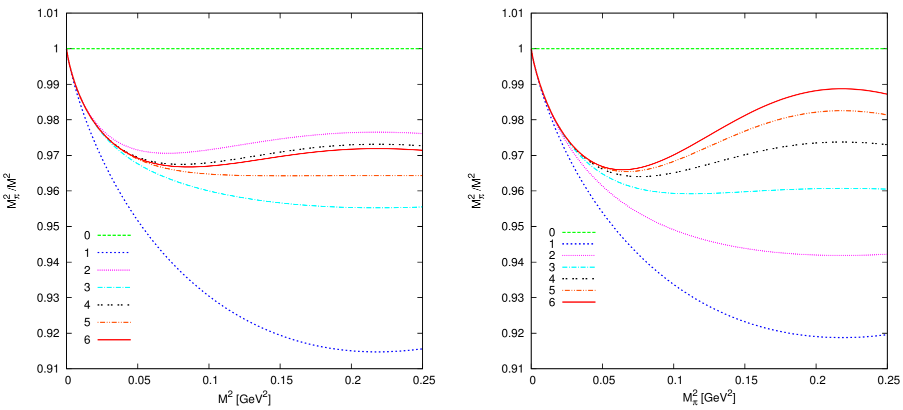

Figure 3: The expansion of the mass in terms of the lowest order or physical mass. Left: $\frac{M^2_{\text{phys}}}{M^2}=1+a_1L_M+a_2L_M^2+a_3L_M^3+\cdots$ with F = 90 MeV, $\mu=0.77$ GeV. Right: $\frac{M^2_{\text{phys}}}{M^2}=1+c_1L_{\text{phys}}+c_2L^2_{\text{phys}}+c_3L^3_{\text{phys}}+\cdots$ with F_π = 92 MeV, $\mu=0.77$ GeV.

quantities. Examples of choices are

$$L_M=\frac{M^2}{16\pi^2F^2}\log\frac{\mu^2}{M^2},\quad \tilde{L}_M=\frac{M^2_{\text{phys}}}{16\pi^2F^2}\log\frac{\mu^2}{M^2_{\text{phys}}},\quad L_{\text{phys}}=\frac{M^2_{\text{phys}}}{16\pi^2F^2_{\text{phys}}}\log\frac{\mu^2}{M^2_{\text{phys}}}. \tag{4.3}$$

For masses the expansion in $\tilde{L}_M$ worked best, but no general obvious best choice was found. How the choice affects the expansion is shown in Fig. 3 when the mass is expressed in L_M or L_{phys}. That the best choice is not universal is illustrated by the same graphs but for the decay constant show in Fig. 4. Here the apparent quality of convergence is the other way round.

As a last example for this case I show the corrections to quantities which are more dominated by the LLs. The $\pi\pi$-scattering lengths a^0_0 and a^2_0 are shown in Fig. 5. A comparison of LL versus the full two-loop results for these is in [22].

More results can be found in the papers, in particular we also discussed vector and scalar form-factors.

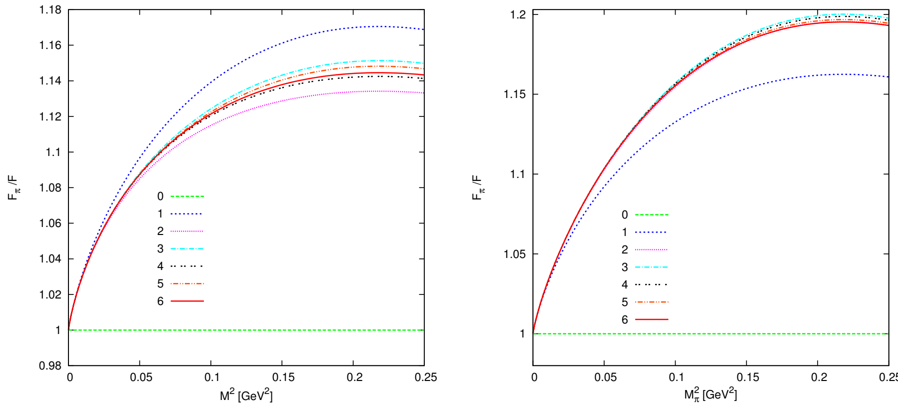

Figure 4: The expansion of the decay constant in terms of the lowest order or physical mass. Left: $\frac{F_{\text{phys}}}{F} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \cdots$ with F = 90 MeV, $\mu = 0.77$ GeV. Right: $\frac{F_{\text{phys}}}{F} = 1 + c_1 L_{\text{phys}} + c_2 L_{\text{phys}}^2 + c_3 L_{\text{phys}}^3 + \cdots$ with F_π = 92 MeV, $\mu = 0.77$ GeV.

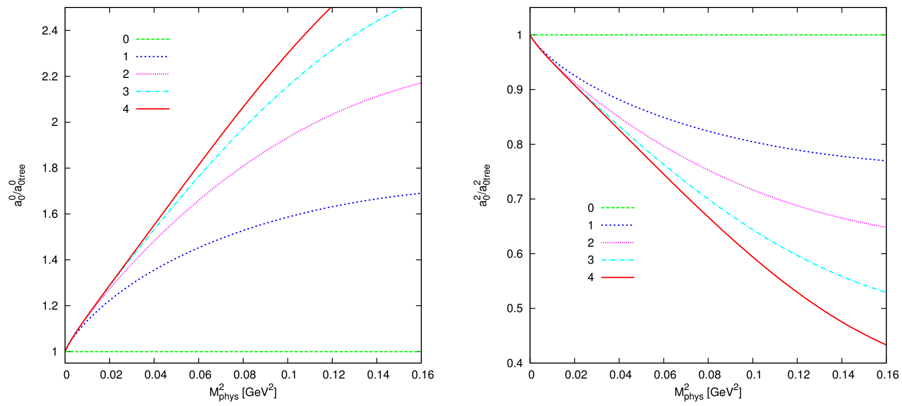

Figure 5: The expansion of the a_0^0 and a_0^2 $\pi\pi$-scattering lengths with the leading logarithms.

In the massless case tadpoles vanish and the proliferation of diagrams does not occur in theories with four- and higher meson vertices only. The reason is that the tadpoles are responsible for the proliferation. This together with a parametrization of vertices using Legendre polynomials lead to recursion relations that can be solved to very high orders for form-factors and meson-meson scattering [23, 24, 25, 26].

For the weak interactions there are also some results, for $K \to n\pi$ in [27, 28] and $K_S \to \gamma\gamma$ and $K_S \to \gamma l^+ l^-$ [29]. They calculated LL to two-loop order for those processes.

5. Anomaly for the case $O(4)/O(3)$ or $SU(2) \times SU(2)/SU(2)$

For anomalous processes we need to add the Wess-Zumino-Witten term to the Lagrangian. After that we can use the same method for calculating leading logarithms in this sector. More results and a deeper discussion can be found in [3]. For the decay $\pi^0 \to \gamma\gamma$ we find a well known zero [30, 31] for the LL at one-loop level but larger contributions at higher orders. Nevertheless, the expansion converges extremely well. Relative to lowest order the LL contributions up to six loops are

$$\frac{A(\pi^0 \to \gamma\gamma)_{LL}}{A(\pi^0 \to \gamma\gamma)_{LO}} = 1 + 0 - 0.000372 + 0.000088 + 0.000036 + 0.000009 + 0.0000002 + \dots . \quad (5.1)$$

Similarly the nonfactorizable part with both photons off-shell is very small and only starts at three-loop order and in the chiral limit only at four-loops.

As a last anomalous example, the amplitude for the $\gamma\pi \to \pi\pi$ vertex in terms of the usual form-factor converges as

$$F_0^{3\pi LL} = (9.8 - 0.3 + 0.04 + 0.02 + 0.006 + 0.001 + \dots)\ \mathrm{GeV}^{-3} \quad (5.2)$$

We found no places with bad convergence in our work on anomalies [3].

6. $SU(N) \times SU(N)/SU(N)$

The work reported in the previous sections was mainly on the $O(N+1)/O(N)$ massive non-linear sigma model. Other symmetry breakings are possible. In particular N-flavour Chiral Perturbation Theory has the symmetry breaking structure of $SU(N) \times SU(N) \to SU(N)$. The methods used above can be readily generalized to this case. In particular, the check with using different parametrizations exists as well. We used up to four different parametrizations for a unitary matrix and agreed with known results at two-loop orders for masses, decay constants, vacuum expectation values [32] and meson-meson scattering [33].

Results for the LL for these quantities up to six loops can be found in [4]. One of the hopes was to see if we could get a useful leading large N result since this is given by planar diagrams only. As an example the LL for the mass coefficients are shown in Tab. 2. Again, we did not see an all-order guess, not even for the leading N coefficients.

7. Nucleon

In the nucleon sector rather little has been done beyond one-loop. Earlier work that we are aware of are the full order p^5 calculation [34, 35] and order p^6 [36, 37] of the nucleon mass. In addition there is the order p^5 LL correction to g_A, the nucleon axial-vector coupling [38].

We use as the main underlying method the heavy baryon approach, see [39] for an early review and references. As a check we also use the relativistic formulation [40] with IR regularization [41]. The known problems with the latter regularization do not affect the LLs. The results agreed. Below we only discuss the heavy baryon method. The results of this section can be found in more detail in [5].

The lowest order Lagrangian is of order p

$$\mathscr{L}_{N\pi}^{(0)} = \bar{N}\left(iv^\mu D_\mu + g_A S^\mu u_\mu\right)N\,. \quad (7.1)$$

i	a_i for $N=2$	a_i for $N=3$	a_i for general N
1	$-1/2$	$-1/3$	$-N^{-1}$
2	$17/8$	$27/8$	$9/2N^{-2}-1/2+3/8N^2$
3	$-103/24$	$-3799/648$	$-89/3N^{-3}+19/3N^{-1}-37/24N-1/12N^3$
4	$24367/1152$	$146657/2592$	$2015/8N^{-4}-773/12N^{-2}+193/18+121/288N^2$ $+41/72N^4$
5	$-8821/144$	$-\frac{27470059}{186624}$	$-38684/15N^{-5}+6633/10N^{-3}-59303/1080N^{-1}$ $-5077/1440N-11327/4320N^3-8743/34560N^5$
6*	$\frac{1922964667}{6220800}$	$\frac{12902773163}{9331200}$	$7329919/240N^{-6}-1652293/240N^{-4}$ $-4910303/15552N^{-2}+205365409/972000$ $-69368761/7776000N^2+14222209/2592000N^4$ $+3778133/3110400N^6$

Table 2: The coefficients of the expansion of the physical mass in terms of L_M for the $SU(N)\times SU(N)/SU(N)$ case. Definitions as in (4.2).

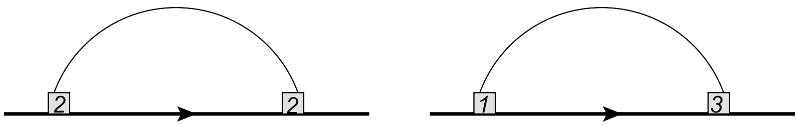

Figure 6: Example of diagrams with different RGO but the same p-order. The thick line is the nucleon, the number in the box indicate p-order of the vertex. Both diagrams are order p^5. The left diagram has RGO 1, the right RGO 2.

As for the meson sector we need checks on our calculation. We can use different parametrizations of the meson field as before but in addition there are two different choices of the p^2 action. The standard BKM [39, 42] and the EM [43] version respectively:

$$\begin{aligned}
\mathscr{L}^{(1)BKM}_{\pi N} =&\bar{N}_v\Big[\frac{(v\cdot D)^2-D\cdot D-ig_A\{S\cdot D,v\cdot u\}}{2M}+c_1\mathrm{tr}(\chi_+)+\left(c_2-\frac{g_A^2}{8M}\right)(v\cdot u)^2+c_3u\cdot u\\
&+\left(c_4+\frac{1}{4M}\right)i\varepsilon^{\mu\nu\rho\sigma}u_\mu u_\nu v_\rho S_\sigma\Big]N_v\\
\mathscr{L}^{(1)EM}_{N\pi} =&\frac{1}{M}\bar{N}\Big[-\frac{1}{2}\left(D_\mu D^\mu+ig_A\{S_\mu D^\mu,v_\nu u^\nu\}\right)+A_1\mathrm{tr}\left(u_\mu u^\mu\right)+A_2\mathrm{tr}\left((v_\mu u^\mu)^2\right)+A_3\mathrm{tr}(\chi_+)\\
&+A_5 i\varepsilon^{\mu\nu\rho\sigma}v_\mu S_\nu u_\rho u_\sigma\Big]N\,.
\end{aligned}\tag{7.2}$$

The propagator is order p but loops still add p^2 in the chiral counting just as for mesons. As a consequence p-counting and loop counting no longer coincide. We solve this by introducing $\hbar^2\sim p^{n+1}$ for nucleons and $\hbar^n\sim p^{n+2}$ for mesons and introduce the concept of renormalization group order (RGO) [5]. This is approximately the same as the maximum power of $1/w$ or $1/(d-4)$, including the parts from the counter-terms, that can show up in a given diagram. Diagrams at the same p-order can differ in RGO, an example is shown in Fig. 6. Note that a look at the equations

k_2	$-4c_1M$
k_3	$-\frac{3}{2}g_A^2$
k_4	$\frac{3}{4}\left(g_A^2+(c_2+4c_3-4c_1)M\right)-3c_1M$
k_5	$\frac{3g_A^2}{8}\left(3-16g_A^2\right)$
k_6	$-\frac{3}{4}\left(g_A^2+(c_2+4c_3-4c_1)M\right)+\frac{3}{2}c_1M$
k_7	$g_A^2\left(-18g_A^4+\frac{35g_A^2}{4}-\frac{443}{64}\right)$
k_8	$\frac{27}{8}\left(g_A^2+(c_2+4c_3-4c_1)M\right)-\frac{9}{2}c_1M$
k_9	$\frac{g_A^2}{3}\left(-116g_A^6+\frac{2537g_A^4}{20}-\frac{3569g_A^2}{24}+\frac{55609}{1280}\right)$
k_{10}	$-\frac{257}{32}\left(g_A^2+(c_2+4c_3-4c_1)M\right)+\frac{257}{32}c_1M$
k_{11}	$\frac{g_A^2}{2}\left(-95g_A^8+\frac{5187407g_A^6}{20160}-\frac{449039g_A^4}{945}+\frac{16733923g_A^2}{60480}-\frac{298785521}{1935360}\right)$

Table 3: The fully calculated coefficients in the LL and odd-power sub-leading log coefficients for the nucleon mass. The coefficients are defined in (7.3).

shows that it is possible to calculate the sub-leading logarithm if there is no contribution from tree-level diagrams at a given order. For nucleon observables where fractional powers of the quark mass can show up this will be the case. The example of the nucleon mass at odd p-orders shows this.

We use here M for the lowest order nucleon mass and m for the lowest order pion mass. Expanding in the lowest-order logarithm $L=\frac{m^2}{(4\pi F)^2}\log\frac{\mu^2}{m^2}$ we define

$$\begin{aligned}M_{\text{phys}} &= M+k_2\frac{m^2}{M}+k_3\frac{\pi m^3}{(4\pi F)^2}+k_4\frac{m^4}{(4\pi F)^2M}\ln\frac{\mu^2}{m^2}+k_5\frac{\pi m^5}{(4\pi F)^4}\ln\frac{\mu^2}{m^2}+\cdots\\ &= M+\frac{m^2}{M}\sum_{n=1}^{\infty}k_{2n}L^{n-1}+\pi m\frac{m^2}{(4\pi F)^2}\sum_{n=1}^{\infty}k_{2n+1}L^{n-1},\end{aligned} \tag{7.3}$$

The coefficients up to k_6 were known from the earlier work. We have fully calculated the coefficients up to k_{11}, these are shown in Tab. 3.

The Lagrangian is invariant under $g_A\leftrightarrow -g_A$ and flipping the sign of the meson field. So only even powers of g_A can show up. This is clearly visible in Tab. 3. The k_{2n} have an even more peculiar structure. That only one power of the p^2-Lagrangian coefficients can show up is a consequence of the RGO counting but why only maximum g_A^2 shows up is not clear to us. If we *assume* that no higher powers of g_A show up we can calculate k_{12} as well. Doing that and rewriting now the logarithms in terms of the physical pion mass leads to the simpler coefficients r_i shown in Tab. 4. Taking a look at these coefficients we *conjecture*

$$M=M_{\text{phys}}+\frac{3}{4}m_{\text{phys}}^4\frac{\log\frac{\mu^2}{m_{\text{phys}}^2}}{(4\pi F)^2}\left(\frac{g_A^2}{M_{\text{phys}}}-4c_1+c_2+4c_3\right)-\frac{3c_1}{(4\pi F)^2}\int_{m_{\text{phys}}^2}^{\mu^2}m_{\text{phys}}^4(\mu')\,\frac{d\mu'^2}{\mu'^2}. \tag{7.4}$$

We can now use the known result for the pion mass discussed before and the conjecture (7.4) to obtain k_{14} and k_{16}. Using this we have calculated the LL and the odd-power sub-leading logarithms fully to five loops and have a conjecture for the LL at 6 and 7 loops for the nucleon mass.

Numerical results for a standard set of input parameters are shown in Fig.7. The convergence

r_2	$-4c_1M$
r_3	$-\frac{3}{2}g_A^2$
r_4	$\frac{3}{4}\left(g_A^2+(c_2+4c_3-4c_1)M\right)-5c_1M$
r_5	$-6g_A^2$
r_6	$5c_1M$
r_7	$\frac{g_A^2}{4}\left(-8+5g_A^2-72g_A^4\right)$
r_8	$\frac{25}{3}c_1M$
r_9	$\frac{g_A^2}{3}\left(-116g_A^6+\frac{647g_A^4}{20}-\frac{457g_A^2}{12}+\frac{17}{40}\right)$
r_{10}	$\frac{725}{36}c_1M$
r_{11}	$\frac{g_A^2}{2}\left(95g_A^8-\frac{1679567g_A^6}{20160}+\frac{451799g_A^4}{3780}-\frac{320557g_A^2}{15120}+\frac{896467}{60480}\right)$
r_{12}	$\frac{175}{4}c_1M$

Table 4: The calculated coefficients in the LL and odd-power sub-leading log coefficients for the nucleon mass in terms of the physical pion mass.. The coefficients are defined similar to (7.3).

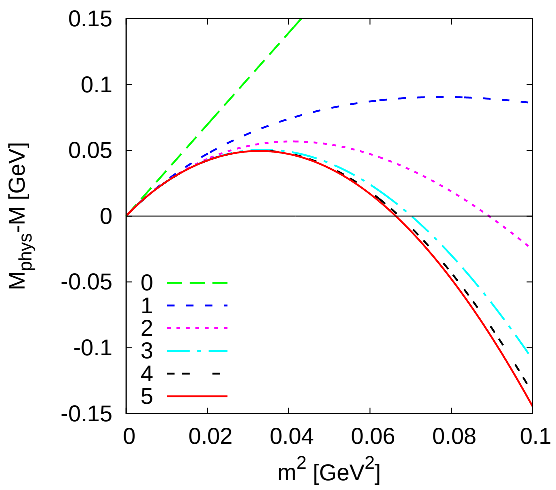

Figure 7: The corrections to the nucleon mass at a given loop-order from the LL and the odd-power sub-leading logarithm. The input parameters used are $M = 938$ MeV, $c_1 = -0.87$ GeV^{-1}, $c_2 = 3.34$ GeV^{-1}, $c_3 = -5.25$ GeV^{-1}, $g_A = 1.25$, $\mu = 0.77$ GeV and $F = 92.4$ MeV.

at the physical pion mass is very good. In Fig. 8 we show the contribution of all the terms we have obtained.

8. Conclusions

We discussed in this talk our recent work on leading logarithms in massive effective field theories. There is a rather large number of results available in the meson sector [1, 2, 3, 4]. We encourage people to have a look at the (very) many tables in those references. We welcome any all-order conjectures.

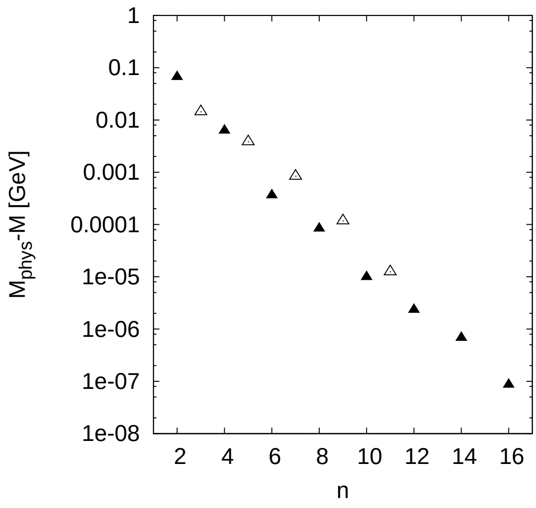

Figure 8: The individual contribution from the r_n term up $n = 16$ to the nucleon mass at the physical pion mass.

The nucleon mass we obtained as the first result using this method in the baryon sector [5]. Work is in progress for other quantities. We had a simple conjecture for the LL at all orders in terms of the pion mass LL but could not find a proof.

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