

Hadronic uncertainties and isospin violation in supersymmetric dark matter models

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Current limits from dark matter direct-detection experiments place a powerful constraint on the parameter space of the Minimal Supersymmetric Standard Model (MSSM). The interpretation of these limits, however, depends sensitively on the hadronic uncertainties associated with the scattering of supersymmetric dark matter particles off nucleons. For spin-independent scattering, we review the role of 2- and 3-flavour chiral perturbation theory in the determination of these hadronic uncertainties, and quantify the amount of isospin violation within several simplified versions of the MSSM. In each case, we identify parameter-space configurations that produce (almost) vanishing cross sections and examine the complementarity of constraints due to direct-detection, flavour, and collider experiments. In the vicinity of these so-called blind spots, we find that isospin violation may be much larger than typically expected in the MSSM.

*The 8th International Workshop on Chiral Dynamics, CD2015
29 June 2015 - 03 July 2015
Pisa, Italy*

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1. The Dark Matter Puzzle

There is nowadays a consensus of evidence from astrophysical observations [1,2] that the total matter density in the Universe $\Omega_M \approx 0.3$ is dominated by a component of non-baryonic origin. The data imply that this exotic form of matter is cold, non-luminous (i.e. “dark”), and has a present-day density of $\Omega_{DM} \approx 0.25$. Among several possible candidates, weakly interacting massive particles (WIMPs) have received considerable attention from both theorists and experimentalists.

The appeal of WIMP dark matter stems from the observation that the contribution to Ω_{DM} from a massive particle χ is determined (up to logarithmic corrections) by the $\chi\chi$ annihilation cross-section σ_{ann} into lighter particles,

$$\Omega_\chi \approx \frac{10^{-13} \text{ barn}}{\langle \sigma_{ann} v \rangle}, \quad (1.1)$$

where v is the relative velocity of the incoming χ s and $\langle \dots \rangle$ denotes a thermal average. If χ has weak-strength interactions, then $\langle \sigma_{ann} v \rangle$ is of the order of a picobarn and Ω_χ can reproduce part or all of the observed relic density Ω_{DM} . Taken at face value,¹ this so-called WIMP miracle provides a surprising connection between dark matter and the electroweak scale, where solutions to the hierarchy problem of the Standard Model (SM) are expected to give rise to new physics.

An explicit realisation of this picture occurs in the Minimal Supersymmetric SM (MSSM), where softly-broken supersymmetry stabilises the electroweak scale, while an exact R -parity ensures that the lightest superpartner (LSP) is a stable neutralino (also denoted by χ) with electroweak couplings and a mass in the range from tens to hundreds of GeV. If dark matter is composed of neutralinos, then galactic rotation curves imply there should be a considerable flux of these particles in the Milky Way. This opens up the possibility of detecting neutralinos directly using terrestrial experiments, where e.g. the elastic scattering of neutralinos off nuclei may leave measurable imprints in the recoil energy of the target. Although no conclusive signal has yet been found in such direct detection experiments, the current exclusion limits from LUX [4], as well as upcoming proposals like XENON1T [5] and LUX-ZEPLIN (LZ) [6], can lead to strong constraints on the parameter space of the MSSM. The interpretation of these limits, however, depends sensitively on the hadronic uncertainties associated with the χ -nucleon scattering cross-section.

Here we review the role of 2- and 3-flavour chiral perturbation theory in the determination of these hadronic uncertainties (Sec. 2), and quantify the amount of isospin violation within several simplified models of the MSSM (Sec. 3). This is a summary of a general analysis [7] concerning parameter space regions where the χ -nucleon scattering cross-section is strongly suppressed. A key feature of our work is that in the vicinity of these so-called blind spots [8,9], isospin-violation may be much larger than typically expected in the MSSM. Concluding remarks are given in Sec. 4.

2. Hadronic Matrix Elements and Neutralino-Nucleus Scattering

In general, the elastic scattering of the LSP off a nucleus $\mathcal{N} = \frac{A}{Z}X$ can involve spin-dependent and spin-independent (SI) interactions. Focusing on the latter, the key observable of interest is the

¹There are well known cases [3] where the estimate in (1.1) no longer applies.

χ - \mathcal{N} scattering cross-section at zero momentum transfer

$$\sigma_{\text{SI}} = \frac{4\mu_\chi^2}{\pi} [Zf_p + (A-Z)f_n]^2, \quad (2.1)$$

where $\mu_\chi = m_\chi m_{\mathcal{N}} / (m_\chi + m_{\mathcal{N}})$ is the reduced mass of the χ - \mathcal{N} system, and f_p and f_n are effective SI couplings of the LSP to the proton and neutron respectively. For nucleons N , the χ - N couplings f_N are defined by

$$\frac{f_N}{m_N} = \sum_{q=u,d,s} f_q^N C_q + f_Q^N \sum_{q=c,b,t} C_q, \quad (2.2)$$

where C_q is the Wilson coefficient of the scalar operator $m_q \bar{\chi} \chi \bar{q} q$ while

$$m_N f_q^N = \langle N | m_q \bar{q} q | N \rangle \quad \text{and} \quad f_Q^N = \frac{2}{27} (1 - f_u^N - f_d^N - f_s^N). \quad (2.3)$$

Traditionally [10–12, 30], the scalar matrix elements of the light quarks (2.3) have been determined by applying the Feynman-Hellmann theorem

$$\langle N | m_q \bar{q} q | N \rangle = m_q \frac{\partial m_N}{\partial m_q} \quad (2.4)$$

to the chiral $SU(3)_L \times SU(3)_R$ expansion of the nucleon mass

$$m_N = m_0 + \sum_{\phi=\pi,K,\eta} c_\phi m_\phi^2 + O(m_{\pi,K,\eta}^3), \quad (2.5)$$

where m_0 is the chiral-limiting value of m_N and c_ϕ is a linear combination of low-energy constants. Then the couplings f_q^N can be expressed in terms of the pion-nucleon sigma-term

$$\sigma_{\pi N} = \langle N | \bar{m} (\bar{u}u + \bar{d}d) | N \rangle, \quad \bar{m} = \frac{1}{2} (m_u + m_d), \quad (2.6)$$

the strangeness content of the nucleon

$$y = \frac{2 \langle N | \bar{s}s | N \rangle}{\langle N | \bar{u}u + \bar{d}d | N \rangle} \quad (2.7)$$

and another parameter

$$z = \frac{\langle N | \bar{u}u - \bar{s}s | N \rangle}{\langle N | \bar{d}d - \bar{s}s | N \rangle} \quad (2.8)$$

that is related to isospin violation. For some time, it has been known that obtaining reliable uncertainty estimates in this framework is problematic because:

1. The up- and down-quark coefficients $f_{u,d}^N$ depend explicitly on the 3-flavour quantities y and z . As a result, the order 30% uncertainties associated with chiral $SU(3)_L \times SU(3)_R$ perturbation theory χPT_3 propagate into the 2-flavour sector;
2. The strange-quark scalar matrix element is defined as

$$f_s^N = \frac{\sigma_{\pi N}}{2m_N} \frac{m_s}{\bar{m}} y, \quad (2.9)$$

where $m_s/\bar{m} = (27.4 \pm 0.4)$ [14], the strangeness content is taken from the relation $y = 1 - \sigma_0/\sigma_{\pi N}$, with $\sigma_0 = (36 \pm 7)$ MeV [15], and $z \simeq 1.49$ is extracted from *leading-order* fits to the baryon mass spectrum [16]. Within the range $\sigma_{\pi N} = 50 \pm 15$ MeV covered in [17–22], large values for the strangeness content of the nucleon² and the corresponding coupling $f_s^N = 0.2 \pm 0.2$ have been inferred. Although still employed in the literature (see e.g. [24,25]), such large values are incompatible with modern lattice QCD calculations, which yield a much smaller and more reliable determination of f_s^N [26]:

$$f_s^N \Big|_{\text{lattice}} = 0.043 \pm 0.011. \quad (2.10)$$

Evidently, the choice of input for f_s^N can have a dramatic effect on the interpretation of experimental limits on σ_{SI} . For example, it has been shown [27] that lattice input for f_s^N can *weaken* bounds on the constrained MSSM parameter space by factors of 5-10 (!) relative to those obtained in the traditional χPT_3 framework.

In Ref. [28], it was demonstrated that problem 1 can be circumvented by using the 2-flavour theory χPT_2 directly, thus avoiding the 3-flavour expansion in the first place. (Problem 2 is avoided by adopting lattice input (2.10) for f_s^N .) Starting from the χPT_2 expansion of the nucleon mass at $O(m_\pi^3)$ and including the effects due to strong isospin violation, one finds [28]

$$\begin{aligned} f_u^N &= \frac{\sigma_{\pi N}(1 - \xi)}{2m_N} + \Delta f_u^N, & f_d^N &= \frac{\sigma_{\pi N}(1 + \xi)}{2m_N} + \Delta f_d^N, \\ \Delta f_u^p &= (1.0 \pm 0.2) \times 10^{-3}, & \Delta f_u^n &= (-1.0 \pm 0.2) \times 10^{-3}, \\ \Delta f_d^p &= (-2.1 \pm 0.4) \times 10^{-3}, & \Delta f_d^n &= (2.0 \pm 0.4) \times 10^{-3}, \end{aligned} \quad (2.11)$$

where

$$\xi = \frac{m_d - m_u}{m_d + m_u} = 0.36 \pm 0.04 \quad (2.12)$$

is taken from [14]. A key feature of this approach is that isospin violation can be systematically accounted for, so it becomes natural to ask whether there exist regions in the MSSM parameter space where such effects may be important. In this context, we observe that the cross section (2.1) may be rewritten as³

$$\sigma_{\text{SI}} = \frac{4\mu_\chi^2}{\pi} f_p^2 \left[A + (A - Z) \left(\frac{f_n}{f_p} - 1 \right) \right]^2, \quad (2.13)$$

so that the departure of f_n/f_p from unity emerges as a convenient measure of isospin violation.

3. Blind Spots and Isospin Violation

Virtually nothing is known about how supersymmetry is broken at the weak-scale, so in general, the MSSM contains more than 100 parameters whose values are to be constrained by experiment. Given the huge parameter space, it is necessary to make additional, simplifying assumptions

²Although see [23] for important higher-order corrections to σ_0 which can reduce the value of y , albeit with large uncertainties.

³Corrections to the single-nucleon picture underlying (2.1) in the form of two-nucleon currents can be systematically taken into account using effective field theory [29–32].

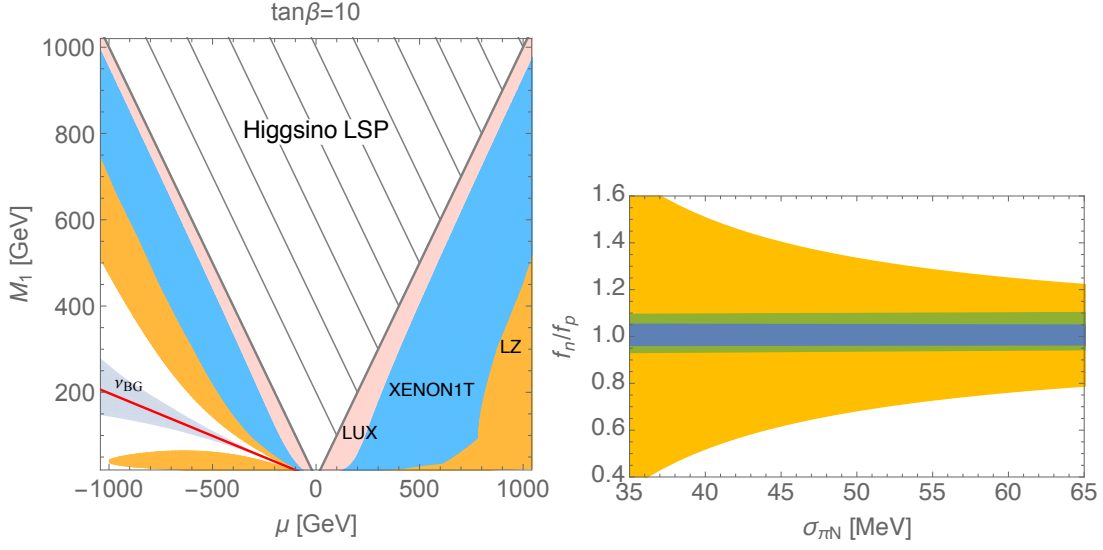


Figure 1: *Left:* Current and projected limits on SI χ -xenon scattering due to h exchange with $\tan\beta = 10$. The pink band shows the existing constraints from LUX [4], while projected limits from XENON1T [5] and LZ [6] are given by the blue and orange regions respectively. The blind spot where the SI cross section vanishes is denoted by the red line and lies within the irreducible neutrino background (v_{BG}) shown in grey. *Right:* Amount of isospin violation in terms of f_n/f_p . The coloured bands correspond to the 1σ uncertainties associated with the different determinations of the scalar matrix elements f_q^N discussed in the text.

in order to undertake phenomenological analyses. In our work [7], we have examined several simplified models where all but a few superpartners are decoupled from the spectrum. Not only does this allow us to focus on dark matter signals of interest, it also allows us to develop an analytical framework with which to explore the underlying parameter space. Here we consider the effects of isospin violation in two simplified models [8,9] where χ -quark scattering is mediated by Higgs exchange in the t -channel: for an analysis of scenarios involving the exchange of light 3rd generation squarks, we refer the reader to [7].

In these models, the lightest neutralino χ is an admixture of bino, wino, and Higgsino interaction eigenstates, obtained by diagonalising the mass matrix

$$\begin{pmatrix} M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u \\ 0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u \\ -\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & -\mu \\ \frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\mu & 0 \end{pmatrix}. \quad (3.1)$$

Here M_1 (M_2) are the soft supersymmetry-breaking masses of the bino (wino), μ is the Higgsino mass parameter, and $v_{u,d}$ are the two Higgs $H_{u,d}$ vacuum expectation values, whose ratio v_u/v_d is denoted by $\tan\beta$. Let us now consider the two simplified models in turn.

Model 1: The simplest possible dark matter model in the MSSM consists of a light Higgs h with properties consistent with the SM, and a neutralino LSP which we assume to be predominantly bino-like, i.e. where $M_1 \ll M_2, \mu$. In this case, the corresponding Wilson coefficients are

$$C_{u_i}^h = C_{d_i}^h = \frac{g_1^2}{4m_h^2} \frac{1}{M_1^2 - \mu^2} (M_1 + \mu \sin 2\beta), \quad u_i = u, c, t, \quad d_i = d, s, b, \quad (3.2)$$

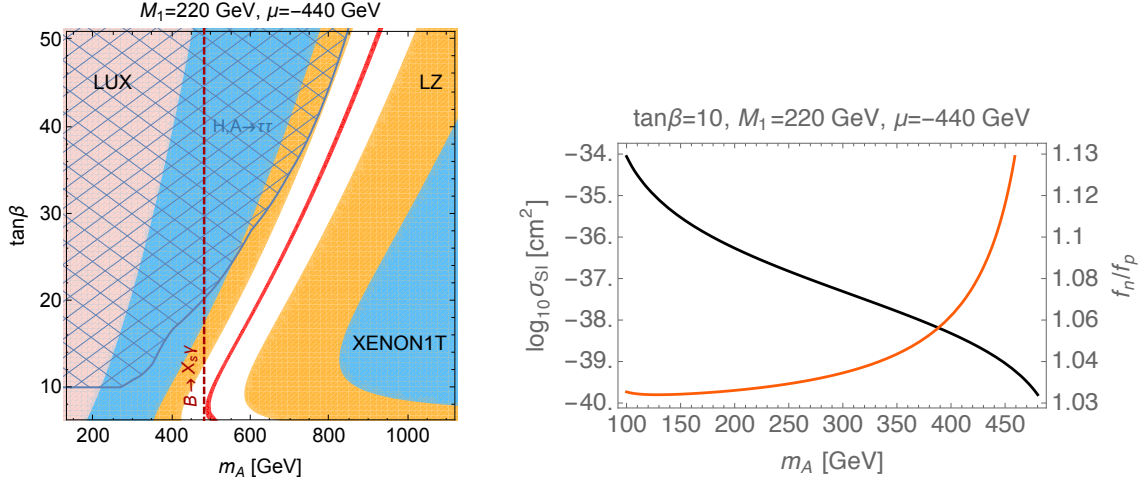


Figure 2: *Left:* Current and projected limits on SI χ -xenon scattering due to h, H exchange with different benchmark values for M_1 and μ . Excluded regions and the blind spot are colour-coded as in Fig. 1, with the cross-hatched region in dark-blue corresponding to CMS limits [33] on $H, A \rightarrow \tau^+ \tau^-$. The region to the left of the dark-red dashed line at $m_A \simeq m_{H^+} \simeq 480$ GeV is excluded by $B \rightarrow X_s \gamma$ [34]. *Right:* Pseudoscalar Higgs mass m_A dependence of the SI χ -xenon cross section (black) and the central value of f_n/f_p (red) as determined by χPT_2 method.

so we conclude that (a) for μ negative, the SI amplitude vanishes in the parameter space regions where $M_1 + \mu \sin 2\beta = 0$ [8]; (b) the C_{q_i} are independent of quark flavour, so isospin violation is determined entirely by hadronic quantities

$$\frac{f_n}{f_p} = \left(\frac{m_n}{m_p} \right) \frac{2 + 7 \sum f_q^n}{2 + 7 \sum f_q^p}. \quad (3.3)$$

Note in particular that f_s^N dominates the sums in (3.3) and can produce the largest source of uncertainty on f_n/f_p . In the left plot of Fig. 1, we show the prospects for constraining the blind spot with current and – assuming no signal is seen – future direct detection experiments. In the right plot, we display the 1σ uncertainties on f_n/f_p as a function of $\sigma_{\pi N}$. Here we have compared three approaches to determine the scalar matrix elements:

- (1) the χPT_2 method (blue) proposed in [28], with f_s^N taken from the lattice (2.10);
- (2) the traditional χPT_3 method [10–12, 30] (orange) where f_s^N is typically large;
- (3) use of the χPT_3 formalism, but with lattice input for f_s^N (green).

As seen in the Figure, the uncertainties associated with the traditional χPT_3 method are large and display a strong sensitivity to $\sigma_{\pi N}$. In contrast, the χPT_2 method is largely independent of $\sigma_{\pi N}$ and yields uncertainties which are *smaller* by factors of ≈ 6 or more. In this case, the 1σ uncertainties on f_n/f_p are at the 5% level, so the naive expectation that isospin violation is a small effect in the MSSM is borne out explicitly.

Model 2: A more interesting example arises when the spectrum is extended to include the heavier Higgs states H, A, H^\pm of the MSSM. In this case, destructive interference between the h

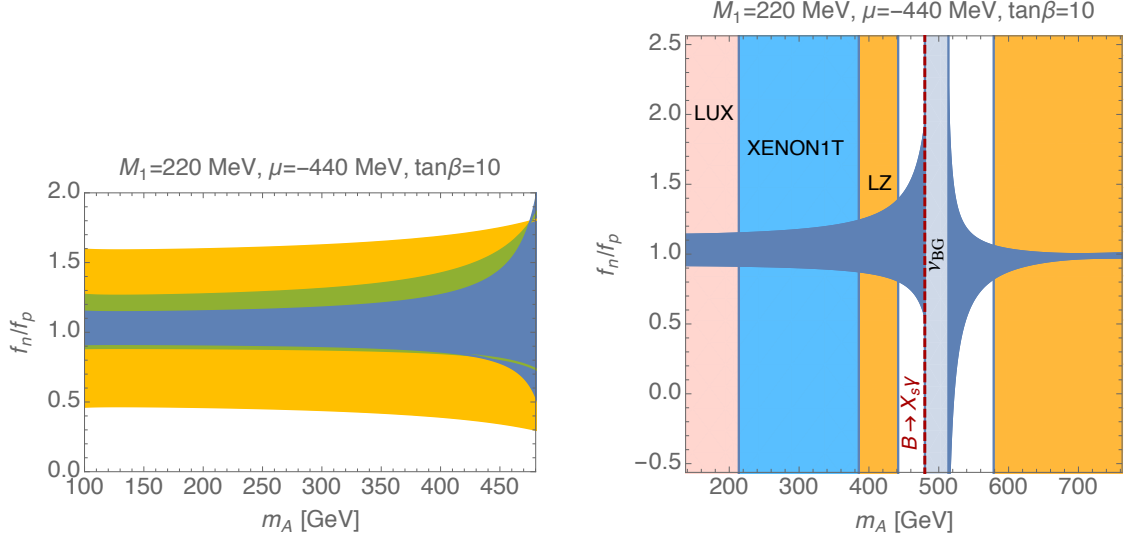


Figure 3: *Left:* Amount of isospin violation in terms of f_n/f_p due to h, H exchange in χ -xenon scattering. The shaded regions show the uncertainty on f_n/f_p due to each determination of the scalar matrix elements discussed in the text. Colour coding as in Fig. 1.

and H amplitudes gives rise to a generalised blind spot [9]

$$\frac{2}{m_h^2}(M_1 + \mu s_{2\beta}) + \mu \tan \beta \frac{1}{m_H^2} \simeq 0, \quad (3.4)$$

for $\tan \beta$ large and $m_A > m_h$. As a result of the extended Higgs sector, a complementarity of constraints from collider, flavour and direct detection arises (left plot of Fig. 2). In particular, increased sensitivity in future LHC searches for the decay $A, H \rightarrow \tau^+ \tau^-$ will be able to cover the blind spot region (3.4), where direct detection limits are necessarily weak.

Another interesting feature of (3.4) is that destructive interference produces a central value for f_n/f_p which becomes large as the blind spot is approached (right plot of Fig. 2). This is reflected in the corresponding uncertainties shown in Fig. 3 (left plot), and highlights the need for complementary constraints from e.g. flavour observables (right plot).

4. Summary

An accurate evaluation of nucleon scalar matrix elements and their uncertainties remains an important ingredient in the interpretation of dark matter direct detection limits. For neutralino dark matter, it is well known [27] that the input value for f_s^N can have a dramatic effect on excluded regions of the MSSM parameter space. Here we have highlighted another area where control over hadronic uncertainties is particularly important, namely, isospin violation in the vicinity of blind spots [8, 9]. If signals of weak-scale supersymmetry remain persistently absent at the LHC and direct detection experiments, reducing these uncertainties will become particularly relevant as the MSSM is cornered in these regions of parameter space.

5. Acknowledgments

LCT thanks the organisers and Hadron Structure & Meson-Baryon working group of Chiral Dynamics 2015 for the opportunity to present this work. Financial support from the Swiss National Science Foundation (SNF) is gratefully acknowledged. AC is supported by a Marie Curie Intra-European Fellowship of the European Community's 7th Framework Programme under contract number (PIEF-GA-2012-326948).

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