

Chiral nuclear forces up to N⁴LO

Hermann Krebs*[†]

Ruhr-Universität Bochum

Institut für Theoretische Physik II

E-mail: hermann.krebs@rub.de

Three-nucleon forces play very important role in few and many-body simulations of nuclei/nuclear reactions at low energy. Knowledge of their precise form might lead to resolution of long standing puzzles in few-nucleon physics (e.g. A_y -puzzle in elastic nucleon-deuteron scattering). Chiral effective field theory provides a systematically improvable tool for their calculation. By now three-nucleon forces have been calculated up to N³LO (partly up to N⁴LO) in chiral expansion. In this proceeding I will discuss the current status of their construction and their ongoing implementation in few-body calculations.

*The 8th International Workshop on Chiral Dynamics, CD2015 ****

29 June 2015 - 03 July 2015

Pisa, Italy

*Speaker.

[†]I would like to express my thanks to my collaborators Evgeny Epelbaum and Ulf-G. Meißner and all members of LENPIC collaboration for sharing their insight on the discussed topics. I also thank the organizers for the invitation and for making this exciting workshop possible. This work was supported in part by the ERC project 259218 NUCLEAREFT and by DFG (SFB/TR 16, "Subnuclear Structure of Matter").

1. Introduction

In the low energy sector, where momenta of hadrons are much smaller than the chiral symmetry breaking scale of the order $\Lambda_\chi \sim 1 \text{ GeV}$, chiral effective field theory (EFT) proved to be a powerful tool for description of various hadronic scattering processes and nuclear spectra. Based on an assumption that approximate chiral symmetry of QCD is spontaneously broken and on the experimental evidence of the mass gap between the masses of the pions (interpreted as Goldstone bosons in the chiral limit) and heavier mesons (like ρ , ω etc.) it is possible to formulate effective field theory of QCD where the expansion parameter is given by small momenta and masses of light mesons divided by the hard scale Λ_χ . Rather than using quarks and gluon degrees of freedom (dof) it is much more efficient to build a low energy EFT of QCD with hadronic degrees of freedom, the only observed dof in the low energy sector. This theory, called chiral perturbation theory (χ PT), was successfully applied in the pure meson and meson-nucleon sector (see [1] for a review). In the two- and more-nucleon sector the existence of bound states does not allow one to use the purely perturbative approach of χ PT. Nevertheless, a perturbative approach can be applied to the effective potential (nuclear forces) which, strictly speaking, can not be uniquely determined from matching to the nuclear observables. The reason for its non-uniqueness is a non-unique off-shell behavior of the nucleons within a certain nuclear process. However, once calculated in χ PT and sandwiched in the Lippmann-Schwinger equation (or its generalization in more than two-nucleon sector) one can numerically compute physical observables which, by definition, do not depend on off-shell information of the nuclear force. The possibility of such a perturbative access to the nuclear forces was first mentioned in pioneering papers of Steven Weinberg [2, 3, 4]. Based on Weinberg's ideas chiral nuclear forces have been calculated up to fifth [5, 6, 7] (partly even up to sixth [8]) order in the chiral expansion within the last two decades (see [9, 10] for reviews).

Chiral EFT approach has its advantages and disadvantages: One weakness of the chiral EFT approach is a fast increase of low-energy constants (LEC) with increasing chiral order. The values of these LEC's are not constrained by chiral symmetry and are fitting parameters in practical calculations. This might lead to decrease of predictive power of the theory. However, due to abundance of experimental data in the two- and three-nucleon sectors all the LEC's which appear in nuclear forces can be fitted to the data and there is a lot of room left for prediction in the three- and more-nucleon sector. Another obvious disadvantage is inherited from perturbative treatment of field theory. The convergence radius of χ PT is a priori unknown. We can only hope (without any proof) that finite-order χ PT calculations converge rapidly enough to final result. Keeping in mind all this we would like also to mention various advantages of chiral approach. The most prominent one is a direct connection to QCD. Since we are dealing with the most general EFT which is constructed inline with symmetries of QCD we can expect that LEC's (at least in meson- and single-nucleon sector where perturbative renormalization of χ PT is possible) can in principle be calculated in the future from lattice QCD. This would obviously increase the predictive power of χ PT. More practical advantage of χ PT is coming from the constraints of chiral symmetry. The processes like pion-pion, pion-nucleon, nucleon-nucleon scattering are not only calculated from one and the same chiral Lagrangian but are also related to each other. LEC's which appear in pion-nucleon scattering e.g. show up in the nucleon-nucleon scattering potential where pion-nucleon amplitude (in an unphysical kinematic region) is a subprocess. The unique relation between all

these processes is directly dictated by the Ward-Takahashi identities of QCD and shows up naturally in the χ PT approach. Due to the power counting, the χ PT approach naturally explains the hierarchy of the nuclear forces: two-nucleon forces start to show up at leading order. In contrast to this, three-nucleon forces start to show up first at next-to-next-to-leading (N^2 LO) and four-nucleon forces start to contribute at next-to-next-to-next-to-leading (N^3 LO). For this reason we have a hierarchy of the forces

$$V_{2N} \gg V_{3N} \gg V_{4N}, \quad (1.1)$$

where V_{2N} , V_{3N} and V_{4N} denote two-, three- and four-nucleon forces, respectively. Another important strength of the χ PT approach is its predictive power of the long range behavior of nuclear forces. Nuclear forces can be decomposed into local and non-local contributions. Non-local one comes from contact interactions and relativistic corrections. The local one (which means that they depend only on relative momenta) comes from pion exchange topologies and contact interactions. When we transform the relative momenta of the local part of the force in coordinate space we can decompose the local part of the nuclear forces into long-range part (relative distance r much larger than Compton wave length of the pion $r \gg \lambda_\pi \sim 1.4$ fm), intermediate-range ($r \sim \lambda_\pi$) and short-range part ($r \ll \lambda_\pi$). The short-range part of the nuclear force is parametrized by LEC's and depends on a regularization scheme adapted in order to regularize the Lippmann-Schwinger equation. The long- (and intermediate-)range part of the force is scheme independent and is entirely predicted by χ PT and so can be tested by confronting various experimental data for a given nuclear process. Nowadays, chiral nuclear forces are known very precisely due to quite high order of their χ PT calculation. This allows one doing precision physics with light nuclei in the low-energy sector.

In the following, I will give a status report on the construction of the nuclear forces and on their partial wave decomposition which is essential for their implementations in Faddeev equations and many-body simulations like no core shell model or coupled cluster approaches. In section 2 I will briefly discuss the novelties in nucleon-nucleon sector. More extensive discussion can be found in the proceeding of Epelbaum and in original publications [5, 6, 7]. In section 3 I will report on the status of the construction of the three-nucleon forces and will describe how partial wave decomposition can be performed in a very efficient way for local forces.

2. Novelties in chiral nucleon-nucleon forces

The long- and intermediate-range contributions to the chiral NN force at N^3 LO have been derived by Norbert Kaiser [11, 12, 13, 14] more than ten years ago. Briefly after these publications the fits of the LEC's have been performed by two groups: Entem and Machleidt (EM) [15] and by Epelbaum, Glöckle and Meißner (EGM) [16]. These efforts in combination with N^2 LO three-nucleon forces lead to very fruitful applications in the three- and more-nucleon sector [10, 17, 18]. Both groups EM and EGM used a non-local version of the regulator. EGM estimated the theoretical uncertainty of calculation by variation of the cut-off in some reasonable region (between 400 and 700 MeV). Further increase of the cutoff would lead to unphysical deeply bound states which would make many-body simulations with these forces impossible. On top of the non-local regularization EGM used spectral function regularization [19] in order to cutoff a too strong attraction of two-pion

exchange contributions at $N^2\text{LO}$. This lead to the introduction of an additional spectral function cut-off. Both cutoffs have been varied independently from each other to give a theoretical error estimate of the calculations. EM used different non-local cutoffs for different partial waves. In this way additional tuning parameters have been introduced in the fit to NN scattering data.

Last year, the last building block (subleading two-pion-exchange contributions at two-loop level) of the $N^4\text{LO}$ NN force has been constructed independently by Entem et al. [7] and our group [5]. Beside increasing of one chiral order, two further improvements have been done in [5] compared with the earlier work of EGM [16]: Nonlocal cutoff regularization cuts off high momentum modes which is not completely equivalent to a regularization of the short-range part. To regularize the short range part, one can use semi-local regulator. The local part of the nuclear force is regularized in coordinate space with the local cutoff and short-range part is regularized with a non-local cutoff in momentum space. This procedure is preferable since semi-local regulator does not affect analytic structure of the amplitude by construction. With this regulator one can fit LEC's to the NN phase shifts by using the value of $c_3 = -4.69\text{GeV}^{-1}$ (which is fixed from pion-nucleon scattering inside the Mandelstam triangle [20]) without having spurious deeply-bound states in the NN spectrum. In EGM, even with additional spectral function regularization, a reduced value of $c_3 = -3.40\text{GeV}^{-1}$ has been used. The use of $c_3 = -4.69\text{GeV}^{-1}$ would directly lead to the appearance of spurious deeply bound states at $N^2\text{LO}$. The second improvement is in the use of uncertainty quantification. The theoretical error quantification based on the variation of the cutoff has some disadvantages: The region where one performs cutoff variation is chosen in a somewhat arbitrary way. From the practical point of view, one does not want to choose the cutoff too low in order not to affect low-energy physics. On the other hand one can not choose cutoff too high due to the appearance of spurious deeply bound states. There is obviously a reflection of some arbitrariness in this choice. Additionally to this issue it is important to mention that new NN LEC's show up only at even chiral orders in NN force. The appearing NN LEC's are responsible for compensating the cutoff dependence. Using cutoff variation as error estimate would lead to no decrease of the theoretical error by going from an even order to next (odd) order in chiral expansion. Proper error quantification should lead to decrease of theoretical order at every chiral order. The idea is to quantify the theoretical error of the given calculated observable at a fixed cutoff. The theoretical error can be estimated from neglected higher order corrections. The method can be easily explained on the following simplified example. Assume we have an observable $X(Q)$ (where Q is an expansion parameter) which is calculated up to certain order n in chiral expansion and which is here oversimplified by a Taylor series:

$$X(Q) = \sum_{i=0}^n a_i Q^i + \mathcal{O}(Q^{n+1}). \quad (2.1)$$

The truncation error can be estimated via

$$\Delta_X^n = \max(|a_0|, \dots, |a_n|) Q^{n+1}. \quad (2.2)$$

In this estimate we hope that the largest LEC already appeared in the performed calculation and the hope is that at higher orders no larger LEC's will appear. If this nevertheless happens, this will flow into the error estimate of the next order. From statistical considerations this approach corresponds to consistent quantitative predictions for 68% degree of belief intervals [21]. In the

true chiral expansion we do not have a simple Taylor series but also non-polynomial peaces like e.g. chiral logarithms. Eq. 2.2 can be replaced in this case by

$$\Delta_X^n = \max(Q^{n+1}|X_0|, Q^n|X_2 - X_0|, Q^{n-1}|X_3 - X_2|, \dots, Q|X_n - X_{n-1}|),^1 \quad (2.3)$$

where X_i denotes an approximation of $X(Q)$ to the chiral order i [6]. Indeed Eq. 2.3 is equivalent to Eq. 2.2 in the case of oversimplified example 2.1:

$$X_0 = a_0, \quad X_2 = a_0 + a_2 Q^2, \quad X_3 = a_0 + a_2 Q^2 + a_3 Q^3, \quad \dots \quad (2.4)$$

Obviously this error estimate can be performed for every fixed cutoff and leads to a decrease of the theoretical error at every chiral order. It even allows one to identify the optimal cutoff: the cutoff for which we get the smallest theoretical error. An illustration of this method is shown in Fig. 1 for the total cross section of neutron-proton scattering at four different laboratory energies for five different cutoffs R_i [5]. One can clearly see the decrease of theoretical error at every chiral order. The error bands overlap in most of the cases and we can see that $R_2 = 0.9\text{fm}$ and $R_3 = 1.0\text{fm}$ are preferable cutoffs for which we get the smallest error bars. Chiral error quantification method can be applied to any observable. As an application of this method we looked at the nucleon deuteron (Nd) scattering and at the spectra of light nuclei with only chiral NN force implemented up to fifth order in chiral expansion. Here I restrict my discussion only to the total cross section of Nd scattering. More extensive discussion (in particular of the spectrum and radii of light nuclei) can be found in [23]. In Fig. 2 we show chiral expansion of Nd total cross section for a fixed cutoff $R = 1.0\text{fm}$. For every chiral order, we also show the theoretical error due to the chiral uncertainty quantification. Comparison with experimental results shows clearly the missing contribution of the three-nucleon forces. Obviously, this was known long before. In [23] however, the statement is sharpened due to the given chiral error estimate: The size of the missing three-nucleon forces appears as large as the error bars of the Q^2 order. Indeed the three-nucleon forces start to contribute at the third order in chiral expansion. So, their expected size agrees with the one estimated by the adopted chiral error quantification.

3. Chiral three-nucleon forces

Chiral three-nucleon forces (3NF) start to contribute at $N^2\text{LO}$. More than twenty years ago they have been worked out by Ordonez and van Kolck [25] see also [26, 27]. The first complete analysis of Nd scattering up to $N^2\text{LO}$ was performed by Epelbaum et al. [28]. In later works the leading $N^2\text{LO}$ 3NF's were combined with the $N^3\text{LO}$ NN forces. Three- and more-nucleon sector has been extensively studied with these forces with fairly good agreement with experimental data, see e.g. [18, 17, 10] for recent reviews. Some deficiencies still remain: in the three-nucleon sector the most prominent one are A_y -puzzle in Nd scattering and the discrepancy for the Nd break up reaction in the space star configuration. It will be interesting to see if the remaining discrepancies will be resolved by further corrections to 3NF at $N^3\text{LO}$ or $N^4\text{LO}$.

¹Note that there is no X_1 since there are no contributions at the order Q to the (parity conserving) chiral nuclear forces.

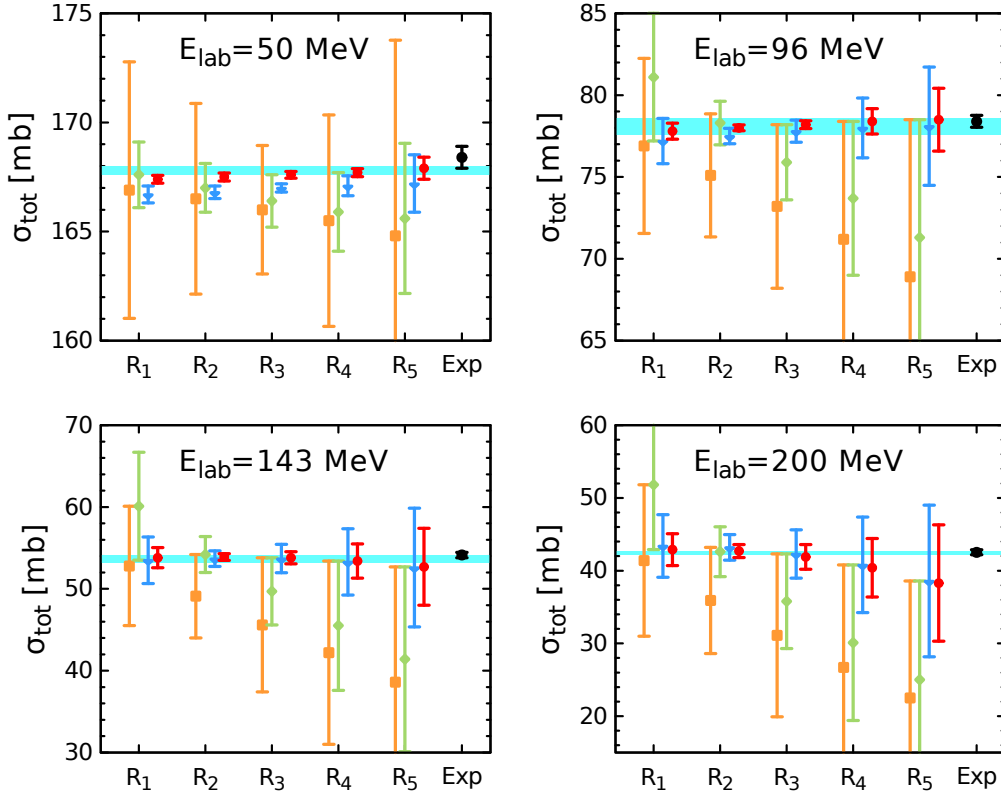


Figure 1: Predictions for the np total cross section based on the improved chiral NN potentials at NLO (filled squares, color online: orange), N²LO (solid diamonds, color online: green), N³LO (filled triangles, color online: blue) and N⁴LO (filled circles, color online: red) at the laboratory energies of 50, 96, 143 and 200 MeV for the different choices of the cutoff: $R_1 = 0.8$ fm, $R_2 = 0.9$ fm, $R_3 = 1.0$ fm, $R_4 = 1.1$ fm and $R_5 = 1.2$ fm. The horizontal band refers to the result of the Nijmegen partial wave analysis with the uncertainty estimated as explained in [5]. Also shown are experimental data of Ref. [22].

The construction of 3NF up to N³LO has been completed in 2011 by Bernard et al. [29, 30]² see also the work by Ishikawa and Robilotta [31] for two-pion-exchange part of N³LO 3NF. It took three more years until they could be implemented numerically in the old non-local cutoff scheme [32]. These results, however, should be considered as an intermediate step. Our current understanding is that the forces in this work were not properly regularized which lead to unnaturally large LECs C_D and C_E from N²LO 3NF's. At the moment we are working on the numerical implementations of the N³LO 3NF's within the novel semi-local regularization scheme with new chiral NN forces. The expectation is that in this scheme, C_D and C_E terms will be of a natural size and all regularization issues will be under control.

For numerical implementations of the 3NF's one has to perform their partial wave decomposition (PWD). The reason for this is that Faddeev equation is solved in the partial wave basis. Also for NCSM simulations one needs partial wave decomposed input. This is a non-trivial task due to

²The long range part of 3NF's has been worked out even upto N⁴LO [33, 34]. Their short and intermediate range parts are still under construction.

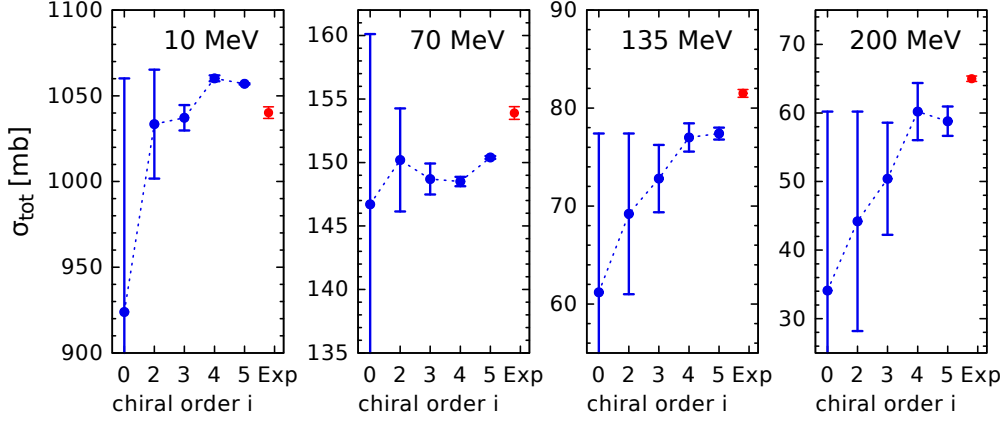


Figure 2: (Color online) Predictions for Nd total cross section based on the NN potentials of Refs. [6, 5] for $R = 1.0$ fm without including the three-nucleon forces. Error bars correspond to the chiral error quantification defined in [23]. Experimental data are from Ref. [24].

the large three-nucleon phase space. Due to abundance of channels one has to calculate a $10^5 \times 10^5$ matrix. Each matrix element involves for the most general 3NF a five dimensional numerical integration over the angles of Jacobi momenta. In this formulation, 3NF PWD is very expensive computationally. There is, however, a way how to perform 3NF PWD in a more efficient way. The most complicated part of the chiral 3NF is local. Non-localities arise from contact interactions and relativistic corrections and have a polynomial form³. The idea is to use the locality feature of a part of the 3NF and reduce the number of numerical angular integrations from five to three. Due to a polynomial form of the non-local parts of chiral 3NF, we can put the polynomial information into spherical harmonics over Jacobi momenta which are anyway present in the calculation. In this way, the same method can be applied to the entire chiral 3NF. The implementation of this idea is documented in Hebeler et al. [35]. In many cases, the calculations are 1000 times faster than in the original five-dimensional formulation. In this way we could perform 3NF PWD and store unregularized $N^3\text{LO}$ 3NF matrix elements for a large set of angular momenta which is sufficient for studies of light and intermediate mass nuclei.

The last technical milestone is a semi-local regularization of the chiral 3NF. We would like to adopt the same regularization for 3NF's as in NN case: The local part which includes pion physics should be regularized in coordinate space and the short range part should be regularized with the standard non-local regulator in momentum space. In this way the long-range physics (prediction of the χPT approach) will not be affected by regularization. However, performing Fourier transform of the $N^3\text{LO}$ 3NF's into coordinate space, applying regularization, and transforming them back to momentum space is a non-trivial task due to the complicated structure of the chiral 3NF at this order. A better idea is to use the fact that the local regularization in coordinate space is a simple multiplication of the force with the regulator function $R(r_{12}, r_{13}, r_{23})$ which depends on relative distances r_{ij} between nucleons i and j . It is well known that the Fourier transform of a product of

³At least up to $N^4\text{LO}$.

two functions ends up to be a convolution integral of Fourier transformed functions. For this reason, the Fourier transform of the product of the chiral 3NF with the regulator in coordinate space will be a convolution integral of the chiral 3NF and the Fourier transformed regulator in momentum space. But in the PWD basis a convolution integral becomes a simple matrix multiplication. For this reason, we can use already stored unregularized matrix elements of the chiral 3NF and multiply them with the partial-wave decomposed regulator matrix. This seems to be a very elegant way of producing semi-local regularized matrix elements. Numerical implementation of this idea is under way.

4. Summary

In this proceeding I gave a brief status report about construction and implementation of the chiral nuclear forces up to $N^4\text{LO}$ in chiral expansion. In the two-nucleon sector, the forces have been developed up to $N^4\text{LO}$. Additionally to construction there are two important novelties. The first one is the introduction of semi-local regularization which does not affect long-range physics by construction. Spectral function regularization becomes with this regularization obsolete, and it becomes possible to use LEC's determined in the pion-nucleon sector without introducing spurious deeply bound states. The second novelty is the introduction of chiral uncertainty quantification which allows one to give a theoretical error estimate at fixed cutoff. This appears to be a better approach than the error estimate through cutoff variation in which even to odd chiral order increase does not lead to decrease of theoretical error and the cutoff region in which one varies the cutoffs appears somewhat arbitrary. Due to its simplicity, it is possible to apply this error quantification to few- and many-body observables and get predictions with quantified theoretical errors. There is also much progress on the construction and implementation of the three-nucleon forces. They have been constructed partly up to $N^4\text{LO}$. New ideas to exploit the locality feature in PWD lead to an increase of efficiency by factor 1000 in very expensive PWD productions of 3NF matrix elements. This allowed us to produce and store a large basis of 3NF matrix elements up to $N^3\text{LO}$ which are sufficient for studies of light and intermediate mass nuclei. Implementation of the semi-local regularization to $N^3\text{LO}$ 3NF is a last technical milestone. The idea of its implementation via a convolution in momentum space is currently under construction. Once finished we will enter a fascinating time of simulation runs where many few- and many-body observables will be confronted with the complete $N^3\text{LO}$ calculations with properly quantified theoretical error. Chiral 3NF's will be extensively tested in light and heavier nuclear systems. This is the main goal of LENPIC (Low Energy Nuclear Physics International Collaboration) which is recently formed out of scientists from the chiral, few-body and many-body communities.

References

- [1] V. Bernard, *Prog. Part. Nucl. Phys.* **60**, 82 (2008) [arXiv:0706.0312 [hep-ph]].
- [2] S. Weinberg, *Phys. Lett. B* **251**, 288 (1990).
- [3] S. Weinberg, *Nucl. Phys. B* **363**, 3 (1991).
- [4] S. Weinberg, *Phys. Lett. B* **295**, 114 (1992) [hep-ph/9209257].

- [5] E. Epelbaum, H. Krebs and U.-G. Meißner, Phys. Rev. Lett. **115**, no. 12, 122301 (2015) [arXiv:1412.4623 [nucl-th]].
- [6] E. Epelbaum, H. Krebs and U.-G. Meißner, Eur. Phys. J. A **51**, no. 5, 53 (2015) [arXiv:1412.0142 [nucl-th]].
- [7] D. R. Entem, N. Kaiser, R. Machleidt and Y. Nosyk, Phys. Rev. C **91**, no. 1, 014002 (2015) [arXiv:1411.5335 [nucl-th]].
- [8] D. R. Entem, N. Kaiser, R. Machleidt and Y. Nosyk, arXiv:1505.03562 [nucl-th].
- [9] E. Epelbaum and U.-G. Meißner, Ann. Rev. Nucl. Part. Sci. **62** (2012) 159 [arXiv:1201.2136 [nucl-th]].
- [10] E. Epelbaum, H.-W. Hammer and U.-G. Meißner, Rev. Mod. Phys. **81**, 1773 (2009) [arXiv:0811.1338 [nucl-th]].
- [11] N. Kaiser, Phys. Rev. C **61**, 014003 (2000) [nucl-th/9910044].
- [12] N. Kaiser, Phys. Rev. C **62**, 024001 (2000) [nucl-th/9912054].
- [13] N. Kaiser, Phys. Rev. C **64**, 057001 (2001) [nucl-th/0107064].
- [14] N. Kaiser, Phys. Rev. C **65**, 017001 (2002) [nucl-th/0109071].
- [15] D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003) [nucl-th/0304018].
- [16] E. Epelbaum, W. Glöckle and U.-G. Meißner, Eur. Phys. J. A **19**, 401 (2004) [nucl-th/0308010].
- [17] N. Kalantar-Nayestanaki, E. Epelbaum, J. G. Messchendorp and A. Nogga, Rept. Prog. Phys. **75**, 016301 (2012) [arXiv:1108.1227 [nucl-th]].
- [18] H.-W. Hammer, A. Nogga and A. Schwenk, Rev. Mod. Phys. **85**, 197 (2013) [arXiv:1210.4273 [nucl-th]].
- [19] E. Epelbaum, W. Glöckle and U.-G. Meißner, Eur. Phys. J. A **19**, 125 (2004) [nucl-th/0304037].
- [20] P. Buettiker and U.-G. Meißner, Nucl. Phys. A **668**, 97 (2000) [hep-ph/9908247].
- [21] R. J. Furnstahl, N. Klco, D. R. Phillips and S. Wesolowski, Phys. Rev. C **92**, no. 2, 024005 (2015) [arXiv:1506.01343 [nucl-th]].
- [22] W. P. Abfalterer *et al.*, Phys. Rev. C **63**, 044608 (2001).
- [23] S. Binder *et al.*, arXiv:1505.07218 [nucl-th].
- [24] W. P. Abfalterer *et al.*, Phys. Rev. Lett. **81**, 57 (1998).
- [25] C. Ordonez and U. van Kolck, Phys. Lett. B **291**, 459 (1992).
- [26] U. van Kolck, Phys. Rev. C **49**, 2932 (1994).
- [27] J. L. Friar, D. Huber and U. van Kolck, Phys. Rev. C **59**, 53 (1999) [nucl-th/9809065].
- [28] E. Epelbaum, A. Nogga, W. Glöckle, H. Kamada, U.-G. Meißner and H. Witala, Phys. Rev. C **66**, 064001 (2002) [nucl-th/0208023].
- [29] V. Bernard, E. Epelbaum, H. Krebs and U.-G. Meißner, Phys. Rev. C **84**, 054001 (2011) [arXiv:1108.3816 [nucl-th]].
- [30] V. Bernard, E. Epelbaum, H. Krebs and U.-G. Meißner, Phys. Rev. C **77**, 064004 (2008) [arXiv:0712.1967 [nucl-th]].

- [31] S. Ishikawa and M. R. Robilotta, Phys. Rev. C **76**, 014006 (2007) [arXiv:0704.0711 [nucl-th]].
- [32] J. Golak *et al.*, Eur. Phys. J. A **50**, 177 (2014) [arXiv:1410.0756 [nucl-th]].
- [33] H. Krebs, A. Gasparyan and E. Epelbaum, Phys. Rev. C **85**, 054006 (2012) [arXiv:1203.0067 [nucl-th]].
- [34] H. Krebs, A. Gasparyan and E. Epelbaum, Phys. Rev. C **87**, no. 5, 054007 (2013) [arXiv:1302.2872 [nucl-th]].
- [35] K. Hebeler, H. Krebs, E. Epelbaum, J. Golak and R. Skibinski, Phys. Rev. C **91**, no. 4, 044001 (2015) [arXiv:1502.02977 [nucl-th]].