

# Quantum Monte Carlo calculations of electromagnetic moments and transitions in $A \leq 10$ nuclei with two-body $\chi$ EFT currents

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We present *ab initio* Quantum Monte Carlo calculations of electromagnetic observables of light nuclei based on a many-body nuclear Hamiltonian consisting of the Argonne-*v*18 and Illinois-7 two- and three-nucleon potentials, respectively. External electromagnetic probes interact with the electromagnetic charge and current distributions of nuclei. Nuclear charge and current operators account for one- and two-body contributions describing, respectively, the coupling of photons with individual nucleons, as well as pairs of interacting nucleons. Two-body operators are derived from a chiral effective field theory with pions and nucleons including up to two-pion exchange corrections. It is found that two-body contributions can be large and crucial to reach agreement with the experimental data, particularly for selected nuclear magnetic moments and M1 transitions.

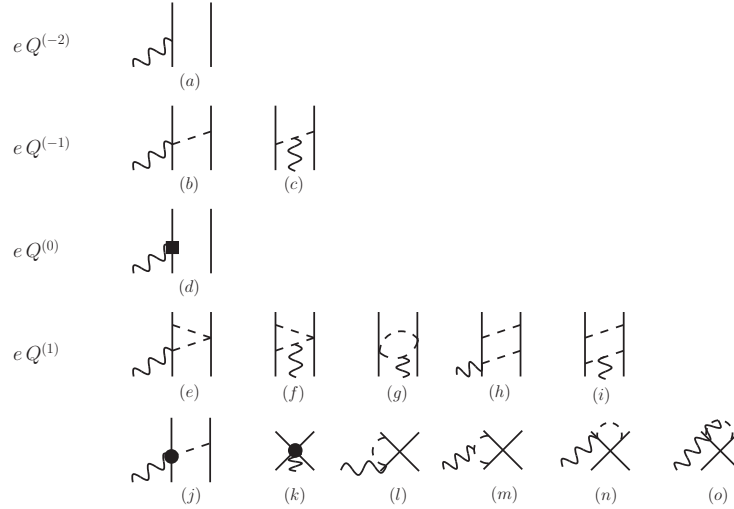
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## 1. The Nuclear Hamiltonian and Electromagnetic Currents



**Figure 1:** Diagrams illustrating one- and two-body electromagnetic currents entering at LO ( $eQ^{-2}$ ), NLO ( $eQ^{-1}$ ), N2LO ( $eQ^0$ ), and N3LO ( $eQ^1$ ). Nucleons, pions, and photons are denoted by solid, dashed, and wavy lines, respectively.

In the *ab initio* framework, the nucleus is described as a system made of  $A$  non-relativistic point-like nucleons interacting among each other via many-body forces and its energy is approximated by the following Hamiltonian:

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} , \quad (1.1)$$

where  $K_i$  is the non-relativistic single-nucleon kinetic energy, while  $v_{ij}$  and  $V_{ijk}$  are two-nucleon (NN) and three-nucleon (3N) potentials, respectively. In the set of calculations presented here the realistic potentials utilized to solve the Schrödinger equation  $H|\Psi\rangle = E|\Psi\rangle$  (where  $|\Psi\rangle$  is the nuclear wave function) are the Argonne v18 [1] (AV18) NN potential in combination with the Illinois-7 [2] 3N potential. Nuclear wave functions are constructed in two steps. First, a trial variational Monte Carlo wave function ( $\Psi_T$ ), which accounts for the effect of the nuclear interaction via the inclusion of correlation operators, is generated by minimizing the energy expectation value with respect to a number of variational parameters. The second step improves on  $\Psi_T$  by eliminating excited states contamination. This is accomplished in a Green's function Monte Carlo (GFMC) calculation which propagates the Schrödinger equation in imaginary time ( $\tau$ ). The propagated wave function  $\Psi(\tau) = e^{-(H-E_0)\tau}\Psi_T$ , for large values of  $\tau$ , converges to the exact one with eigenvalue  $E_0$ . Ideally, the matrix elements should be evaluated in between two propagated wave functions. In practice, we evaluate mixed estimates in which only one of them is propagated, while the remaining one is replaced by  $\Psi_T$ . The calculation of diagonal and off-diagonal matrix elements is discussed at length in Ref. [3] and references therein.

The nuclear electromagnetic current operator is also expressed as an expansion in many-body operators. The current utilized in these calculations accounts for, in addition to the standard one-

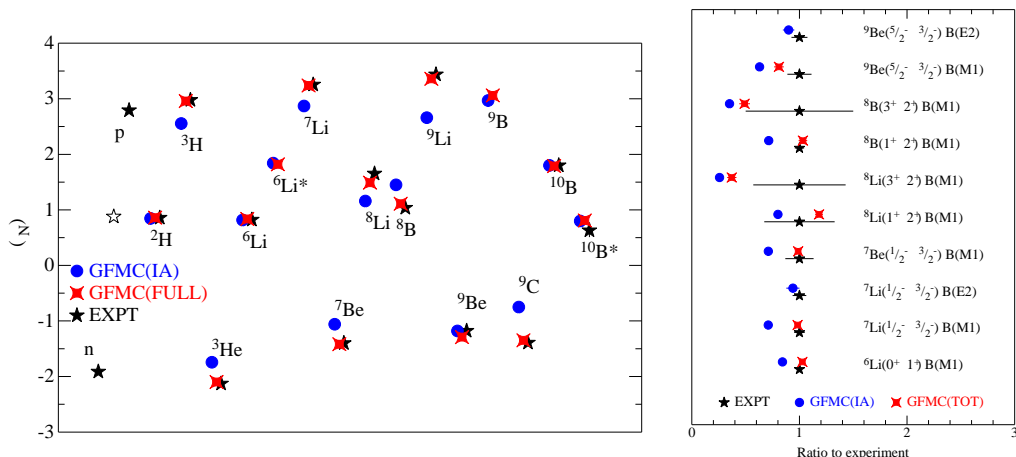
body Impulse Approximation (IA) operators, two-body corrections, and is written as:

$$\mathbf{j}(\mathbf{q}) = \sum_i \mathbf{j}_i(\mathbf{q}) + \sum_{i<j} \mathbf{j}_{ij}(\mathbf{q}), \quad (1.2)$$

where  $\mathbf{q}$  is the momentum associated with the external electromagnetic field. Two-body nuclear electromagnetic charge and current operators have been derived from a  $\chi$ EFT with pions and nucleons in Refs. [4, 5, 6, 7] and consist of multiple-pion exchange and contact-like operators hierarchically arranged in accordance with the scaling imposed by the chiral expansion. The electromagnetic current operators we utilized are diagrammatically illustrated in Fig. 1. In particular, the leading order (LO) contribution to the electromagnetic current is illustrated in panel (a) and it is simply the non-relativistic one-body current used in IA calculations, while the N2LO one-body operator of panel (d) is a relativistic correction to the LO IA current. Currents of one- and two-pion range, describing long and intermediate range dynamics, enter at NLO and N3LO—panels (b), (c), (e)—(j). Short-range dynamics is encoded by the contact currents of panel (k). Unknown low energy constants (LECs) enter the tree-level diagram of panel (e) and contact currents of panel (k). LECs entering the contact terms are of two kinds, namely minimal and non-minimal. The former enter also the NN chiral potential at NLO, and can then be constrained to NN scattering data; the latter need to be fixed from electromagnetic experimental data. A common procedure implemented to reduce the number of unknown non-minimal LECs (there are 5 of them) is to impose that the two LECs entering the isovector part of the tree-level current illustrated in panel (e) are in fact saturated by the  $\Delta$ -couplings entering the  $\Delta$  transition electromagnetic current [5, 7]. The remaining three LECs are here fixed so as to reproduce the magnetic moments of the deuteron, triton, and  $^3\text{He}$  [7].

## 2. Magnetic Moments and M1 transitions in $A \leq 10$ nuclei

Two-body electromagnetic currents have been utilized in GFMC calculations of magnetic moment of light nuclei [8]. These calculations are summarized in the left panel of Fig. 2, where IA results are given by blue dots, while calculations that include the full chiral electromagnetic current operator are indicated by red diamonds to be compared with the experimental data represented by black stars. First, we note that corrections from two-body currents are found to be small where the IA picture is satisfactory (see, *e.g.*,  $^6\text{Li}$ ,  $^9\text{Be}$ ,  $^{10}\text{B}$ ), and large where the IA picture is incomplete (see, *e.g.*,  $^7\text{Li}$ ,  $^7\text{Be}$ ,  $^9\text{C}$ ,  $^9\text{Li}$ ). Corrections from two-body components can be as large as 40%, as seen in the case of  $^9\text{C}$ , and are crucial to reach (or improve) the agreement with the experimental data. It is also interesting to note that two-body effects, while being significant for the  $^9\text{C}$  and  $^9\text{Li}$  magnetic moments, are found to be negligible for those of  $^9\text{Be}$  and  $^9\text{B}$ . This behavior can be explained considering the dominant spatial symmetries of the nuclear wave functions for these  $A = 9$  systems. For example, the dominant spatial symmetry of  $^9\text{Be}$  ( $^9\text{B}$ ) corresponds to an  $[\alpha, \alpha, n(p)]$  structure [9]. Therefore, the unpaired nucleon outside the  $\alpha$  clusters does not interact with other nucleons. As a consequence, two-body currents, that describe the coupling of external electromagnetic probes with pair of interacting nucleons, produce a negligible correction. On the other hand, the dominant spatial symmetry of  $^9\text{C}$  ( $^9\text{Li}$ ) corresponds to an  $[\alpha, ^3\text{He} (^3\text{H}), pp (nn)]$  structure, and two-body correlations contribute in both the trinucleon clusters and in between the trinucleon clusters and the valence  $pp (nn)$  pair, resulting in a large two-body current contribution.

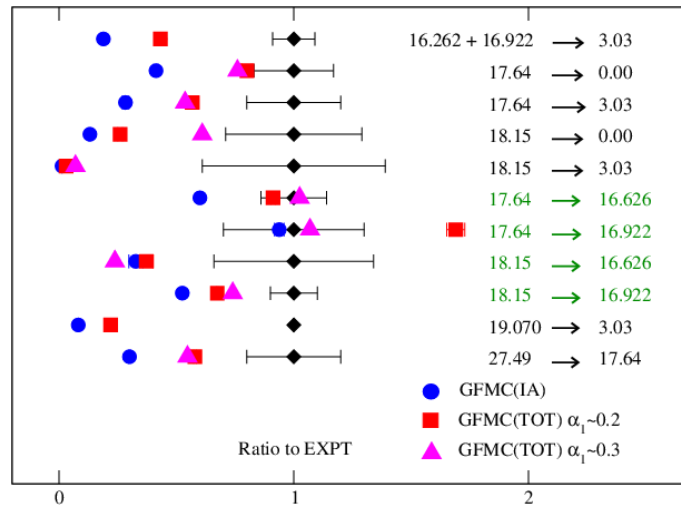


**Figure 2:** **Left:** Magnetic moments in nuclear magnetons for  $A \leq 10$  nuclei from Ref. [8]. Black stars indicate the experimental values, while blue dots (red diamonds) represent GFMC calculations which include the LO or IA (N3LO) e.m. currents from  $\chi$ EFT. **Right:** Transition widths from Ref. [8] normalized to the experimental values for  $A = 6-9$  nuclei. The notation is as in left panel.

In the right panel of Fig. 2, we show the calculated widths normalized to the experimental data of a number of selected electromagnetic E2 and M1 transitions occurring in low-lying nuclear states. Calculations for E2 transitions implicitly include the effect of two-body currents via the Siegert theorem, where the charge density is used in IA. Also for these observables the effect of two-body electromagnetic currents can be large, and for cases in which the experimental errors are relatively small, *e.g.*,  ${}^7\text{Li}(1/2^- \rightarrow 3/2^-)$ ,  ${}^8\text{B}(1^+ \rightarrow 2^+)$ , it is found that their inclusion leads to agreement with the experimental data.

This scheme has been most recently applied to study E2 and M1 transitions occurring in  ${}^8\text{Be}$  [10, 11, 12]. Calculated M1 transition widths normalized to the experimental data are summarized in Fig. 3. Here, experimental data are represented by black diamonds while IA (full) results are indicated with blue dots (red squares). The theoretical description of these transitions is unsatisfactory. The theory seems to systematically underpredict the data. A poor knowledge of the small components entering the calculated GFMC nuclear wave functions is possibly among the causes of this behavior. Nevertheless, the correction due to two-body components in the electromagnetic current is found to be at the 20-30% level. These calculations are affected by an additional uncertainty associated with a poor knowledge of the mixing coefficients implemented to construct the isospin-mixed states at  $\sim 17$  MeV and  $\sim 19$  MeV [11, 12], which, in the calculations indicated by blue dots and red squares, have been taken from the analysis of Barker [13]. In order to assess the sensitivity of the calculated matrix elements to the mixing coefficients we performed independent GFMC calculations in which the coefficient of the  $J^\pi = 1^+$  states at  $\sim 17$  MeV has been fixed so as to reproduce the experimental widths of the transitions highlighted in green in Fig. 3. Calculations obtained using the new (and larger) mixing coefficient are indicated by the pink triangles of Fig. 3. Requiring more isospin-mixing in the  $J^\pi = 1^+$  states at  $\sim 17$  MeV leads, in some cases, to a better

agreement with experimental data.



**Figure 3:** M1 transition widths from Ref. [12] normalized to the experimental values for  $^8\text{Be}$ . Black diamonds indicate the experimental values, while blue dots (red squares) represent GFMC calculations which include the LO or IA (N3LO) electromagnetic currents from  $\chi$ EFT. Pink triangles indicate GFMC results with full electromagnetic currents obtained utilizing isospin-mixing coefficients fixed so as to reproduce the experimental widths of the transitions highlighted in green (see text and Ref. [12] for more details).

### 3. Conclusion and Outlook

In this talk, I presented a set of GFMC *ab initio* calculations of electromagnetic moments and transitions in  $A \leq 10$  nuclei. These calculations account for many-body effects in both nuclear Hamiltonians utilized to generate the wave functions, and electromagnetic current operators. In particular, they use two-body operators that have been derived from a  $\chi$ EFT formulation with pions and nucleons, including up to corrections of two-pion range [4, 5, 6, 7]. The *ab initio* prescription is extremely successful in explaining the experimental data, provided that many-body effects in both the electromagnetic currents and nuclear Hamiltonians are accounted for (for a recent review on *ab initio* calculations of electromagnetic reactions in light nuclei see, *e.g.*, Ref. [14]). Electromagnetic two-body current operators provide a 40% correction in the calculated magnetic moment of  $^9\text{C}$  [8], and corrections at the 20%-30% level in M1 transitions occurring in  $^8\text{Be}$  [12]. There are many interesting electromagnetic observables that can be accessed within this formalism. For example, few (or no) *ab initio* calculations of electromagnetic (charge and magnetic) form factors in  $A > 4$  nuclei currently exist [14], and it would be interesting to perform them to have a deeper insight on nuclear electromagnetic structures. A complete microscopic profile of nuclei includes also studies

of electromagnetic reactions such as radiative captures and photonuclear reactions. From the theoretical point of view, *i*) the construction of chiral potentials compatible with Quantum Monte Carlo computational calculations [15], opens up the possibility of performing consistent Quantum Monte Carlo calculations that use chiral potentials and chiral currents; *ii*) the construction of the chiral NN potential with the explicit inclusion of  $\Delta$ -excitation [16], allows study of the effects of  $\Delta$ -isobars in chiral two-body electromagnetic current operators [4]. Weak processes are also being vigorously studied within the  $\chi$ EFT formulation. Among these studies are the recent derivation of the axial two-body current operator up to one-loop [17] carried out within the same formalism developed for the chiral electromagnetic currents utilized in the discussed *ab initio* GFMC calculations.

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