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Trace elements in WD long term evolution

Giora Shaviv[*†]
Dep. of Physics
Israel Institute of Technology
Haifa, Israel 32,000
E-mail: giora@shaviv.org

The problem: Can the diffusion of trace elements in cooling white dwarfs yield sufficiently high concentration to ignite proper nuclear species and produce thermonuclear reactions?
Method: We discuss the diffusion of trace elements in cooling White Dwarf from the formation of WDs to interior solidification. Trace elements can be leftover elements from the synthesis of hydrogen and helium or elements from the formation of WD that were synthesized in a previous stellar generation. Among the trace elements we assume ^{238}U which emits α particles.
Motivation: The motivation is to see if helium released from the α decay of ^{238}U and the entire radioactive decay changes the structure of the WD and if the radioactive elements which concentrate in the core can eventually ignite a nuclear runaway.

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[*]Speaker.

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1. Introduction

We are interested in long term evolution of CVÕs. The question we pose: can long term diffusion and gravitational settling increase the concentration of trace elements to the point that affects the nuclear evolution of the WD?

The trace elements can have three principal sources: primordial, nuclear leftover from a previous evolution of the star or accretion of the outer layers of a companion star with its trace elements.

Primordial: A single star has a certain amount of trace element with atomic weights that extend from iron to uranium. The White Dwarf may contain trace elements that exist since the formation of the star and diffuse (and gravitationally settle) inward starting at the formation of the star. Here we are interested in what happens to the trace elements during the long WD evolution. We assume that the diffusion continues until the ions crystalize. Can diffusion lead to a critical helium concentration in the star?

Accretion: Suppose that the WD gains mass in a nova cycle. Furthermore, assume that each thermonuclear runaway leaves a small mass on the White Dwarf and this mass contains heady elements. Next, one extrapolates over about $(M_{ch} - M_{WD})/m_l \approx 10^6$ eruptions where M_{ch} in the Chandrasekhar mass and m_l is the mass left on the White Dwarf. The claim is that this is the composition of a progenitor of a SN. Such an extrapolation brings up fundamental questions like: Is the accretion rate constant over this period? does the accretor change its character during its change in mass? What about the donor star? What happens after a very large number of cycles? It the model still identical? The classical calculations carried so far extended over at most 10^3 cycles and above all assumed that the donor does not change despite losing its envelope and the White Dwarf remains unchanged despite its accretion[1]. We note that the starting White Dwarf has a mass of say, $0.8M_{\odot}$ and the M_{ch} has a vanishing radius and infinite density. Yet, the authors assumed that nothing in the parameters of the stars changes during this many cycles in particular the rate of accretion remains constant.

Idan carried such a calculation with a Main Sequence star as a donor and found that after about 4000 cycles, the leftover helium, which was the result of hydrogen burning, ignited and ejected the entire mass accumulated so far on the star. Also, each eruption of the nova heat generated in the outer parts of the star flows inward. If you start a fresh a calculation and calculate a single eruption, you ignore this small but steady inward heat flow. This steady heat flow can become significant over 10^6 cycles.

We define a trace element as one with an abundance of $[X] < 10^{-10}$. The standard treatment of such elements in stellar evolution calculation is to ignore them and put their abundance equal to zero. Here we follow the idea of Chiosi (Chiosi et al. 2015) discuss nuclear reactions leftovers. In particular these authors were interested in the abundance of H and He in very old WDs. Their basic hypothesis is that H as a trace element might trigger a thermonuclear runaway, not necessarily in the center of the star, provided the concentration exceeds a certain limit.

In this first paper on the subject we are interested in the fate of the heavy elements like Fe and U. Nothing happens to these elements save diffusion and gravitational settling during the stellar evolution. Preliminary calculation show that Hydrogen as the lightest element, reaches the surface very quickly and leaves the interior with a mass fraction below $X(H) \leq 10^{-20}$ after 10^{10} years of diffusion. More accurately, as the core crystalizes, the diffusion at the center ends well before the

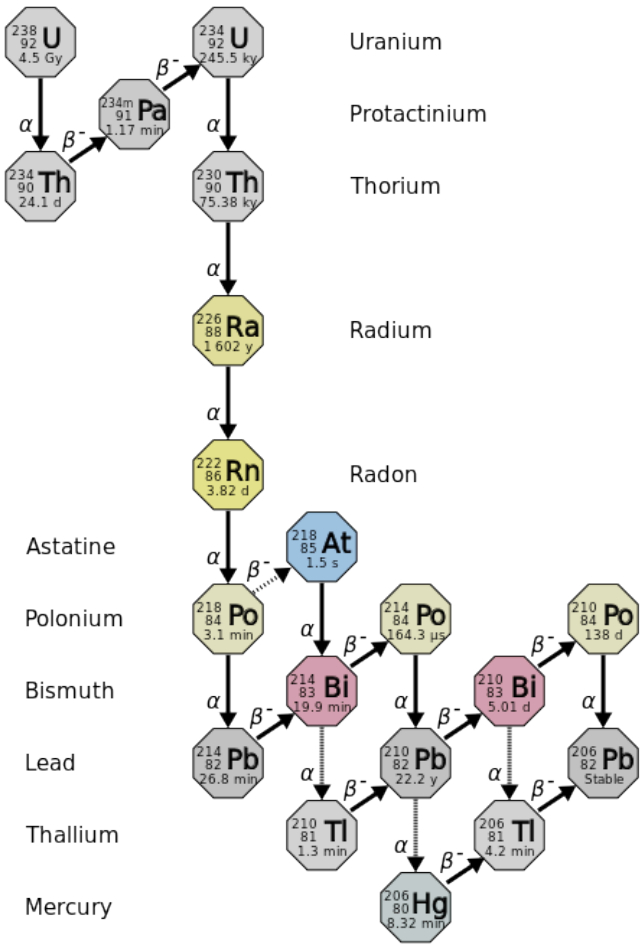

Figure 1: The radioactive decay chain. All the elements are included in the decay of ^{238}U

above time and the above abundance is the abundance when the diffusion and gravitational settling ends.

2. The evolution calculations

We use the stellar evolution program evolve adopted to the current problem. We start the evolution when the star becomes a White Dwarf. From this point on there is practically no convection that can mix the convective zone and erase all the effects of diffusion. Also, in the present calculation we carry the evolution without nuclear reaction, say somehow the star passed the maximum temperature and cools. This is the optical assumption because we let the settling processes operate all the time and teach the maximal effect.

The elements heavier than Lead are all radioactive and decay. Thus, as the settling process takes place, the radioactive elements decay and release energy. This radioactive energy is treated as a local energy source. The element with the longest lifetime is ^{238}U, with a half lifetime for α decay of $4.468 \times 10^9 yrs$ and this is inverted into probability for decay per unit time per ^{238}U

nucleus. All other decays have a significantly shorter half time. Consequently we can assume that all energies from subsequent decays towards Lead are leased instantaneously. The energy of all decays is summed up and released with the decay of ^{238}U.

We have the following decay sequence:

$$^{238}\mathrm{U} \rightarrow^{234} \mathrm{Th} + \alpha + 4.27 MeV$$

which has a decay time $4.468 \times 10^9 yrs$.

$$^{234}\mathrm{Th} \rightarrow^{234n} \mathrm{Pa} + \alpha + 4.54 Mev$$

with a decay time 24.4 day.

As a matter of fact, the entire ^{238}U decay down to ^{206}Pb takes place while the WD cools and the total energy release adds up to about 51.77 MeV per ^{238}U nucleus. There are (238-206)/4 = 32 $\alpha\tilde{\mathrm{O}}$s nuclei and the assumed cosmic abundance of ^{238}U is $10^{-11.5}$ for (X(H)=1). As these elements do not participate in nuclear reaction (only if there are neutrons) they are Òleft aloneÓ to diffuse into the center of the WD or out of it.

The particular case of a secondary being an AGB star producing *s* process elements will be treated in a forthcoming paper. For us here, the *s* process does not produce elements beyond ^{206}Pb and to produce ^{238}U we probably need a SN, which is outside the scope of this paper.

In principle there can be numerical problems in computing elements with abundances below computer accuracy. However, as these elements only diffuse there is no problem in the theory of diffusion: we have just to ignore trace-element - trace element interaction.

Moreover: as the abundance of the trace element enters once only in the diffusion equation, the equation is linear! Hence, one can calculate quite low abundances. Still, even with double precision we are restricted to an abundance of 10^{-14} due to computer truncation. The alternative quadruple precision is prohibitive in computer time and the possible addition in accuracy is not warranted. So the calculation is carried out assuming representative elements with the right atomic weights and atomic numbers and then normalize the results to the assumed initial values.

Once the ions in the plasma crystalize the nature of the diffusion changes. Effectively it almost disappears. Hence it make sense to follow the evolution as long as $\Gamma \leq 175$ the point of starting crystallization. We assume $\Gamma = Z_1 Z_2 e^2 / < r > kT = 175$ where $< r >$ is the mean distance between the relevant ions.

The ^{238}U has a half life time of $4.5 \times 10^9 yr$ while the cooling time of a WD is about $\approx 10^{10}$ years depending on the mass of the White Dwarf. The amount of U left after an evolution for 10^{10} years would be e^{-2} or about 10% of the ^{238}U.

3. Characteristics of diffusion

The characteristics of the heady element diffusion and settling are as follows:

a) The WD is basically isothermal and the main effects are abundance distribution against density profile. As the rate of settling is inversely proportional to density and the temperature does not vary, the process in inversely proportional to the density squared. Hence, the rate of the process slows down as we approach the center of the star.

b) The geometry is spherical so the surface area is proportional to r^2 so the effect increases as r decreases and what we see is a balance between the above and the current trends.

c) The interaction between the ions depends on Z_1Z_2/A_1A_2 so more neutron rich elements have a lower Z/A and diffuse more slowly. The gravitational settling depends on the total atomic weight. However, the difference are small and we did not investigate in any detail the effect of neutron rich element (isotopes) which should sink towards the center faster in proportion to their atomic weight.

d) The abundance of uranium is small and the effect of splitting the uranium into lighter elements on the Fermi energy is below the accuracy in the equation of state.

H as a trace element, floats very quickly and concentrates in the outermost layers. It take $< 10,000yrs$ (more accurately $5000yrs$) for all the hydrogen to reach the surface. On the ther hand, the trace Helium sinks with the same time scale (it provides room for the Hydrogen). However, Helium does not reach the center of the star because the heavy elements. In principle, always the lightest element floats while all other sink. The heaviest element sinks fastest despite the fact that its interaction with other elements is the strongest. Thus, in the sinking towards the center the ^{238}U wins and the helium remains further out bur never builds up a concentration.

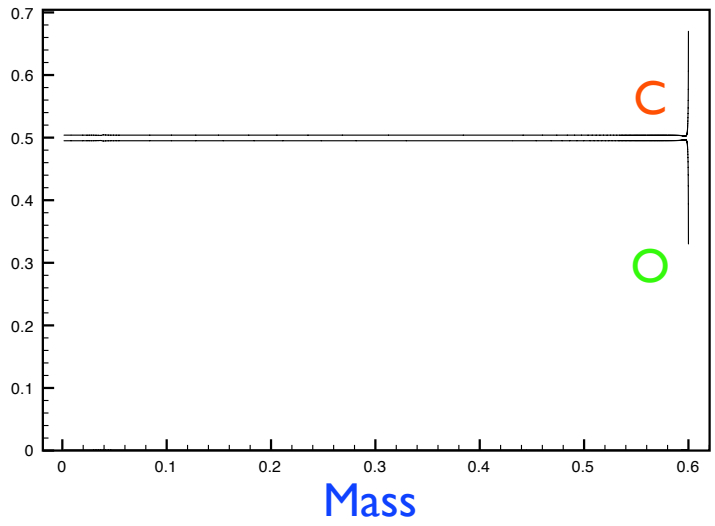

Figure 2: The distribution of C and O in the WD after $10^{10}yrs$.

4. Models run

We run two cases: $0.6M_\odot$ and $1.0M_\odot$ models of C/O WDÕs. We ignored in these first runs nuclear reactions and radioactive decay. So no hydrogen is left in the core.

We continues the calculation for $10^{10}yrs$ and stopped it when the luminosity decreased to $L \approx few 10^{-5}L_\odot$.

We note that crystallization takes place during the diffusion (and cooling). This slows the process further and we ignore it here. Hence the results can be regarded as upper limit for the diffusion.

5. Temporary conclusions

No hydrogen is left the core. The hydrogen concentrates near the surface. He sinks to the center. Uranium disappears from the surface but does not really accumulate in the core. All the mass between Lead and Uranium converts into He

Helium sinks to the core as log as there is H around but appears to float if it is the lowest in atomic weight

Do all SNIa explosion start from the core?

The story has not yet ended, the next chapter in the coming meeting

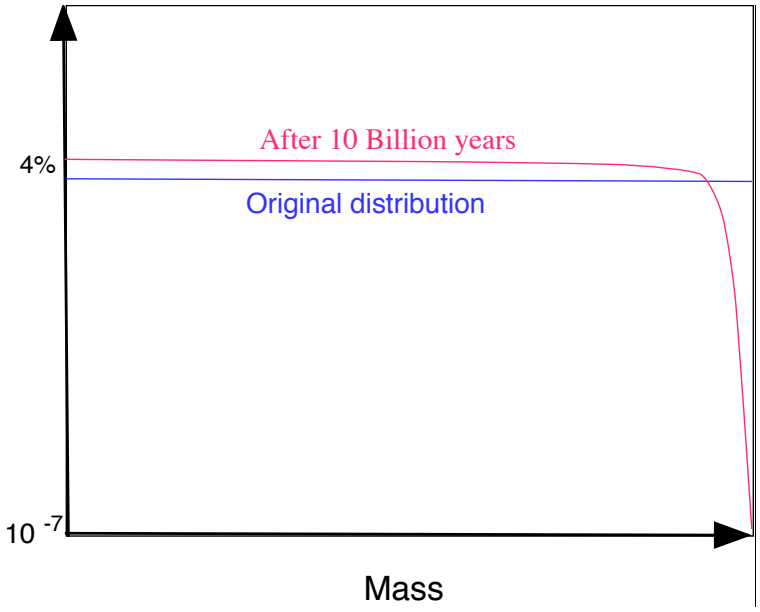

Figure 3: The distribution of ^{238}U in the WD after $10^{10} yrs$.

6. Appendix

We assume the electrons to form a uniform sea of negative charge in which the positive ions move. We follow Chapman and Cowling 1970), namely we assume the ions to behave as an ideal classical gas. Then the i ions satisfy the following diffusion-hydrostatic equation

$$\nabla p_s - \rho_s F_s + n k_B T k_{Ti} \nabla \ln T = -k_B T \sum_j \frac{n_s n_j (C_s - C_j)}{n D_{sj}} \tag{6.1}$$

where $p_, \rho_s$ and n_s are the partial pressure, the density and the number density of the s specie which may be an ion i or an electron e. The force acting on the ion or electron is:

$$F = g + \frac{e_s}{m_s} E. \tag{6.2}$$

C_i are the velocities relative to the center of mass so that $\sum \rho_s C_s = 0$. The coefficients k_{Ts} and D_{sj} are the generalized thermal and binary diffusion coefficients. These coefficients satisfy the conditions:

$$\sum k_{Ts} = \sum k_{Ti} + k_{Te} = 0, \quad D_{si} = Dis \tag{6.3}$$

The index imply electrons, i, j imply ions while all other indexes imply any specie.

The condition of charge neutrality yields: $\sum n_s e_s = 0$. Summing eq. 1 yields then the hydrostatic condition: $\nabla p = \rho g$.

The diffusion coefficients are given by:

$$nDst = \frac{3}{16A_1(s,t)} \left(\frac{2k_BT}{\pi\mu_{st}}\right)^{1/2} \left(\frac{2k_BT}{e_s e_t}\right)^2 \tag{6.4}$$

$$k_{Ts} = -\sum_t \frac{n_s n_t}{n^2} \left(f_{ts} M_{st} b_t - f_{st} M_{ts} b_s\right) \tag{6.5}$$

$$f_{st} = \frac{5}{2\sqrt{2}} \left(\frac{m_{st}}{m_s}\right)^{1/2} \left(\frac{e_t}{e_s}\right)^2 \frac{A_1(s,t)}{A_2(s,t)} \tag{6.6}$$

$$\mu_{st} = \frac{m_s m_t}{m_s + m_t}, \qquad M_{st} = \frac{M_{st}}{m_t} \tag{6.7}$$

$$A_1(s,t) = \ln(1+v_{st}^2); \qquad A_2(s,t) = 2\left[A_1(s,t) - \frac{v_{st}^2}{(1+v_{st})^2}\right] \qquad v_{st} = \frac{4dk_BT}{e_s e_t} \tag{6.8}$$

A_1 and A_2 are the Coulomb logarithms and should include corrections of the order of unity due to local deviations in the distributions. We apply the integrals in regions where $A_1 \gg 1$.

The coefficient b_t is found by solving the linear scheme:

$$\left(n_s \sum_{t\neq s} f_{st} n_t \left[6M_{st}^2 + 2.6M_{ts}^2 + 1.6M_{st}M_{ts}A_2(s,t)/A_1(s,t)\right]\right) b_s$$
$$-\sum_{t\neq s} f_{ts} n_t M_{st} M_{ts} [8.6 - 1.6\frac{A_2(s,t)}{A_1(s,t)}] b_t = n \tag{6.9}$$

The fact that $m_e/m_i \leq 5\times 10^{-4}$ allows to reduce the thermal diffusion coefficients to

$$n^2 k_{Te} = 3n_3 \sum_j f_{ej} n_j; \quad b_e = \frac{n}{n_e + 2.6\sum_j f_{ej} n_j},$$
$$n^2 k_{Ti} = -3n_i \left[\sum_j (f_{ji} M_{ij} bj - f_{ij} M_{ji} + bi) + f_{ei} n_e b_e\right] \tag{6.10}$$

Furthermore, as we are interested in trace elements, we ignored the $trace - trace$ interaction and kept only the $trace - n_i$ where $n_i \geq 10^{-2}$.

We further assume that the radial electric current vanishes so that the electron velocity $C_e \approx C_i$. This is supposed to be correct in our case because despite the lower mass of the electron and the extra speed of the electron due to degeneracy. What is clear is that the global conditions imply it. As $D_{ij}/D_{ie} \leq (m_e/m_i)^{1/2}$ the basic system 6.1 splits into two parts:

$$\nabla p_e - \rho_e F_e + nk_BT k_{Te} \nabla \ln T = 0$$
$$\nabla p_i - \rho_i F_i + nk_BT k_{Ti} \nabla \ln T = -n_i k_B T \sum_j \frac{n_j(C_i - C_j)}{nD_{ij}} \tag{6.11}$$

The first equation determines the microscopic electric which is substituted into the second equation. The equation is then solved under the conditions $\sum \rho_i C_i = 0$.

The above method was used to dove for the velocities of all the abundant element. Fyor the remaining *trace* elements their diffusion velocity was determined from eq. 6.9 to 6.11 by keeping the first order terms in X_s and neglecting any binary encounter betaeen any tao trace ions.?

References

[1] Epelstain, N., Yaron, O., Kovetz, A. & , D., "A thousand and one nova outbursts", 2007, MNRAS, 374, 1449.

[2] Chiosi, E. , Chiosi, C., Trevisan, P. , Piovan, L. & Orio, M., "Exploring an alternative channel of evolution towards SNa Ia explosion", 2015, MNRAS, 448, 2100. 2015AcPPP...2..103S 1.000 01/2015

[3] Shaviv, G.; Idan, I.; Shaviv, N. J. Why a New Code for Novae Evolution and Mass Transfer in Binaries? 2014arXiv1411.5300S 1.000 11/2014

[4] Shaviv, Giora; Idan, Irit; Shaviv, Nir J. Long term evolution of an interacting binary system 2013MNRAS.433.2884I 1.000 08/2013

[5] Idan, Irit; Shaviv, Nir J.; Shaviv, Giora The fate of a WD accreting H-rich material at high accretion rates 2012JPhCS.337a2051I 1.000 02/2012

[6] Idan, Irit; Shaviv, Nir J.; Shaviv, Giora The Fate of a WD Accreting H-Rich Material at High Rates 2012MmSAI..83..477I 1.000 00/2012

[7] Idan,I.; Shaviv, N. J.; Shaviv, G. The fate of a WD accreting H-rich material at high ratesa (in preparation)