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Nuclear Physics

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Lattice QCD is making good progress toward calculating the structure and properties of light nuclei and the forces between nucleons. These calculations will ultimately refine the nuclear forces, particularly in the three- and four-nucleon sector and the short-distance interactions of nucleons with electroweak currents, and allow for a reduction of uncertainties in nuclear many-body calculations of nuclei and their reactions. After highlighting their importance, particularly to the Nuclear Physics and High-Energy Physics experimental programs, I will discuss the progress that has been made toward achieving these goals and the challenges that remain.

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1. Introduction

Nuclear physics is on the brink of being changed in remarkable ways by the use of Lattice QCD (LQCD) to provide reliable calculations of low-energy strong interaction processes that cannot be reliably obtained by any other means. The crucial step of verifying LQCD as a rigorous source of strong-interaction observables, complementary to experiment, is beginning to happen by the reproduction of nuclear physics quantities that are already precisely known. As this verification stage is in process, genuine predictions of QCD performed with LQCD are beginning to emerge. There are many important strong interaction quantities, impacting a broad array of research programs and technologies, that are required to be more precisely known than will be possible experimentally or through known analytical theory methods. One class of such quantities is the multi-nucleon forces, such as the three-neutron and four-neutron interactions. Figure 1 shows the present uncertainties in

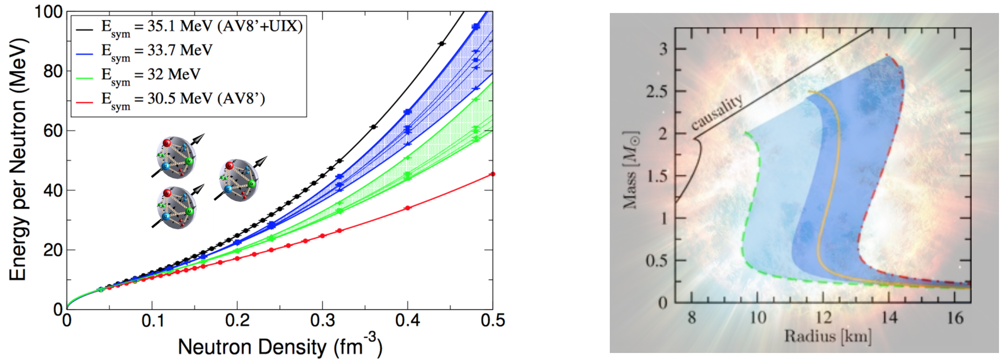


Figure 1: The energy per neutron as a function of neutron density for a range of parameters defining nuclear forces (left panel) [1], and the allowed region of mass and radius of neutron stars for a similar range parameters (right panel) [2].

the energy per neutron as a function of neutron density from current nuclear forces, which is seen to become significant beyond nuclear matter densities [1]. This, and other uncertainties, impact our ability to calculate basic properties of neutron stars, such as the mass-radius relation [2], also shown in Figure 1. The US is building FRIB (Facility for Rare Isotope Beams), and other countries are building similar facilities, to study nuclei and nuclear systems that exist for a very short time. These nuclei participate in explosive astrophysical environments, and have so far proven difficult to examine in the laboratory. FRIB, currently under construction at Michigan State University, will make major inroads into the detailed study of these exotic systems. The anticipated measurements will refine our understanding of multi-neutron forces, particularly the three-neutron interaction, and more generally three-body and four-body forces. LQCD calculations of few-nucleon systems will, combined with nuclear many-body calculations, will provide complementary information and constraints on the multi-nucleon forces. The discovery of gravitational waves emitted from inspiraling black-hole binary systems [3] was a major accomplishment, opening a new era of exploration of the universe, and the gravitational wave signals expected from inspiring binary neutron star systems will be sensitive to the nuclear equation of state (EoS), and hence the three-neutron and higher forces.

A second class of quantities that need to be known with higher precision are the interactions of nuclear systems with electroweak probes, and candidate particles beyond the standard model (BSM). Such processes are critical to much of the US's nuclear and particle physics experimental programs, including the planned ton-scale $0\nu\beta\beta$-decay experiment, the Deep Underground Neutrino Experiment (DUNE), neutron electric dipole moment searches, and cross sections for simple fusion processes. For electromagnetic interactions, there is a well-established hierarchy for the

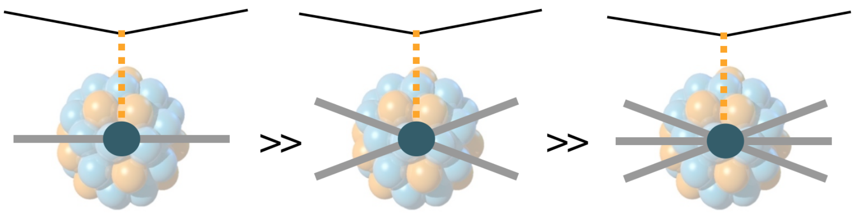


Figure 2: A cartoon of the relative importance of multi-nucleon interactions with external probes.

relative contributions of multi-nucleon effects, as shown in Figure 2. Low-energy processes are dominated by one-body interactions, with two-body interactions typically at the few percent level, and three-nucleon interactions further suppressed. For nuclear matrix elements of the axial current, calculations with truncated model spaces require a reduced (quenched) value of g_A for the one-body interaction. This is most likely due to inconsistent treatments in matching from chiral effective field theories to the nuclear many-body space, where multi-nucleon interactions with the axial current will likely compensate the quenched g_A. This highlights the importance of appreciating that a nucleus is not simply a collection of non-interacting nucleons, but a complex system with multi-nucleon interactions, including with external probes.

A third class of quantities is directly related to the US's planned electron-ion collider (EIC) that is designed to precisely measure the gluonic structure of the nucleon and of nuclei. This was identified as a long-term priority for nuclear physics [5], which may lead to an EIC in the United States. LQCD gluonic calculations are notoriously difficult, but are essential to the success of an EIC program. Figure 3 shows a classical calculation of the time dependence of the gluon density in a nucleus [4].

The grand plan for using the numerical technique of LQCD in nuclear physics is quite simple. It is to be expected that systems involving up to twelve nucleons or so will be accessible to LQCD calculations in the not so distant future. For small and modest lattice volumes, the energy splittings between levels is sufficient so that there is the possibility of isolating them with techniques such as the variational method, and the binding energies, and more generally observables associated with the ground states and low-lying excited states, can be compared with experiment. More generally, the energy eigenvalues computed in a range of lattice volumes will be used to refine effective nuclear (many-body) forces through appropriate finite-volume matching calculations that can then be used to make predictions in Minkowski space, as sketched in Figure 4.

One of the challenges facing nucleon and multi-nucleon systems is the signal-to-noise problem. At asymptotically large times, the even moments of a nucleus correlation function is dominated by multi-pion states, which fall exponentially with a multiple of the pion mass, while the

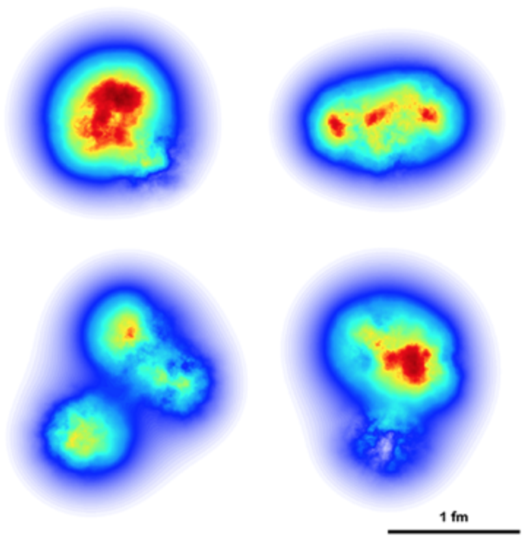


Figure 3: Time-dependence of the gluon field in a nucleus [4].

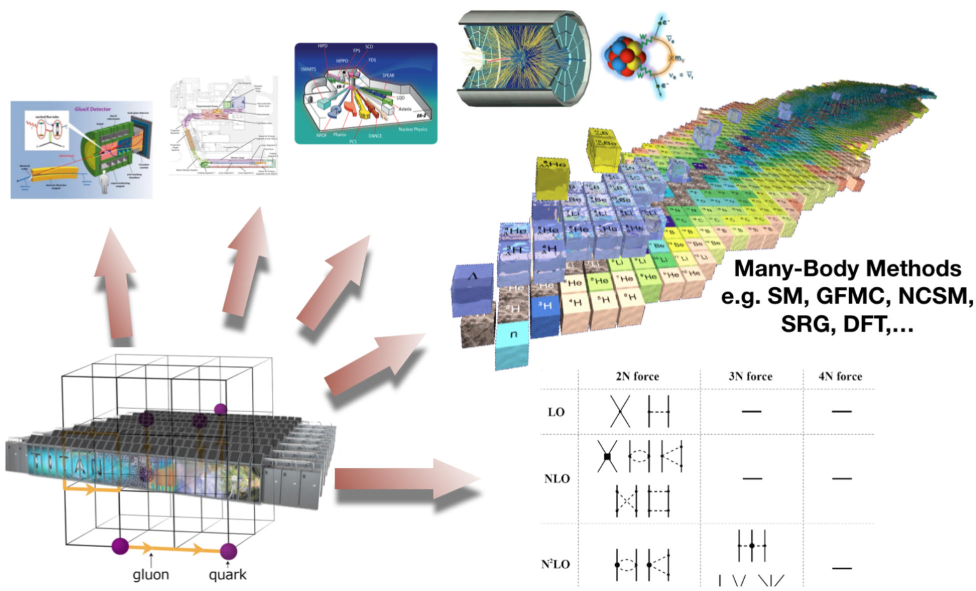


Figure 4: Cartoon of the grand plan for LQCD in nuclear physics.

odd moments are suppressed by at least one factor of the nucleon mass. At short times, for appropriate source structures, all moments of the correlation functions are determined by the nucleon mass. At intermediate times, the signal-to-noise ratio is degrading but with an exponent that is significantly less than the canonical $A(M_N - \frac{3}{2}m_\pi)$ [6], and it is in such an intermediate time interval ("Golden Window") that plateaus in multi-nucleon correlation functions can be identified, and binding energies and scattering parameters determined.

Until about five years ago, the quark contractions required to form nuclear correlation functions required excessive computational resources to evaluate. For light nuclear systems, the contraction problem was solved through understanding the symmetry of the contractions, implementing recursion algorithms, and automated code generation. This reduced the impact of the quark contractions on the overall computational resources required for production of s-shell nuclei and hypernuclei. [7, 8, 9].

The methodology for extracting phase-shifts, scattering parameters and two-nucleon bound states from LQCD calculations was formulated in detail by Luscher [10, 11]. The two-hadron wave function, $\psi(r)$ satisfies

$$-\frac{1}{2\mu}\Delta\psi(r) + \frac{1}{2}\int d^3\mathbf{r}'\, U_E(\mathbf{r},\mathbf{r}')\psi(r') = E\psi(r) \quad , \tag{1.1}$$

where $U_E(\mathbf{r},\mathbf{r}')$ is an energy-dependent non-local "potential", and where the total energy of the system is $W = 2\sqrt{M^2+ME}$. $U_E(\mathbf{r},\mathbf{r}')$ depends analytically on E in the region below the inelastic threshold and is a smooth function of the coordinates. This construction readily reveals the Lüscher relations which, after truncation in angular momentum space, relates energy splittings (to non-interacting states) to the parameters defining the S-matrix in that system.

The HALQCD collaboration uses wall sources to create two-nucleon states and forms correlation functions that depend upon spatial separation by a composite product sink comprised of two nucleon field operators, $G_{NN}(r) = Z_{NN}(r)e^{i\mathbf{p}\cdot\mathbf{R}}\psi(r)$. At the energy eigenvalues of the lattice calculation, this object satisfies the Schrodinger equation and a resulting potential, $U_E(r)$, can be identified [12]. HALQCD has been performing calculations at the physical point and have derived such $U_E(r)$'s. The interpretation of the derived $U_E(r)$'s remains to be determined as they do not have sufficient statistics to identify plateaus in their correlation functions. The single nucleon correlation function from these wall sources plateaus after $t \sim 18$, and the $U_E(r)$'s in the two-nucleon sector, an example of which is shown in Figure 5, are contaminated by excited states. It would be helpful for the HALQCD collaboration to extract the masses associated with the long-distance behavior of the $U_E(r)$'s.

A well-defined procedure for extracting scattering parameters and S-matrix elements from two-hadron energy eigenvalues is Lüsher's method, which has been used extensively to study hadron-hadron interactions. The PACS, NPLQCD and Mainz collaborations use effective masses to directly measure the energy eigenvalues of multi-nucleon states in a range of lattice volumes, to directly measure binding energies and use Lüsher's method to extract scattering parameters and phase shifts. In simple systems, by extracting $k^*\cot\delta$ below the inelastic threshold from systems with a range of boosts and boundary conditions [14, 15, 16], single-channel phase shifts can be determined from energy eigenvalues along with scattering parameters in the effective range expansion (ERE) which is applicable below the t-channel cut, an example of which is shown in Figure 6.

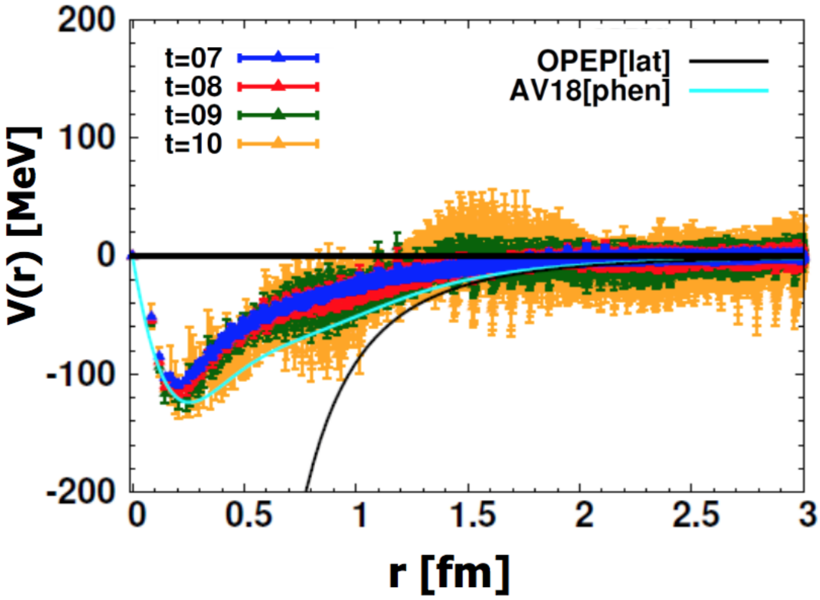


Figure 5: A tensor $U_E(r)$ derived from correlation functions in the $^3S_1$-$^3D_1$ coupled channels. [I thank Takumi Doi for providing this figure.]

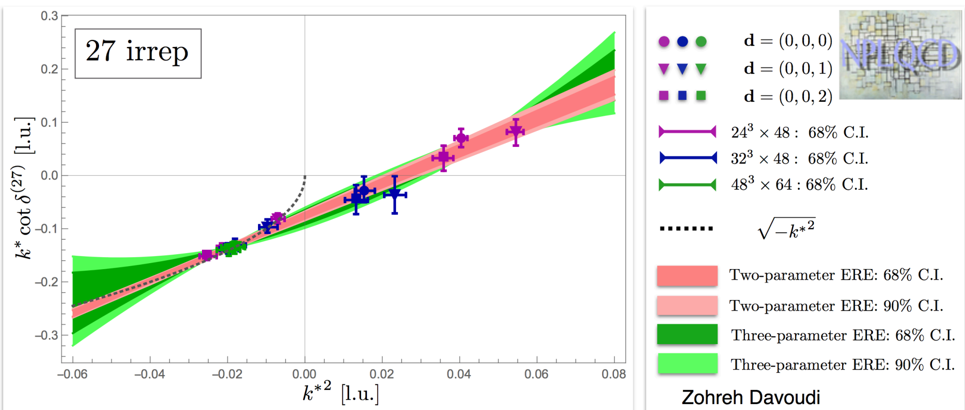


Figure 6: $k^* \cot\delta$ as a function of k^{*2} in the **27** irrep. of flavor SU(3) calculated at a pion mass of $m_\pi \sim$ 805 MeV [13]. **d** denotes the boost vector in lattice units. Two- and three-parameter ERE fits are shown by the shaded regions.

Multi-channel systems can also be addressed, and the corresponding S-matrix elements can be constrained from finite-volume energy eigenvalues, but the analysis is more complex [17].

The PACS collaboration has been calculating the binding energies of s-shell nuclei for a number of years, and, impressively, are now performing calculations at the physical pion mass [9, 18]. Figure 7 shows the current status of these calculations. The two-nucleon systems are showing en-

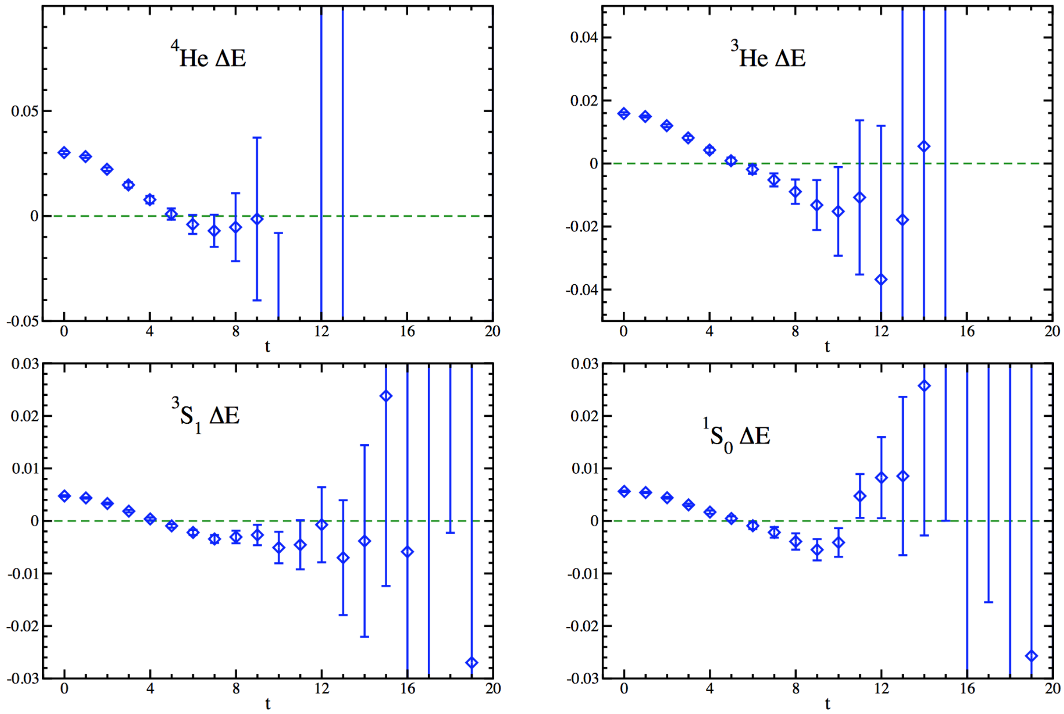
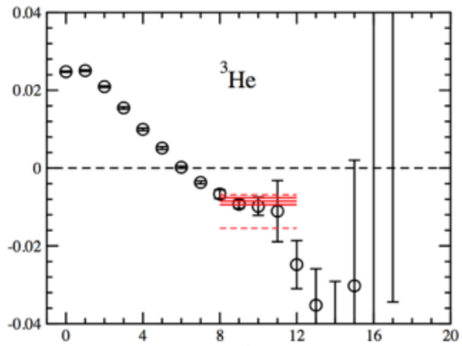


Figure 7: Effective mass plots of the s-shell nuclei calculated at the physical pion mass, $m_\pi \sim 140$ MeV, by the PACS collaboration. [I thank Takeshi Yamasaki for providing these figures.]

couraging signs of developing plateaus. Given the lattice volumes currently employed, the finite-volume effects for these levels are expected to be significant [19] in connecting to experiment. It is also encouraging to see correlation functions for ^{3}He and ^{4}He, and one hopes that increased statistics will reveal the binding energies of these systems that are measured in experiment (modulo electromagnetic and isospin breaking effects). Unfortunately, it appears that the correlation functions are entering the exponentially-degrading signal-to-noise region at the times when plateaus are forming in ^{3}He and ^{4}He. The PACS correlation functions shown in Figure 8 are created from smeared sources for the quark propagators, from which the single nucleon correlation functions are observed to plateau around time-slice $t = 7$.

HALQCD has been calculating $U_E(r)$ [12], Eq. (1.1), over a range of pion masses for a large number of two-baryon systems, including NN and hypernuclear systems, for example Refs. [22, 23], and a small number of three-baryon systems [24]. The signal-to-noise issues are less severe for the heavier systems and consequently the signals for the systems with large strangeness are better than for the NN systems. They have high precision determinations of the $U_E(r)$ in the H-dibaryon coupled-channels system, from which the draw the preliminary conclusion that the H-dibaryon is



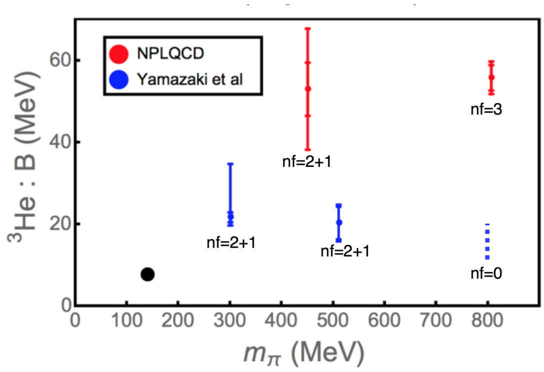


Figure 8: The left panel shows a ^{3}He effective mass plot from the PACS collaboration at $m_\pi \sim 300$ MeV, while the right panel is a summary of calculations of ^{3}He binding energy as a function of the pion mass [20, 18, 21].

a resonance above $\Lambda\Lambda$ threshold at the physical point. In deriving these $U_E(r)$, the energy of the two-baryon system is required, and HALQCD acts with time-derivatives at time slices where the effective mass has not plateau'd and attributes the non-plateau'ing to two-baryon excitations in the lattice volume, ignoring the possibility of single baryon excitations. As the single nucleon effective masses from the wall sources do not plateau until time-slice $t \sim 18$, it seems that such extractions are contaminated by single nucleon excitations until $t \sim 18$, as shown in Figure 9 [25], and deriving $U_E(r)$ in this way introduces uncertainties for $t < 18$ that are not accounted for. The $U_E(r)$ that have been calculated by HALQCD are derived from the $t \leq 10$ range of the correlation functions. It is very encouraging that HALQCD is performing calculations at the physical point, and early results are shown in Figure 5, and we look forward to them being able to extract $U_E(r)$ with precision in time intervals where the single-baryon and two-baryon correlation functions have both established plateaus in the effective mass.

The Mainz Lattice collaboration has been calculating the binding energy of two-baryon systems, in the SU(3) limit, and including SU(3) breaking, over a range of pion masses [26, 27]. Extrapolating the results of these calculations with an ERE to locate the bound states, they find that the H-dibaryon is bound for $m_\pi \gtrsim 450$ MeV, as shown in Figure 10. They find binding energies that are somewhat deeper than those of other calculations [28, 29].

The CalLatt team has calculated p-wave and higher partial wave phase shifts [30] on two ensembles of NPLQCD gauge-field configurations with $m_\pi \sim 805$ MeV, as shown in Figure 11. For the p-waves, they find non-zero phase shifts in the different cubic irreps. which agree within uncertainties, and which are seen to be dominated by the scattering volume. In the s-wave channels, they find binding energies that agree with the previous NPLQCD results [20, 13] within their uncertainties. In addition, using a truncated ERE, they find a state shifted by ~ 3 MeV in the deuteron channel, that is consistent with zero within uncertainties. It is likely that this state is an artifact of working with only two lattice volumes, unlike the three used by NPLQCD, and of their analysis methods. Despite the defects associated with the suggestion of there being a second bound state in the deuteron channel at this pion mass, some have speculated that this is a cause for concern in

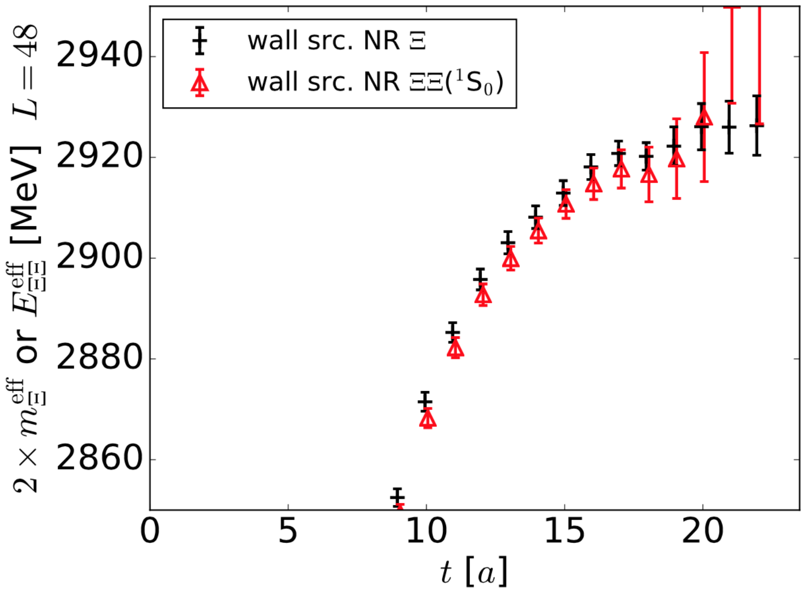


Figure 9: Effective mass plots for the Ξ and $\Xi\Xi(^1S_0)$ from wall sources generate by HALQCD [25].

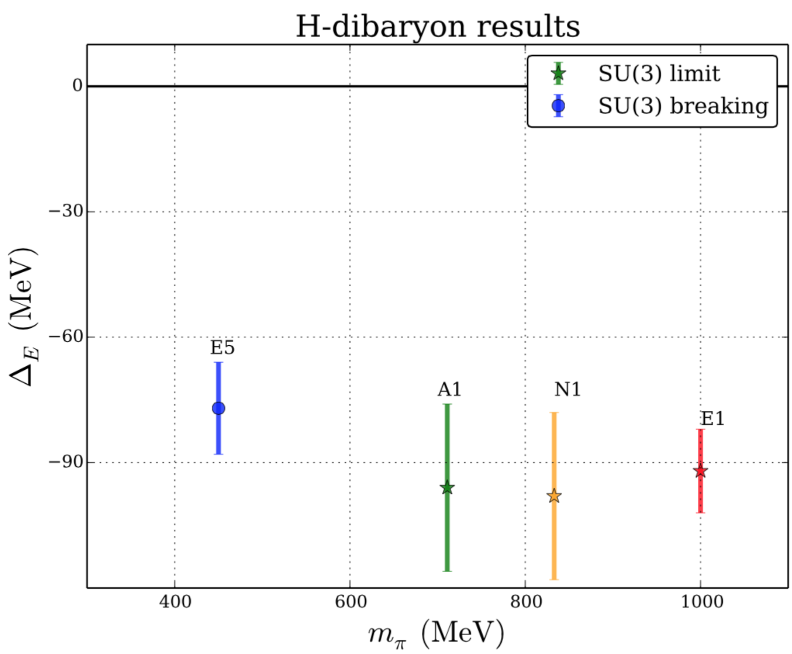


Figure 10: The binding energy of the H-dibaryon over a range of pion masses calculated by the Mainz Lattice Collaboration [26, 27].

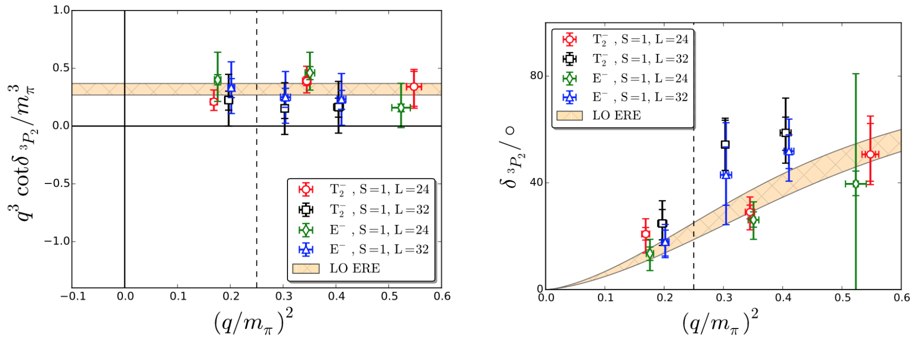


Figure 11: The left panel shows the results of CalLatt's calculation of the real part of the inverse scattering amplitude in the ^{3}P$_2$ channel, while the right panel shows the associated phase shift [30].

using Lüsher's method to extract two-baryon information from LQCD calculations [31]. I believe this not to be the case.

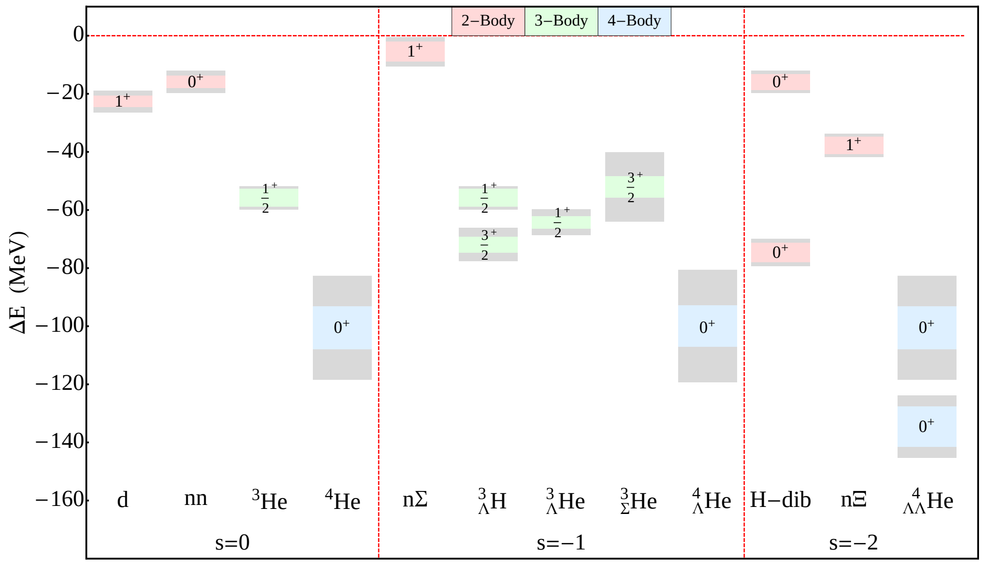


Figure 12: Low-lying states in s-shell nuclei and hypernuclei at a pion mass of $m_\pi \sim 805$ MeV [20, 13].

Since 2004, the NPLQCD collaboration has been calculating the properties and interactions of $A = 2,3,4,5$ systems. The first comprehensive analysis of light nuclei and hypernuclei at $m_\pi \sim 805$ MeV was performed in 2011-2013 [20, 13], the results of which are shown in Figure 12. Generically, one finds that the nuclei are more deeply bound at heavier pion masses. Surprisingly, the scattering parameters extracted from the phase-shift analysis, indicates that the deuteron remains a "fluffy" nucleus over a large range of pion masses and is unlikely to be fine-tuned. It appears that this is a generic feature of a Yang-Mills theory with three "light" quarks. The s-wave scattering phase shifts have been determined at $m_\pi \sim 805$ MeV [13] and $m_\pi \sim 450$ MeV [21] using Lüsher's method, and it is observed that the phase shifts exhibit zero-crossings, and it is close to that of nature in both spin channels at $m_\pi \sim 450$ MeV. The appropriate low-energy effective field theory (EFT) counterterms were constrained, permitting chiral extrapolations. Hyperon-nucleon scattering has been investigated using both Lüsher's method and also by fitting the coefficients of a low-energy effective Hamiltonian at leading order in Weinberg's power counting [32, 33] to the finite-volume energy eigenvalues. The later was done by explicitly diagonalizing the finite-volume Hamiltonian matrix to fit the energy eigenvalues, and then using this Hamiltonian to predict the continuum bound state energies and scattering amplitude [33]. The uncertainties associated with the extrapolated quantities are somewhat larger than experiment, and these are in the process of being reduced through further calculations.

There have been some recent suggestions of a "Plateau Crisis" by the HALQCD collaboration [31, 25], suggesting that results obtained using Lüsher's method have identified false plateaus in their energy spectra. Let me make a few comments about such statements. The implicit suggestion is that the HALQCD method is superior and reliable because it does not require plateaus. My own experience suggests that the only "crisis" that has been encountered is not requiring plateaus in effective masses prior to extracting observables using the HALQCD method. PACS has reproduced the effect from correlation functions that are claimed to demonstrate the "crisis", as shown in Figure 13. As already discussed, the effective masses from the wall sources plateau much latter,

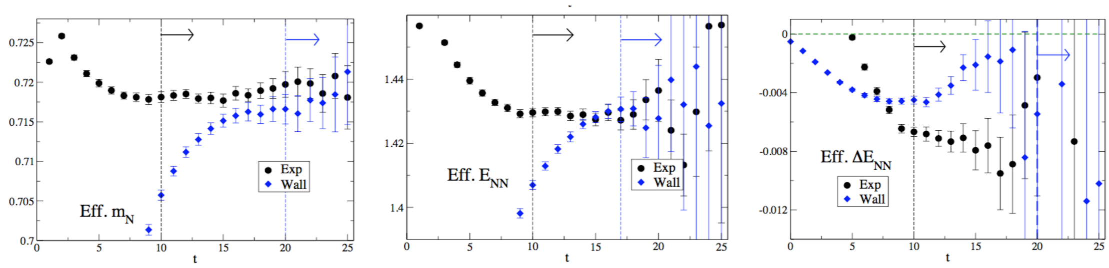


Figure 13: A comparison between correlation functions generated from wall-sources and exponentially-smeared sources by the PACS collaboration. The left panel shows nucleon effective masses, the middle panel shows the two-nucleon effective energies and the right panel shows the binding energies formed from the ratio of correlation functions. [I thank Takeshi Yamasaki for allowing me to show these clarifying figures.]

as shown by the blue points. The binding energy formed from the ratios of wall-source correlation functions exhibits a false plateau in a time interval (around $t \sim 10$) due to the significant contamination from excited states, and should be discarded. Plateau'ing in both wall-source correlation functions occurs for $t \gtrsim 18$, and significantly more statistics are required to extract a statistically meaningful binding energy from those higher time-slices. In contrast, the localized sources, such as the exponentially-smeared sources used to generate the black points in Figure 13, provide plateaus in the effective masses at much earlier times. A plateau can be extracted from the ratio of correlation functions at an earlier time than from the wall-sources, from $t \gtrsim 12$, and is statistically significant. Both NPLQCD and PACS employ smeared local sources for the quark propagators, and both the single-hadron and two-baryon energies have plateaued in the time-slices from which the binding and continuum energies are determined. It would be helpful if collaborations showed effective mass plots associated with the single-hadron and two-hadron systems, along with energy differences, in future publications.

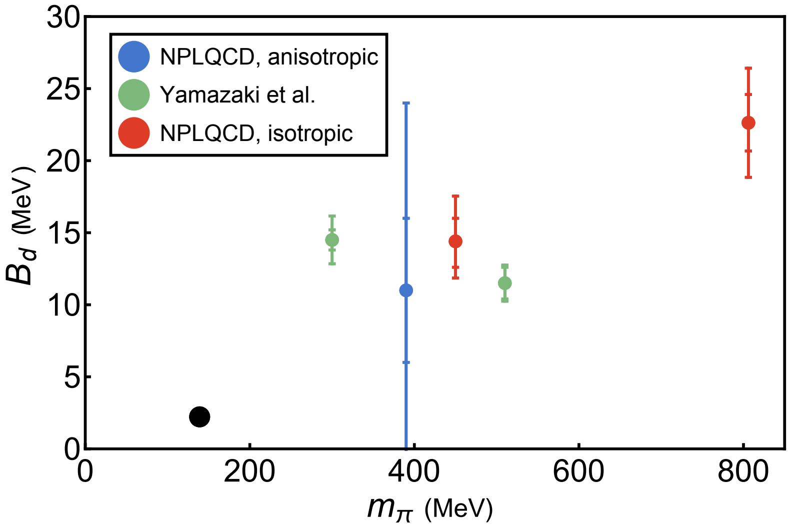


Figure 14: The deuteron binding energy as a function pion mass [34, 20, 18]. In contract, HALQCD does not find a bound deuteron at the heavier pion masses.

Due to its unnaturally small binding energy, resulting from a delicate cancellation between short-, medium- and long-range physics, calculating the deuteron binding energy at the physical point will be challenging, requiring large volumes and fine lattice spacings. A compilation of independent calculations of the deuteron binding energy is presented in Figure 14. The number of calculations is small, and calculations over a range of pion masses is required to extrapolate to the physical point.

One of the exciting recent developments in the field is the first serious efforts to match the results of LQCD calculations to a low-energy EFT, and then predict the properties of nuclei beyond those of the lattice results [35, 36]. This is putting in place, albeit at unphysical quark masses, one of the critical components of the program to be able to make QCD predictions for elements of the Periodic Table. They used the results of the two-nucleon and three-nucleon energy eigenvalues (scattering parameters and binding energies) to constrain the two-nucleon and three-nucleon effective interactions in the pionless EFT, which is appropriate to use in the case of $m_\pi \sim 805$ MeV pions. A comparison between the predicted four-nucleon binding energy and that calculated with the EFT showed that the four-nucleon interactions are small (within the uncertainty of the calculation) and verified the two nucleon and three-nucleon forces. The EFT was then used to predict the binding of ^{5}He, ^{5}Li and ^{6}Li at this pion mass, as shown in Figure 15. This is a major development in the field, and guides the way for connecting future LQCD calculations of multi-nucleon systems to elements far into the Periodic Table.

Another major recent development was the first LQCD calculation of an inelastic nuclear reaction cross section [37]. Using background magnetic fields, the NPLQCD collaboration calculated the low-energy cross section for the radiative capture process $np \to d\gamma$, which is dominated by the M1 amplitude in this energy regime. Performing calculations of the energy splittings of two-nucleon systems in background magnetic fields at pion masses of $m_\pi \sim 805$ MeV and $m_\pi \sim 450$ MeV, the NPLQCD collaboration isolated the correlated two-nucleon interaction in the

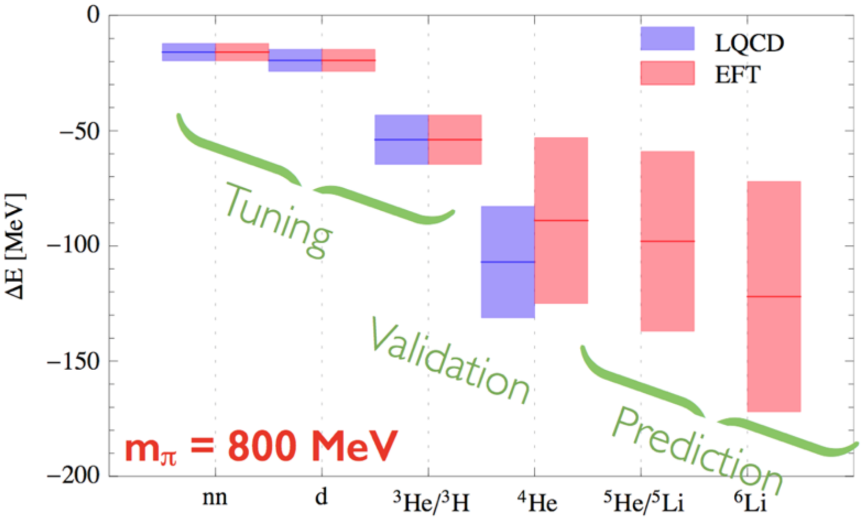


Figure 15: The two-nucleon and three-nucleon scattering parameters and binding energies calculated at $m_\pi \sim 805$ MeV were used to constrain interactions in the pionless EFT. Predictions for the $A = 5,6$ systems are a prediction at this pion mass [35, 36].

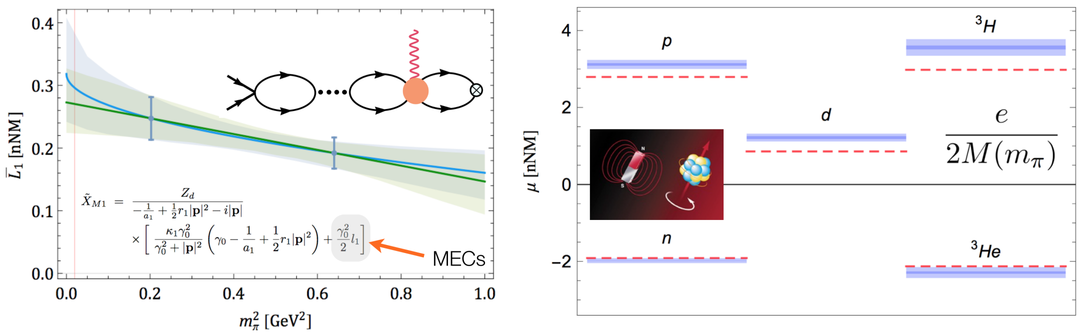


Figure 16: The correlated two-nucleon interaction (meson-exchange currents) extrapolated to the physical pion mass [37] (left panel), and the magnetic moments of the s-shell nuclei at $m_\pi \sim 805$ MeV [38, 39]. The red-dashed horizontal lines correspond to experimental values.

pionless EFT, attributed to meson-exchange currents, and found only mild quark-mass dependence, similar to that observed for the magnetic moments, as shown in Figure 16. Using this quantity extrapolated to the physical pion mass, and the experimentally determine scattering parameters, a cross section of $\sigma^{\rm lccd} = 334.9(5.3)$ mb was predicted at an incident neutron speed of $v = 2,200$ m/s, which is to be compared with the experimental cross section of $\sigma^{\rm lccd} = 334.2(0.5)$ mb. The magnetic moments of the light nuclei have also been calculated and $m_\pi \sim 805$ MeV [38, 39]. When expressed in units of natural Nuclear Magnetons, they agree remarkably well with the corresponding experimental values, as shown in Figure 16.

On a more exotic topic, it is conceivable that the dark matter in our universe are composite particles from a confining gauge theory. It is then plausible that the dark matter is not simply single "hadrons" of this exotic gauge interaction, but also the nuclei that are likely to be created also.

Interesting work in this area was done by Detmold and collaborators at MIT [40, 41], in which they calculated the spectrum of light nuclei resulting from a SU(2) gauge group as a candidate for the dark matter. They found some interesting results with regard to multi-scale dark matter with the possibility of inelastic reactions.

There are several calculations being pursued in multi-baryon systems that are of importance. In the two-baryon systems, calculations of the binding energies are continuing at the physical point by PACS and HALQCD using distinct methods. PACS is focused on nucleon-nucleon interactions from localized smeared sources by direct calculation of the ground state energies, while HALQCD is deriving energy-dependent non-local interactions from wall-sources without requiring plateaus in effective mass plots. NPLQCD and the Mainz LQCD collaboration are pursuing calculations in multiple volumes at unphysical pion masses. As the strange baryons have a less severe signal-to-noise problem than the nucleons, results in the high-strangeness systems are more precise. The progress in calculating electroweak matrix elements in multi-nucleon systems will continue, with first calculations of axial-current matrix elements expected in the near future, including those dictating the cross section for proton-proton fusion and the Gamow-Teller matrix element for tritium β-decay [42]. One also expects to see progress in calculating matrix elements related to $\beta\beta$-decay of nuclei [43].

2. Conclusions

The field of nuclear physics is about to be revolutionized by the ability to reliably calculate low-energy strong interaction quantities using LQCD. The low-lying spectra and simple properties of light nuclei, along with simple nuclear reactions, are now being calculated over a range of pion masses directly from QCD - which was unthinkable just 15 years ago. With exascale computing resources arriving in the near future, precise calculations of light nuclei and their interactions from QCD, at the physical quark masses and including electromagnetism, will become straightforward, and are critical to the success of the experimental programs in both high-energy physics and nuclear physics. One of the really exciting developments witnessed during the last year or so is groups working at the physical point, and one hopes that these efforts can accumulate adequate statistics to reproduce the experimental values of the two-nucleon scattering parameters and the deuteron binding energy.

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