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Viscosity of the pure $SU(3)$ gauge theory revisited

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We compute the Euclidean correlators of the energy-momentum tensor in Yang-Mills theory at finite temperature at zero and finite spatial momenta with lattice simulations. We perform continuum extrapolations of these quantities using $N_\tau = 10, 12, 16$ lattices. We use these correlators to estimate the shear viscosity of the gluon plasma in the deconfined phase.

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1. Introduction

Since relativistic hydrodynamics is quite successul in the interpretation of heavy ion experiments [1], it would be of great interest to calculate the shear viscosity of the quark gluon plasma from first principles. One possible route to determine the viscosity is through the Kubo-formula, relating transport coefficients to the zero frequency behavior of spectral functions:

$$\eta(T) = \pi \lim_{\omega \to 0} \lim_{\mathbf{k} \to 0} \frac{\rho^{12,12}(\omega, \mathbf{k}, T)}{\omega}, \tag{1.1}$$

$$\tag{1.2}$$

paired with the inversion of the integral transformation:

$$C_{\mu\nu,\rho\sigma}(\tau, \mathbf{p}) = \int_0^\infty \mathrm{d}\omega \rho_{\mu\nu,\rho\sigma}(\omega, \mathbf{p}, \mathrm{T}) \frac{\cosh(\omega(\tau - 1/(2\mathrm{T})))}{\sinh(\omega/(2\mathrm{T}))}, \tag{1.3}$$

relating the Euclidean correlators of the energy-momentum tensor $\langle T_{\mu\nu} T_{\rho\sigma} \rangle$ calculable on the lattice to the spectral function appearing in the Kubo formula. Calculations of this kind face great difficulties, as can be clearly seen from Figure 1, that illustrates why this inversion is a well-known ill-posed problem. The inversion being such a hard problem, it is very important that at least the

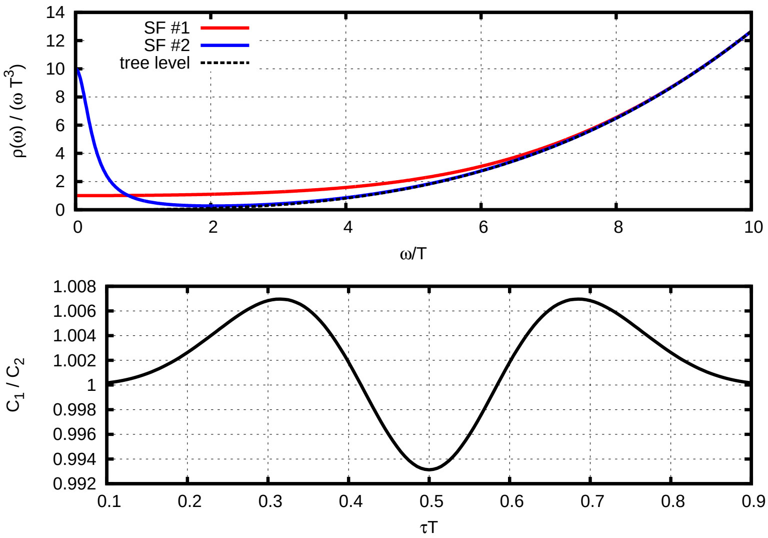

Figure 1: To illustrate the insensitivity of the Euclidean correlators to the IR features of spectral functions, we show the different spectral functions, with the same UV, but different IR features. The corresponding viscosities are different by a factor of 10, but the Euclidean correlators differ by less than 1%. Therefore to have any chance of estimating the viscosity a high precision on the correlators is of great importance.

input information on the correlators is reliable. It is also well known that the stess-energy tensor correlators have a severe sign problem in lattice gauge theory. This for the quenched case has a solution in the multilevel algorithm [2]. This algorithm depend crucially on the locality of the action, and therefore it is hard to generalize for dynamical fermions. Some progress in the regard has been made recently in [3]. Nevertheless, at least in the quenched case, high statistical precision can be achieved with the multilevel algorithm.

Some progress in the problem of viscosity have also been made by H. Meyer [4, 5, 6]. Of particular importance is the following. The following Ward identity:

$$-\omega^2 \rho_{01,01} = \mathbf{q}^2 \rho_{13,13},$$

shows that the UV behavior of $\rho_{01,01}$ is milder, only ω^2 unlike the ω^4 behaviour of $\rho_{13,13}$. This means less UV contamination of the IR signal, and therefore an easier inversion of the integral transform. The thermodynamic identity $-C_{01,01}(\tau, \mathbf{q}=0)/T^5 = s/T^3$ however means we need nonzero momenta to obtain information about the viscosity using this correlator.

In this conference contribution, we look at the continuum extrapolation of the relevant correlators $C_{01,01}$ and $C_{13,13}$ in the quenched theory, since the cut-off effects on previous studies were largely unknown, and as we argued, a great precision on the correlators is very important in this area.

2. Lattice details

For our study, we use anisotropic lattices, with renormalized anisotropy: $\xi_R = 2$. For anisotropy tuning we use the Wilson flow technique introduced in [7]. We use the multilevel algorithm to reduce errors near $\tau T = 0.5$, and the Tree level Symanzik improved gauge action to reduce cut-off effects. We use the clover discretization of the energy momentum tensor, maily because the center of the operator is always on a site, therefore the separation is always an integer in lattice units. We have ensembles at two different temperatures: $1.5T_c$ and $2T_c$, and the following lattice geometries:$80\times 20^2 \times 20$, $64\times 16^2\times 16$, $48\times 12^2\times 12$, $40\times 10^2\times 10$. The long direction is needed so that we can have small spatial momenta, to use hydrodynamics prediction for our fits.

2.1 Anisotropy tuning

The bare anisotropy $\xi_0(\beta)$ is tuned so that $\chi_R \equiv 2$. For the tuning we define a *spatial* and a *temporal* w_0 scale[7]:

$$\left[\tau \frac{d}{d\tau} \tau^2 \langle E_{ss}(\tau)\rangle\right]_{\tau=w_{0,s}^2} = 0.15\,, \tag{2.1}$$

$$\left[\tau \frac{d}{d\tau} \tau^2 \langle E_{ts}(\tau)\rangle\right]_{\tau=w_{0,t}^2} = 0.15\,, \tag{2.2}$$

with

$$E_{ss}(\tau) = \frac{1}{4}\sum_{x,i\neq j} F_{ij}^2(x,\tau)\,, \tag{2.3}$$

$$E_{st}(\tau) = \xi_R^2 \frac{1}{2}\sum_{x,i} F_{i4}^2(x,\tau)\,. \tag{2.4}$$

The tune the anisotropy we need to perform simulations with several bare anisotropies and interpolate to the value where $w_{0,s} = w_{0,t}$ is satisfied.

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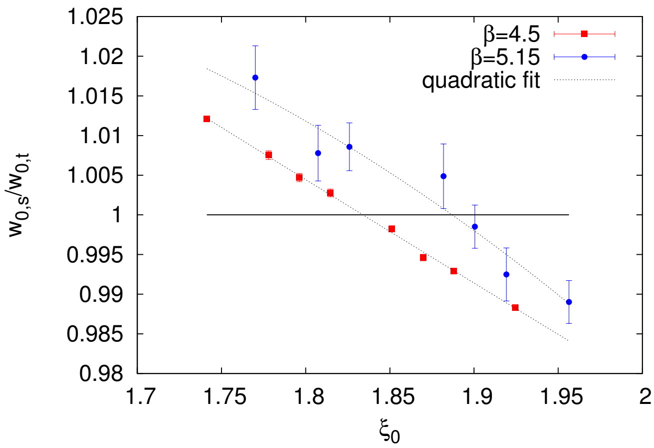

Figure 2: Anisotropy tuning with simulations at different bare anisotropies.

2.2 Renormalization

The overall constant Z_6^{ts} can be determined form the thermodynamic identity:

$$C_{01,01}(\tau,\mathbf{q}=\mathbf{0})/T^5 = s/T^3$$

For the renormalization of $C_{13,13}$, we will use shifted boundary conditions. The bare energy momentum tensor requires multiplicative renormalization, with separate factors of the sextet, triplet and singlet components.

$$T^R_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 (T^{[1]}_{\mu\nu} - T^{[1]}_{\mu\nu}(T=0))$$

with the definitions (no sum over μ and ν):

$$\begin{aligned}
T^{[6]}_{\mu\nu} &= \frac{1}{g_0^2}\sum_{\sigma} F^a_{\mu\sigma}F^a_{\nu\sigma} \\
T^{[3]}_{\mu\nu} &= \delta_{\mu\nu}\frac{1}{g_0^2}\left\{\sum_{\rho} F^a_{\mu\rho}F^a_{\nu\rho} - \frac{1}{4}\sum_{\rho,\sigma} F^a_{\rho\sigma}F^a_{\rho\sigma}\right\} \\
T^{[1]}_{\mu\nu} &= \delta_{\mu\nu}\frac{1}{g_0^2}\sum_{\sigma,\rho} F^a_{\rho\sigma}F^a_{\rho\sigma}
\end{aligned}$$

We use the clover definition of $F^a_{\mu\nu}$ and define our correlators from the sextet (off-diagonal) components. In the presence of an anisotropy Z_6 splits into three different renormalization constants:

$$\begin{aligned}
T_{01} &= \frac{Z_6^{ts}}{g_0^2} F^a_{02}F^a_{12} + \frac{Z_6^{ts}}{g_0^2} F^a_{03}F^a_{13} \\
T_{12} &= \frac{Z_6^{tt}}{g_0^2} F^a_{01}F^a_{02} + \frac{Z_6^{ss}}{g_0^2} F^a_{13}F^a_{23}
\end{aligned}$$

For an isotropic gauge action the renormalization constants have been worked out with shifted boundary conditions in [8]. Using shifted boundary conditions with shift vector $\vec{\xi} = (\xi_1, \xi_2, \xi_3) = (1,1,1)$ the off-diagonal T_{0i} components develops a non-vanishing expectation value. Since the directions are equivalent, requiring $T_{01} = T_{02} = T_{03}$ gives:

$$2Z_6^{tt}\frac{1}{g_0^2}F_{02}^aF_{12}^a = 2Z_6^{ss}\frac{1}{g_0^2}F_{03}^aF_{13}^a = Z_6^{st}\frac{1}{g_0^2}(F_{01}^aF_{21}^a + F_{03}^aF_{23}^a)$$

Therefore the ratios Z_6^{ss}/Z_6^{ts} and Z_6^{tt}/Z_6^{ts} can be calculated from a single simulation with $L_0^{-1} = \sqrt{1+|\vec{\xi}|^2}T = 2T$.

Thus, e.g. to renormalize $T_{\mu\nu}$ in a $N_\tau = 12$ simulation with $\xi_R = 2$, we make an auxilliary run on a $48 \times 96 \times 48 \times 3$ lattice with the same bare parameters. The resulting factors will depend on β and N_τ, and the method requires that $N_\tau/4$ is an integer. We observe an $1/N_\tau^2$ scaling. For the renormalization of $N_\tau = 10$ we can interpolate in N_τ.

3. Results on the correlators

Some renormalized correlators can be seen in Figure 3. Continuum limit extrapolations at the middle point $\tau T = 1/2$ can be seen in Figure 4. As can be seen, for $C_{13,13}$ we found cut-off errors of approx. 3% for $N_t = 16$ at $\tau T = 1/2$. Thus, by contemplating Figure 1, it is easy to see that the loss of precision from not doing a continuum extrapolation of this quantity could potentially be fatal for the estimate of the viscosity.

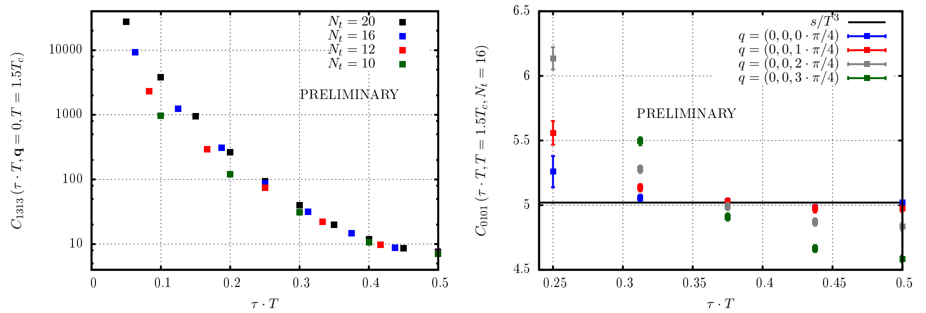

Figure 3: Renormalized correlators $\langle T_{13}T_{13}\rangle$ at zero spatial momentum and different lattice spacings(left) and renormalized correlators $\langle T_{01}T_{01}\rangle$ for $N_t = 16$ and different spatial momenta(right). Both are at $T = 1.5T_c$.

4. Esimation of the viscosity

To fit the spectral function we will use and ansatz that use the prediction of hydrodynamics at low ω [9]:

$$-\frac{\rho_{01,01}^{(hydro)}}{\omega} = \frac{\eta}{\pi}\frac{q^2}{\omega^2 + (\eta q^2/(sT))^2}, \tag{4.1}$$

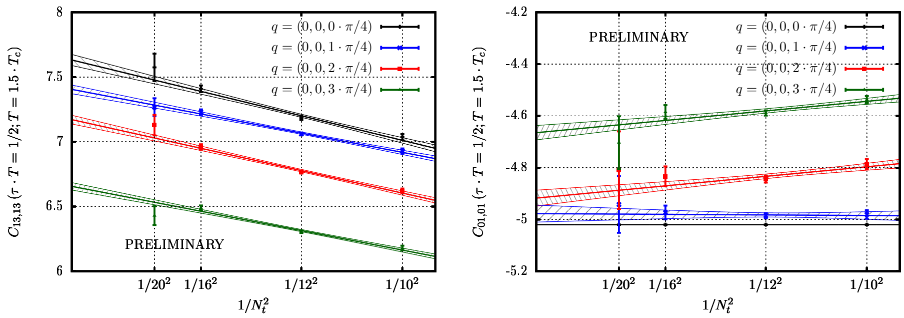

Figure 4: Continuum limit extrapolations at the point $\tau T = 1/2$ for $\langle T_{13}T_{13}\rangle$(left) and $\langle T_{01}T_{01}\rangle$(right).

and leading order perturbation theory at high frequency [5]:

$$-\rho^{(pert)}_{01,01} = \frac{d_A}{8(4\pi)^2} q^2(\omega^2 - q^2)\mathscr{I}([1-z^4], \omega, q, T) \tag{4.2}$$

$$\mathscr{I}([P[z]], \omega, q, T) = \theta(\omega - q)\int_0^1 dz \frac{P(z)\sinh(\omega/2T)}{\cosh(\omega/2T) - \cosh(qz/2T)} + \tag{4.3}$$

$$+\theta(-\omega + q)\int_1^\infty \frac{-P(z)\sinh(\omega/2T)}{\cosh(\omega/2T) - \cosh(qz/2T)} \tag{4.4}$$

where we only take the part $\omega > q$, since the $\omega < q$ part described the transport properties of a free gas of gluons, which we substitute with the ansatz from hydrodynamics(strong coupling). Our ansatz assumes the hydrodynamic prediction for the spectral function, strictly only valid for $\omega \ll T$, can be extended to higher frequencies. This is true for $\mathscr{N} = 4$ SYM theory, where AdS/CFT can be used to calculate the spectral function[9]. Our ansatz can not produce a quasiparticle peak, that would appear in weak coupling treatements of QCD, like kinetic theory [10, 11]. Also, our ansatz only has two parameters: C and η/s. C describes the extent to which the leading order prediction for the UV part of the spectral function gets changes, while η/s is the transport coefficient we want to estimate.

We present two different fits. First we use C_{0101} as a function of τ and $\mathbf{q}$ at $N_t = 16$. The choice of channel is motivated by the smaller cut-off errors compared to C_{1313}. Out preliminary results are:

T	η/s	C
$1.5T_c$	0.18(2)(2)(?)	0.69(3)(0)(?)
$2.0T_c$	0.16(2)(3)(?)	0.72(6)(0)(?)

Here the first error is statistical only. The second error is systematic error coming from the choice of τ_{min} and q_{max}. The (?) is stand-in for unkown sytematic errors coming from the choice of the ansatz. For the second fit we use the $q_3/(\pi/4) = 0, 1, 2$ dependence of $C_{0101}(\tau T = 0.5)$ and $C_{1313}(\tau T = 0.5)$ in the continnum. Our results are:

T	η/s	C
$1.5T_c$	0.13(2)(?)	0.67(2)(?)
$2.0T_c$	0.11(2)(?)	0.72(3)(?)

This is the first estimate of η/s using continuum extrapolated data. It is nevertheless consistent with earlier estimates [4, 12].

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PoS(LATTICE2016)073