PROCEEDINGS OF SCIENCE

Two-flavor simulations of the $\rho(770)$ and the role of the $K\bar{K}$ channel

Bin Hu*
Department of Physics, The George Washington University
E-mail: binhu@gwmail.gwu.edu

Raquel Molina
Department of Physics, The George Washington University
E-mail: ramope71@email.gwu.edu

Michael Doring
Institute for Nuclear Studies and Department of Physics, The George Washington University,
Thomas Jefferson National Accelerator Facility
E-mail: doring@gwu.edu

Andrei Alexandru
Department of Physics, The George Washington University
E-mail: aalexan@gwu.edu

Dehua Guo
Department of Physics, The George Washington University
E-mail: guodehua@email.gwu.edu

The $\rho(770)$ meson is the most extensively studied resonance in lattice QCD simulations in two ($N_f = 2$) and three ($N_f = 2+1$) flavors. We analyze all available phase shifts from $N_f = 2$ simulations using unitarized Chiral Perturbation Theory, allowing not only for the extrapolation in mass but also in flavor, $N_f = 2 \to N_f = 2+1$. The $K\bar{K}$ channel can have a significant effect within the model and leads to $\rho(770)$ masses surprisingly close to the experimental one.

34th annual International Symposium on Lattice Field Theory
24-30 July 2016
University of Southampton, UK

*Speaker.

PoS(LATTICE2016)096

1. Introduction

The development of algorithms and increasing computing resources makes it feasible to extract phase shifts from lattice-QCD simulations, like the simulation of the simplest hadronic system containing a resonance, $I = 1$ elastic $\pi\pi$ scattering via the $\rho(770)$ resonance, providing a more and more accurate determination of the amplitude and a test ground to benchmark new techniques. These proceedings report our work on two-flavor simulations of the $\rho(770)$; for more details, see Ref. [1].

A recent lattice QCD study from the GWU lattice group [2] indicates that the ρ mass extracted from a $N_f = 2$ simulation at $m_\pi = 227$ MeV is lighter than its physical value. This result is supported by an independent calculation using a pion mass very close to the physical value from the RQCD Collaboration [3].

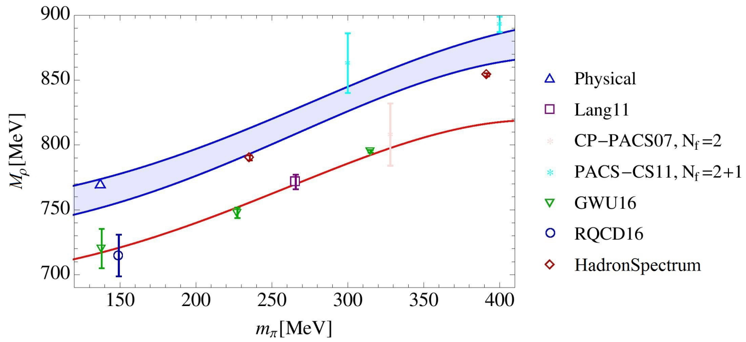

Figure 1: Collection of lattice simulations in (M_π, m_ρ) plane. For the GWU16 data, also the SU(2) extrapolation to the physical mass is shown. Note that this simulation and the RQCD16, Lang11, and CP-PACS07 results are based on $N_f = 2$ simulations while the PACS-CS11 and HadronSpectrum results are based on $N_f = 2+1$ simulations. Red curve: an $N_f = 2$ extrapolation based on the fit to data from Ref. [2]; Blue band: inclusion of the $K\bar{K}$ channel (systematic error only).

The ρ mass discrepancy is clearly shown in Fig. 1, where we collect $N_f = 2$ and $N_f = 2+1$ lattice simulation results from [2, 3, 4, 5, 6, 7, 8, 9] in the (M_π, m_ρ) plane. All the points of $N_f = 2$ simulations seem to follow the same trend, exhibiting too light of a ρ mass at the physical pion mass. The (M_π, m_ρ) values of $N_f = 2+1$ simulations also seem to follow a common trend, but with higher ρ mass that seems to be compatible with the physical ρ mass at physical pion mass. The simulations distinguished by the presence or absence of the strange quark are obviously separated by a gap. This suggests studying the hypothesis that the problem of too light m_ρ can be explained by the missing strange flavor in $N_f = 2$ calculations, or, formulated in terms of hadrons as degrees of freedom, the absence of the $K\bar{K}$ channel.

On the other hand, the $K\bar{K}$ phase shift is small [6, 10], and so is the inelasticity of the $\pi\pi$ amplitude in the ρ channel [11]. It seems then natural to conclude an effectively decoupling of the ρ from the $K\bar{K}$ channel that apparently does not play a significant role in $I = L = 1$ $\pi\pi \to \pi\pi$ scattering. Indeed, the contribution of real kaons are suppressed through the centrifugal barrier. The effects of the virtual kaons below the threshold, however, can not be a priori neglected. A simple analogy of the $\pi\pi$ scattering mechanism with the insertion of an intermediate $K\bar{K}$ states is shown in Fig. 2, the size of which is approximately proportional to $|p_{cm}^2(\pi\pi)p_{cm}^2(K\bar{K})G_{K\bar{K}}|$. The

$p^2_{cm}(\pi\pi)p^2_{cm}(K\bar{K})$ factor comes from the $p-$wave contribution, where p_{cm} stands for momenta in the center of mass frame. The quantity $G_{K\bar{K}}$ denotes the $K\bar{K}$ loop, including the dispersive parts. This simple picture shows a peak around the 770 MeV that can effectively shift the ρ mass. The full, unknown interaction is certainly more complicated and different from this simple analogy that serves here only for illustration. Besides, the substantial shift of the ρ mass might induce a unphysically large inelasticity and a large $K\bar{K}$ phase shift above the threshold which is why one has to check the effects of the $K\bar{K}$ channel in our calculation with all phenomenologically available constraints.

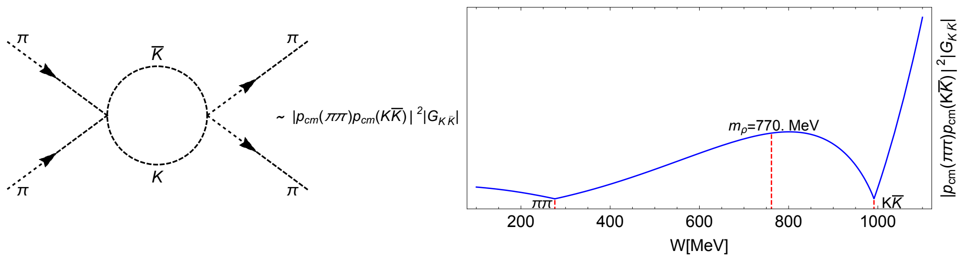

Figure 2: Simple analogy to motivate the possible effect from the $K\bar{K}$ channel. Left: insertion of an intermediate $K\bar{K}$ state in $\pi\pi$ scattering; picture on the right: prediction of a mass shift around the ρ position.

2. Method

We use a model based on unitarized chiral perturbation theory (UCHPT) first developed in Ref. [10], allowing not only for extrapolations to the physical pion mass, but also for transformations between flavors of $N_f = 2$ and $N_f = 2+1$ to study the role of the strange quark. This UCHPT model includes the $K\bar{K}$ contribution in a SU(3) coupled-channel formulation and resums it to infinite order. To satisfy the unitarity relation, the next to leading order (NLO) contact terms are taken into account in the inverse amplitude formulation. In our work, we use dimensional regularization instead of cut-off. Also, there is a minor modification compared to Ref. [10] in $I = 0$ πK, $\pi\eta$ scattering at high energies [12]. For the extrapolation of f_π, the M_π dependence of Refs. [13, 14], summarized in Ref. [15], is used. The workflow of the extrapolation to the three-flavor, physical point is as follows (see also App. B of Ref. [2] for a similar procedure):

1. To fit the $N_f = 2$ lattice shifts, the $K\bar{K}$ channel is removed from the coupled channel $\pi\pi$ and $K\bar{K}$ system, getting a pure SU(2) model. The low-energy constants (LECs) in the $I = 1$ $L = 1$ SU(2) $\pi\pi \to \pi\pi$ transition are expressed in linear combinations $\hat{l}_1 = 2L_4 + L_5$, $\hat{l}_2 = 2L_1 - L_2 + L_3$, which are the two parameters to be fitted.

2. Lattice $N_f = 2$ phase shift data are fitted with the combined LECs $\hat{l}_1$ and $\hat{l}_2$, taking into account the internal correlation between energy eigenvalues when available. The errors from both axes (ΔW, $\Delta\delta$), namely inclined errors in the direction of the Lüscher function, are also considered. In the fit, the data points are chosen in the maximal range around the resonance position fulfilling Pearson's χ^2 test at a 90% upper confidence limit.

3. The fit results are extrapolated to the physical pion mass and the three physical flavors by switching on the $\pi\pi \to K\bar{K}$ and $K\bar{K} \to K\bar{K}$ channel transitions, to obtain a postdiction of experi-

mental phase shifts. The unknown LECs that appear in the SU(3) model but not in the SU(2) model are taken from a global fit to $\pi\pi$ and πK experimental data.

4. The results of fitted LECs and their uncertainties at the physical point are converted to commonly used notations. All phase shifts are fitted with the usual Breit-Wigner (BW) parameterization in terms of g and m_ρ (see, e.g., Ref. [2]).

3. Results

The $N_f = 2$ lattice simulations from Refs. [2, 3, 4, 5, 16, 17] are analyzed. The 68% confidence error ellipses in $\hat{l}_1, \hat{l}_2$ from RQCD [3], GWU [2], Lang et al. [4], and CP-PACS [5] are shown together in Fig. 1 in the supplement of Ref. [1], as a consistency check of the fits. Most of these ellipses have common overlaps except the ellipse from QCDSF [17] that is slightly off, and the ellipse from ETMC [16] that is clearly incompatible.

In Fig. 3, the pictures on the left show the lattice data and the UCHPT fits. The complete lattice data are plotted in black while the selected data points are highlighted in red. The pictures on the right are combinations of experimental ρ phase shifts [11, 18] (data with error bars), $N_f = 2$ extrapolations to the physical pion mass (blue dashed curves), and $N_f = 2+1$ extrapolations to the physical pion mass (red solid curves) as UCHPT prediction of the effects of the $K\bar{K}$ channel. Experimental data are postdicted.

In the following discussion, we exclude data from Aoki *et al.*/PACS-CS [5] (2 measured phase shifts fitted with 2 parameters) and Feng *et al.*/ETMC [16] because the uncertainties are very large, even when simultaneously fitting data from two different pion masses. The $N_f = 2$ extrapolations always lead to ρ resonance lighter than its physical position. When the strange flavor is included in the $N_f = 2+1$ physically extrapolation, the $K\bar{K}$ channel includes a significant shift of the ρ mass to the heavier direction and leads to a good postdiction of experimental data. These effects of extrapolations are apparent in the (m_ρ, g) plane, see Fig. 4, that shows the Breit-Wigner coupling g against the Breit-Wigner mass. The shift of the ρ mass also explains the gap shown in Fig. 1, where the red curve is an $N_f = 2$ extrapolation based on the fit to data from Ref. [2], while the blue band is the $N_f = 2+1$ extrapolation including the $K\bar{K}$ channel, showing only the systematic error. For a detailed discussion of the systematic uncertainty see Ref. [1].

The checks of inelasticity from the $K\bar{K}$ channel up to $W = 1.15$ GeV were made for all the fits we have done in this work, and none of the results exceed the total inelasticity observed [11], as is shown in Fig. 5. Our prediction is of similar size as the $K\bar{K}$ inelasticity derived in Ref. [19] from the Roy-Steiner solution of Ref. [20] (dashed line in the central panel of Fig. 5). To further check the size of inelasticities in the present model, we take the extrapolation of the $N_f = 2$ simulation of Ref. [4] as a representative for our different results and extrapolate it to $N_f = 2+1$ at $M_\pi = 236$ MeV, which is the pion mass corresponding to the $N_f = 2+1$ lattice phase shifts from Ref. [6]. The first and third picture combine our predictions of $K\bar{K}$ phase shift and elasticity (red curve) and the results of the two-channel fits of Ref. [6] to lattice eigenvalues obtained at this pion mass (green and gray bands). The middle picture compares our predicted elasticity with the results from experiments [11] (data with error bars) and the Roy-Steiner determination [19] (black dashed line). There is no conflict of our results with the observed small inelasticity and $K\bar{K}$ phase shift in experiments and lattice simulations.

PoS(LATTICE2016)096

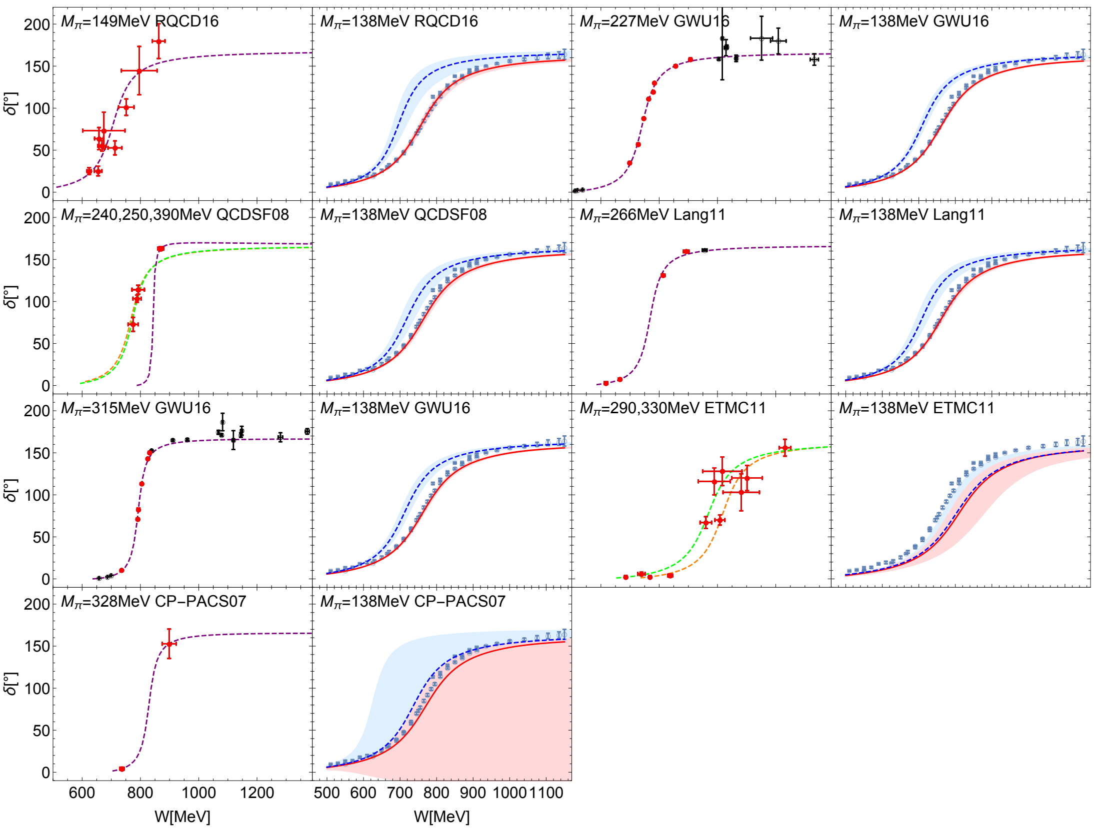

Figure 3: Results for the $N_f = 2$ lattice simulations (ordered by pion mass) of Bali *et al.*/RQCD [3], Guo *et al.*/GWU [2], Göckeler *et al.*/QCDSF [17], Lang *et al.* [4], Feng *et al.*/ETMC [16], Aoki *et al.*/CP-PACS [5]. For each result, the left picture shows the lattice data and fit (dashed line, multiple colors (purple, green, orange) stand for lattice data at different M_π in combined fit), the right figure shows the $N_f = 2$ chiral extrapolation (blue dashed line/light blue area). Without changing this result, the $K\bar{K}$ channel is then included to predict the effect from the missing strange quark (red solid line/light red area). Experimental data (blue circles from [18], squares from [11]) are then postdicted.

4. Summary

As discussed in Ref. [1], the $K\bar{K}$ channel improves the chiral extrapolations of the ρ mass to the physical point significantly except when the lattice data have large uncertainties. The systematically small lattice masses at the physical point after the chiral SU(2) extrapolations of $N_f = 2$ lattice QCD simulation can be explained with the model (that discrepancy is apparent even without invoking any chiral extrapolation for the recent $N_f = 2$ simulation by the RQCD group carried out almost at the physical mass). Although the role of the $K\bar{K}$ channel provides a possible explanation, we do observe slight over-extrapolations in some fits, as shown in Fig. 4. A possible explanation lies in the omission of $t-$ and $u-$channel loops in the present study that will be addressed in future work.

Acknowledgments: This work is supported by the National Science Foundation (CAREER grants PHY-1452055 and PHY-1151648, PIF grant No. 1415459) and by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under contract DE-AC05-06OR23177 and

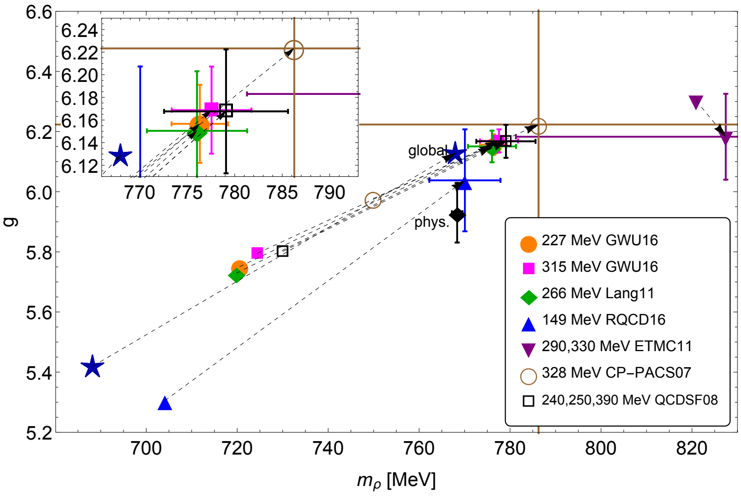

Figure 4: Effect of the $K\bar{K}$ channel in the (m_ρ, g) plane indicated with arrows, after chiral extrapolation to the physical pion mass. The case for the experimental point is indicated as "phys.", the case for a global fit to experimental $\pi\pi$ and πK phase shifts is indicated as "global". Only statistical uncertainties are shown, and only for the case after including $K\bar{K}$.

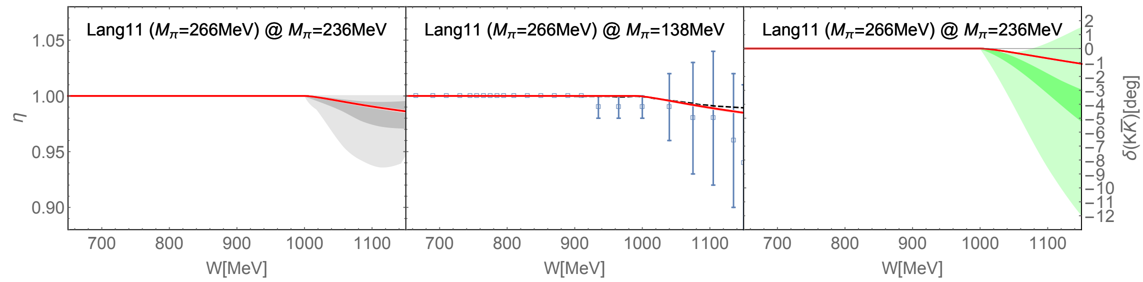

Figure 5: First and third picture: elasticity η and $K\bar{K}$ phase shift at $M_\pi = 236$ MeV. The green and gray bands show the results of the two-channel fits of Ref. [6] to lattice eigenvalues obtained at this pion mass. The dark red bands show the predictions of this study with systematic uncertainties (very narrow light red bands). The middle picture shows the elasticity at the physical pion mass, together with the elasticity determined from experiment [11] (data points) and the $K\bar{K}$ contribution to the inelasticity evaluated in Ref. [19] from the Roy-Steiner determination of Ref. [20] (black dashed line).

grant DE-FG02-95ER40907.

References

[1] B. Hu, R. Molina, M. Döring and A. Alexandru, *Two-flavor Simulations of the $\rho(770)$ and the Role of the $K\bar{K}$ Channel*, *Phys. Rev. Lett.* **117**, no. 12, 122001 (2016) [arXiv:1605.04823 [hep-lat]].

[2] D. Guo, A. Alexandru, R. Molina, and M. Döring, *Rho resonance parameters from lattice QCD*, *Phys. Rev. D* **94**, no. 3, 034501 (2016) [arXiv:1605.03993 [hep-lat]].

[3] G. S. Bali *et al.* [RQCD Collaboration], *ρ and K^* resonances on the lattice at nearly physical quark masses and $N_f = 2$*, *Phys. Rev. D* **93**, 054509 (2016) [arXiv:1512.08678 [hep-lat]].

[4] C. B. Lang, D. Mohler, S. Prelovsek and M. Vidmar, *Coupled channel analysis of the rho meson decay in lattice QCD*, *Phys. Rev. D* **84**, 054503 (2011); Erratum: [Phys. Rev. D **89**, 059903 (2014)] [arXiv:1105.5636 [hep-lat]].

[5] S. Aoki *et al.* [CP-PACS Collaboration], *Lattice QCD Calculation of the rho Meson Decay Width*, *Phys. Rev. D* **76**, 094506 (2007) [arXiv:0708.3705 [hep-lat]].

[6] D. J. Wilson, R. A. Briceño, J. J. Dudek, R. G. Edwards and C. E. Thomas, *Coupled $\pi\pi, K\bar{K}$ scattering in P-wave and the ρ resonance from lattice QCD*, *Phys. Rev. D* **92**, 094502 (2015) [arXiv:1507.02599 [hep-ph]].

[7] J. J. Dudek *et al.* [Hadron Spectrum Collaboration], *Energy dependence of the ρ resonance in $\pi\pi$ elastic scattering from lattice QCD*, *Phys. Rev. D* **87**, 034505 (2013); Erratum: [Phys. Rev. D **90**, 099902 (2014)] [arXiv:1212.0830 [hep-ph]].

[8] N. Ishizuka [PACS-CS Collaboration], *ρ meson decay width from 2+1 flavor lattice QCD*, *PoS LATTICE* **2011**, 125 (2011) [arXiv:1111.0377 [hep-lat]].

[9] S. Dürr *et al.* [Budapest-Marseille-Wuppertal Collaboration], *Lattice QCD at the physical point meets SU(2) chiral perturbation theory*, *Phys. Rev. D* **90**, no. 11, 114504 (2014) [arXiv:1310.3626 [hep-lat]].

[10] J. A. Oller, E. Oset and J. R. Peláez, *Meson meson interaction in a nonperturbative chiral approach*, *Phys. Rev. D* **59**, 074001 (1999); Erratum: [Phys. Rev. D **60**, 099906 (1999)]; Erratum: [Phys. Rev. D **75**, 099903 (2007)] [hep-ph/9804209].

[11] S. D. Protopopescu *et al.*, *Pi pi Partial Wave Analysis from Reactions pi+ p —> pi+ pi- Delta++ and pi+ p —> K+ K- Delta++ at 7.1-GeV/c*, *Phys. Rev. D* **7**, 1279 (1973).

[12] M. Döring, U.-G. Meißner and W. Wang, *Chiral Dynamics and S-wave Contributions in Semileptonic B decays*, *JHEP* **1310**, 011 (2013) [arXiv:1307.0947 [hep-ph]].

[13] J. Gasser and H. Leutwyler, *Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark*, *Nucl. Phys. B* **250**, 465 (1985).

[14] J. Nebreda and J. R. Peláez, *Strange and non-strange quark mass dependence of elastic light resonances from SU(3) Unitarized Chiral Perturbation Theory to one loop*, *Phys. Rev. D* **81**, 054035 (2010) [arXiv:1001.5237 [hep-ph]].

[15] R. Molina and M. Döring, *Pole structure of the $\Lambda(1405)$ in a recent QCD simulation*, Phys. Rev. D **94**, no. 5, 056010 (2016) Addendum: [Phys. Rev. D **94**, no. 7, 079901 (2016)] [arXiv:1512.05831 [hep-lat]].

[16] X. Feng, K. Jansen and D. B. Renner, *Resonance Parameters of the rho-Meson from Lattice QCD*, *Phys. Rev. D* **83**, 094505 (2011) [arXiv:1011.5288 [hep-lat]].

[17] M. Göckeler *et al.* [QCDSF Collaboration], *Extracting the rho resonance from lattice QCD simulations at small quark masses*, *PoS LATTICE* **2008**, 136 (2008) [arXiv:0810.5337 [hep-lat]].

[18] P. Estabrooks and A. D. Martin, *pi pi Phase Shift Analysis Below the K anti-K Threshold*, *Nucl. Phys. B* **79**, 301 (1974).

[19] F. Niecknig, B. Kubis and S. P. Schneider, *Dispersive analysis of $\omega -> 3\pi$ and $\phi -> 3\pi$ decays*, *Eur. Phys. J. C* **72**, 2014 (2012) [arXiv:1203.2501 [hep-ph]].

[20] P. Büttiker, S. Descotes-Genon and B. Moussallam, *A new analysis of pi K scattering from Roy and Steiner type equations*, *Eur. Phys. J. C* **33**, 409 (2004) [hep-ph/0310283].