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Testing the hadro-quarkonium model on the lattice

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Recently the LHCb experiment found evidence for the existence of two exotic resonances consisting of $c\bar{c}uud$ quarks. Among the possible interpretations is the hadro-charmonium model, in which charmonium is bound "within" a light hadron. We test this idea on CLS N_f=2+1 lattices using the static formulation for the heavy quarks. We find that the static potential is modified by the presence of a hadron such that it becomes more attractive. The effect is of the order of a few MeV.

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1. Motivation

The LHCb collaboration analysed the decay $\Lambda_b \to J/\psi\ p\ K$ [1, 2]. A satisfactory description of the data is obtained by adding to the $\Lambda^* \to p\ K$ resonances two additional resonances of exotic quark content $uudc\bar{c}$ labeled by $P_c^+(4380)$ ($J^P = \frac{3}{2}^-$) and $P_c^+(4450)$ ($J^P = \frac{5}{2}^+$) and decaying strongly into $J/\psi\ p$. Flipping the two parities can also explain the data [1, 2]. Attractive forces between charmonium and pp systems have been previously conjectured, e.g., to explain the rapid change in the behavior of polarized pp scattering around $\sqrt{s} = 5\,\mathrm{GeV} \approx m_p + m_p + m_{J/\psi}$ [3].

Five (4 q, 1 $\bar{q}$) quark systems are very difficult to study directly on the lattice. For example see [4] for a study of a charmonium-nucleon system. Here we test a particular model instead, hadro-quarkonium. In this model quarkonia are bound "within" ordinary hadrons [5]. Examples of charmonium-baryon systems which are close in energy to the LHCb pentaquark candidates are $m(\Delta) + m(J/\psi) \approx 4329\,\mathrm{MeV}$ for $J^P = \frac{3}{2}^-$ and $m(N) + m(\chi_{c2}) \approx 4496\,\mathrm{MeV}$ for $J^P = \frac{5}{2}^+$.

2. Hadro-quarkonia in the static limit

The hadro-quarkonium model can be tested in the static quark limit. To leading order in potential non-relativistic QCD, quarkonia can be approximated by the non-relativistic Schrödinger equation with a static quark-antiquark potential $V_0(r)$. The question we want to answer in our study [6] is whether the static potential becomes more or less attractive, when light hadrons are "added". For this we create a zero-momentum projected hadronic state $|H\rangle$ at the time 0. We let it propagate for an interval δt and we then create a quark-antiquark "string". They propagate together for a time interval t. We destroy the string at time $t + \delta t$ and finally the light hadron at time $t + 2\delta t$. We compute the correlator

$$C_H(r, \delta t, t) = \frac{\langle W(r,t) C_{H,\mathrm{2pt}}(t + 2\delta t)\rangle}{\langle W(r,t)\rangle \langle C_{H,\mathrm{2pt}}(t + 2\delta t)\rangle}\,, \tag{2.1}$$

where we average over the spatial positions of the Wilson loop $W(r,t)$ and over the hadronic sink positions in the hadronic two-point function $C_{H,\mathrm{2pt}}$. The difference between the static potential in the presence of a hadron V_H and the potential in the vacuum V_0 can be obtained from

$$\Delta V_H(r, \delta t) \equiv V_H(r, \delta t) - V_0(r) = -\lim_{t\to\infty} \frac{\mathrm{d}}{\mathrm{d}t} \ln[C_H(r, \delta t, t)] \tag{2.2}$$

and extrapolating $\delta t \to \infty$.

3. Lattice results

We analyse the $N_\mathrm{f} = 2 + 1$ CLS ensemble "C101" which has 96×48^3 sites, $m_\pi = 220\,\mathrm{MeV}$, $m_\mathrm{K} = 470\,\mathrm{MeV}$, $Lm_\pi \approx 4.6$, $L \approx 4.1\,\mathrm{fm}$, $t_0/a^2 = 2.9085(51)$ [7]. It has been simulated using the publicly available openQCD package [8]. The lattice spacing $a = 0.0854(15)\,\mathrm{fm}$ is determined from the scale $\sqrt{8t_0}/a$ [9] extrapolated to physical point [10] and using $\sqrt{8t_0} = 0.4144(59)(37)\,\mathrm{fm}$ [11]. We perform a large statistics calculation consisting of 1552 configurations separated by 4 MDUs, times 12 hadron sources (providing 10 forward and backward propagating two-point functions and for the two sources closest to the open temporal boundaries a forward and a backward

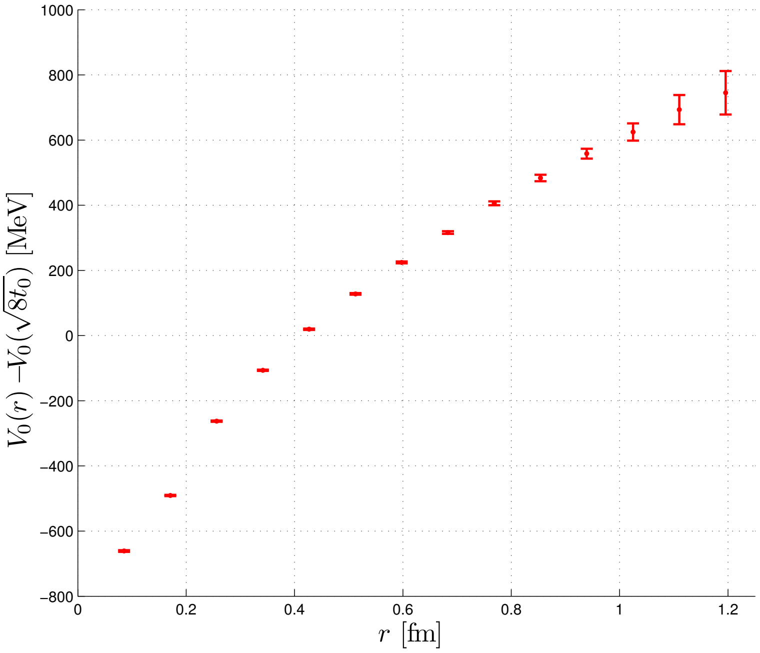

Figure 1: The static potential in the vacuum measured on the CLS ensemble "C101".

two-point function for a total of 22 correlation functions). Wilson loops are measured at all positions and in each direction separately. Hadronic two-point functions and Wilson loops are smeared to optimize their overlap with the respective ground states. We measure ΔV_H for π, K, ρ, $K^\star$ and ϕ mesons; for N, Σ, Λ and Ξ octet baryons with $J^P = \frac{1}{2}^\pm$; and for Δ, $\Sigma^\star$, $\Xi^\star$ and Ω decuplet baryons with $J^P = \frac{3}{2}^\pm$.

We begin the presentation of the results by showing in Fig. 1 the static potential $V_0(r)$ in the vacuum. It has been determined using the methods of [12]. We plot $V_0(r)$ for distances $r \leq 1.2\,\mathrm{fm}$ below the string breaking region.

In order to extract the energy difference ΔV_H in Eq. (2.2) for a given hadron labeled by $H(J^P)$, for each combination of r and δt, we perform linear fits in t to $\ln[C_H(r,\delta t,t)]$. The range of t for the fits is chosen in the region where the effective energy $a^{-1}\ln[C_H(r,\delta t,t)/C_H(r,\delta t,t+a)]$ exhibits a clear plateau. In Fig. 2, Fig. 3, Fig. 4 and Fig. 5 we show the results for the positive parity nucleon, Δ and Σ^* as well as for the negative parity Σ^*, respectively. Notice that different colors in the plots correspond to different values of δt which are slightly displaced horizontally for clarity. We display the statistical errors only. In [6] we also give estimates of the systematic error by changing the range of the fits.

In Fig. 2 we show $\Delta V_H(r,\delta t)$ for the nucleon $N(\frac{1}{2}^+)$. We observe $\Delta V_H(r,\delta t) < 0$. The results agree for $\delta t \gtrsim 3a$ and we take the values for $\delta t = 5a$ as good approximation to the limit $\delta t \to \infty$. The data are well described by a fit to the Cornell parametrization

$$\Delta V_H(r,\delta t = 5a) = \Delta\mu_H - \frac{\Delta c_H}{r} + \Delta\sigma_H r \tag{3.1}$$

with the parameters $\Delta\mu_H$, Δc_H and $\Delta\sigma_H$, also shown in Fig. 2. We find that the size of the effect is $\Delta V_H(r) \approx -1\,\mathrm{MeV}$ to $-2\,\mathrm{MeV}$ at a distance $r \simeq 0.3\,\mathrm{fm}$ and grows to $\Delta V_H(r) \approx -4\,\mathrm{MeV}$ to $-7\,\mathrm{MeV}$

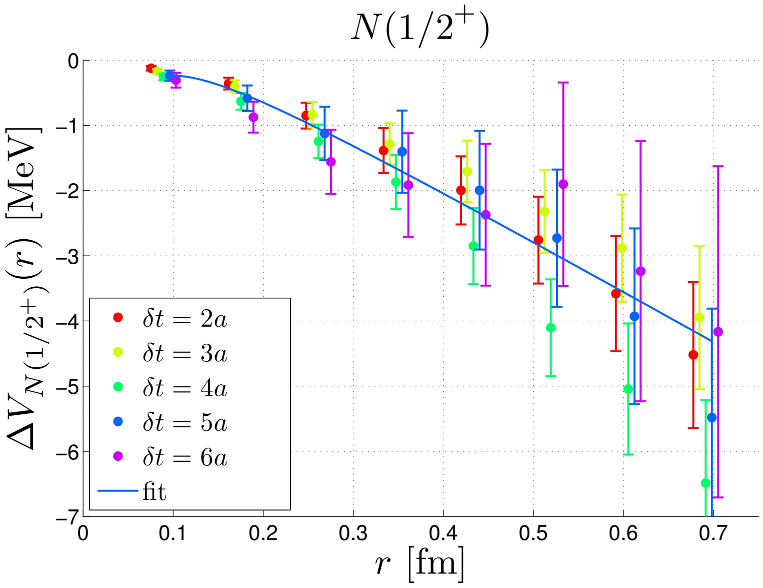

Figure 2: Modification of the static potential "within" a nucleon $N(\frac{1}{2}^+)$.

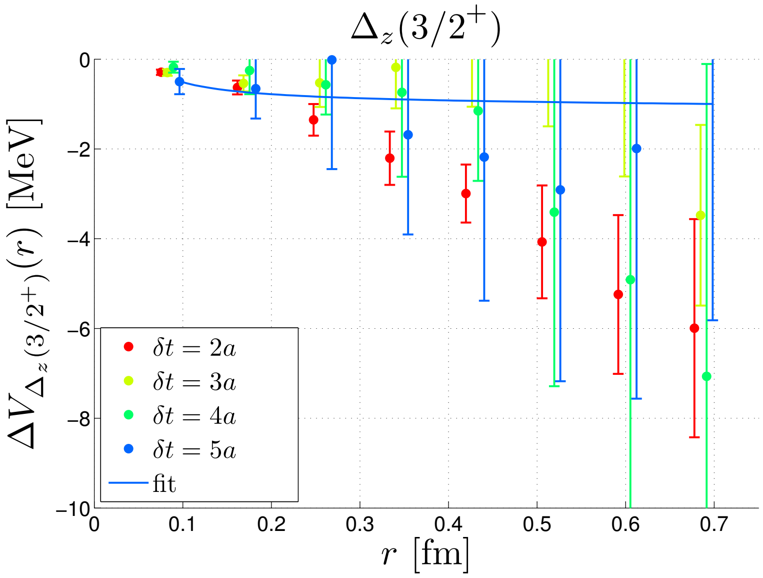

Figure 3: Modification of the static potential "within" a $\Delta(\frac{3}{2}^+)$.

at our largest shown distance $r \simeq 0.7\,\mathrm{fm}$. Notice that a bound state of the nucleon $N(\frac{1}{2}^+)$ with a $\chi_{c2}(2^+)$ could explain the $J^P = \frac{5}{2}^+$ pentaquark resonance.

In Fig. 3 we show $\Delta V_H(r,\delta t)$ for the $\Delta(\frac{3}{2}^+)$. In this case, in Eq. (2.1) we correlate the Δ polarized in z direction with Wilson loops taken in z direction only, to guarantee that we project onto spin $\Lambda = |J_z| = 3/2$ along the distance in z-direction between the static sources. We find similar results as for the nucleon, albeit with rather large errors. Notice that a bound state of a $\Delta(\frac{3}{2}^+)$ with a $J/\psi(1^-)$ could explain the $J^P = \frac{3}{2}^-$ pentaquark resonance. As another example with the same spin and parity assignment but with a strange quark content, in Fig. 4 we show $\Delta V_H(r,\delta t)$

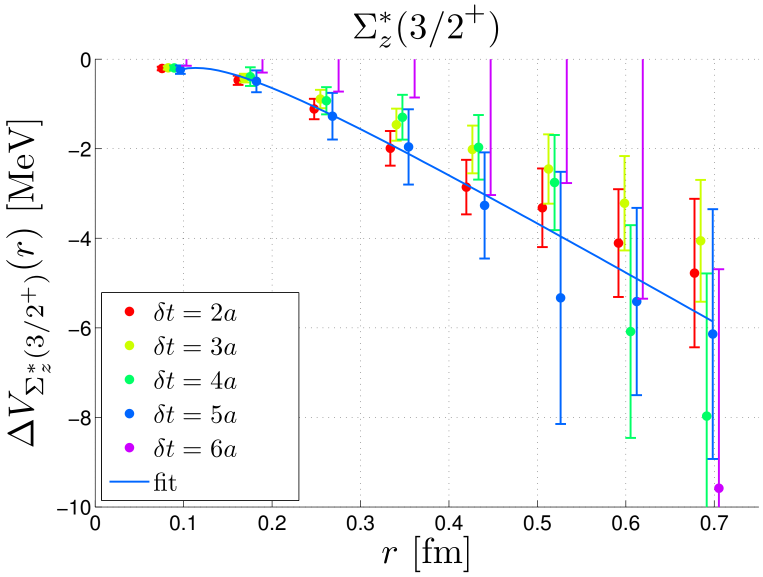

Figure 4: Modification of the static potential "within" a $\Sigma^*(\frac{3}{2}^+)$.

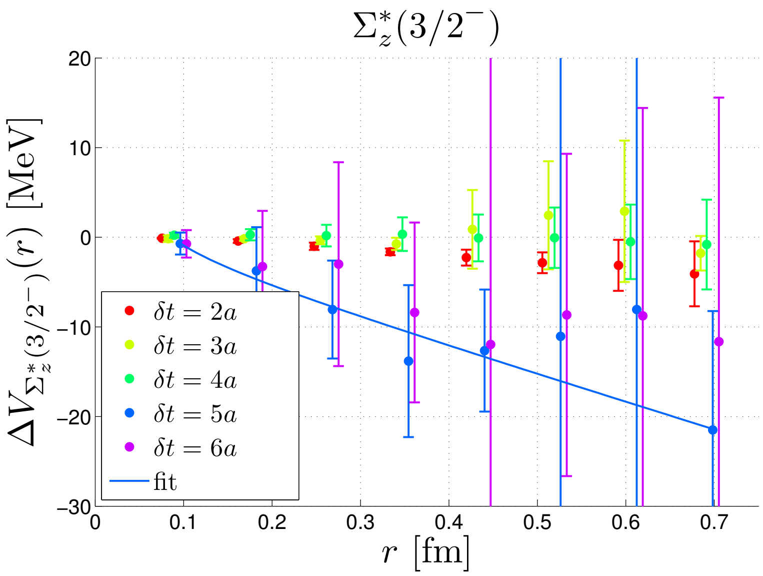

Figure 5: Modification of the static potential "within" a $\Sigma^*(\frac{3}{2}^-)$.

for the decuplet $\Sigma^*(\frac{3}{2}^+)$. The results are very similar to those for the nucleon.

In Fig. 5 we show an example of $\Delta V_H(r,\delta t)$ for a negative parity state, the decuplet $\Sigma^*(\frac{3}{2}^-)$. The statistical errors are much larger than for the positive parity case shown in Fig. 4. Within the errors the values of ΔV_H are consistent with the positive parity case but even larger negative values cannot be excluded for the negative parity case. Notice that a bound state of a $\Sigma^*(\frac{3}{2}^-)$ with a $J/\psi(1^-)$ could give a $J^P = \frac{5}{2}^+$ pentaquark resonance. However in this case it contains a strange quark and also the resulting mass is too large and does not match the mass of the $P_c^+(4450)$ pentaquark.

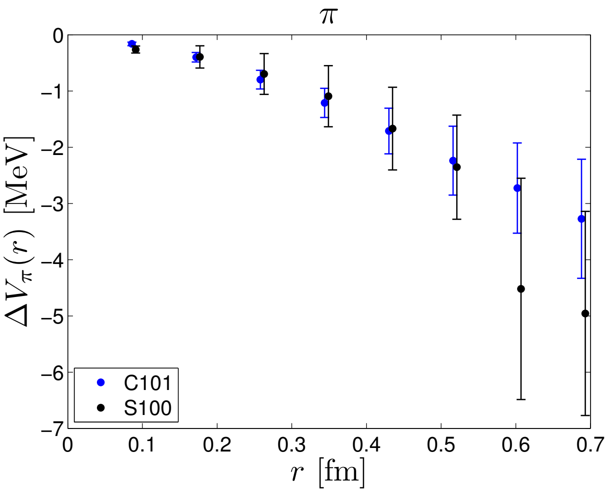

Figure 6: Comparison of the modification of the static potential "within" a pion for two different volumes.

4. Volume check

In order to check for finite volume effects we analysed a second CLS ensemble "S100" with a smaller volume of 128×32^3 sites but with the same lattice spacing and quark masses as the ensemble "C101". At the time of the lattice conference the statistics were insufficient to draw a conclusion. By the time these proceedings were written, data for 478 configurations of "S100" were available. In Fig. 6 we compare the results for $\Delta V_\pi(r, 5a)$ on the "C101" and "S100" ensembles and the conclusion is that there are no significant finite volume effects.

5. Conclusions

We have numerically established the modification ΔV_H of the static quark-antiquark potential in the presence of a hadron, see Eq. (2.2). We find $\Delta V_H(r) < 0$. At a distance of 0.5 fm the size of the effect varies between 2 MeV and 3 MeV for all the hadrons we investigated. The main effect can be parametrized as a reduction of the linear slope of the static potential. We emphasize that we do not see finite volume effects, comparing $Lm_\pi \approx 4.6$ ("C101") with $Lm_\pi \approx 3.1$ ("S100").

In order to answer the question, whether this modification leads to a larger binding energy of charmonium states, we have compared the energy levels that result from solving the Schrödinger equation with the vacuum static potential V_0 and with the modified potential $V_0 + \Delta V_H$. Details of this calculation can be found in [6]. The result is a stronger binding of charmonium $1S$ state by -1 MeV to -2.5 MeV, of $1P$ state by -1 MeV to -5 MeV and of $2S$ state by -1 MeV to -6.5 MeV. These binding energies are similarly small in size as in the deuterium system and may be somewhat inconsistent with the original hadro-charmonium picture.

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