PROCEEDINGS OF SCIENCE

Charmonium and charmed meson spectroscopy from lattice QCD

David Tims[∗ a] **Gavin K. C. Cheung**[b] **Cian O'Hara**[a] **Graham Moir**[b]
Michael Peardon[a] **Sinéad M. Ryan**[a] **Christopher E. Thomas**[b]
[a] *School of Mathematics, Trinity College, Dublin 2, Ireland*
[b] *DAMTP, University of Cambridge, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, UK*

E-mail: timsd@tcd.ie

(For the Hadron Spectrum Collaboration)

Spectra of highly excited hidden and open-charm mesons calculated on dynamical lattice QCD ensembles with a pion mass of $M_\pi \sim 240$ MeV are presented and compared to previous results obtained on a lattice where $M_\pi \sim 400$ MeV. The distillation technique was employed in order to compute the necessary correlation functions, allowing the use of a large basis of interpolating operators with various spatial structures. This basis included operators proportional to the gluonic field strength, allowing the identification of possible hybrid states. We conclude that reducing the light quark mass has little effect on the overall pattern and structure of the spectra.

34th annual International Symposium on Lattice Field Theory
24-30 July 2016
University of Southampton, UK

∗Speaker.

PoS(LATTICE2016)137

1. Introduction

Several charmonium-like states, commonly referred to as the 'X','Y' and 'Z', have been observed experimentally which cannot be explained as conventional mesons (composed of a quark-antiquark pair). The nature of these states is an open question, they have been conjectured to be hybrid in nature (gluonic excitation), or to consist of two quarks and two antiquarks as either "compact" tetraquarks, or hadro-quarkonium, or even to be "extended" meson-meson molecular states. Similarly, in the open charm sector the observed $D_{s0}^*(2317)^{\pm}$ and $D_{s1}(2460)^{\pm}$ states are of particular interest; their masses and widths are respectively smaller and narrower than expected from conventional quark models. For some recent reviews see Refs. [1, 2, 3].

In these proceedings we briefly summarise the results presented in Ref. [4]. Calculations of hidden and open-charm spectra on ensembles with $M_\pi \sim 240$ MeV are compared to results obtained from a previous study (Ref. [5, 6]) where $M_\pi \sim 400$ MeV. We examine the effect of lowering the pion mass on the overall qualitative picture of the spectra. We are particularly interested in whether reducing the pion mass will significantly change the pattern and ordering of states determined to be hybrid in nature [5, 6].

This study also lays the foundation for scattering calculations. The relative positions of states of interest to relevant thresholds at the lower pion mass allows for the determination of regions of interesting physics.

Calculations are performed on dynamical anisotropic ensembles with $2+1$ flavours of dynamical quarks (up, down and strange) generated by the Hadron Spectrum Collaboration [7, 8]. The anisotropy, $\xi \equiv a_s/a_t$, is roughly 3.5, ensuring that $a_t m_c \ll 1$ while keeping $a_s m_c < 1$, where m_c is the mass of the charm quark. The scale was set using the Ω baryon mass via $a_t^{-1} = M_\Omega^{phys}/(a_t M_\Omega)$, where M_Ω is the Ω baryon mass measured on the lattice. Table 1 summarizes the lattice ensembles used.

2. Lattice Spectroscopy

To perform calculations the procedure presented in Ref. [9] is followed. Briefly, spectral information can be extracted from an analysis of the time dependence of two-point Euclidean correlation functions. However, on the lattice, because of reduced symmetry, states at rest are not labelled by continuum spin, J, but rather by the *irreps* Λ, of the octahedral group, O_h. The identification of the spin of a state is therefore complicated and the method used for ameliorating this issue is described

Lattice Volume	M_π (MeV)	N_{cfgs}	N_{tsrcs} for $c\bar{c}$, $c\bar{s}$, $c\bar{l}$	N_{vecs}
$24^3 \times 128$	391	553	32, 16, 16	162
$32^3 \times 256$	236	484	1, 1, 2	384

Table 1: The lattice gauge field ensembles and parameters used. The volume is given as $(L/a_s)^3 \times (T/a_t)$ where the spatial and temporal extents of the lattice are L and T respectively. The number of gauge field configurations used, N_{cfgs}, the number of perambulator time-sources used per configuration, N_{tsrcs}, and the number of eigenvectors used in the distillation framework [11], N_{vecs}, are shown.

in Refs. [9, 10]. Spectra are presented with states labelled by their continuum spin, parity, P, and relevant flavour quantum numbers, e.g. charge-conjugation C.

For each lattice irrep, $\Lambda^{P(C)}$, the distillation technique [11] is used to compute correlation functions involving a large basis of interpolating operators with various spatial structures. To analyse the resulting matrices of correlation functions, $C_{ij}(t)$, a variational procedure is employed [9, 12, 13]; a generalised eigenvalue problem is solved,

$$C_{ij}(t)v_n^j = \lambda_n(t,t_0)C_{ij}(t_0)v_n^j, \tag{2.1}$$

where t_0 is a carefully chosen reference time-slice. For sufficiently large times, the eigenvalues, $\lambda_n(t,t_0)$, known as principal correlators, are proportional to $e^{-E_n(t-t_0)}$ where E_n is the energy of the n^{th} state in lattice irrep $\Lambda^{P(C)}$. The eigenvectors, v_n^j, are related to the operator-state overlaps, $Z_n^{(k)} \equiv \langle n|\mathcal{O}_k^\dagger|0\rangle$, and provide information on the structure of a state; in particular these operator-state overlaps form the basis of the spin identification approach.

3. Spectra and Comparisons

In this section plots are presented which compare spectra computed on ensembles with a pion mass of 240 MeV and spectra computed on ensembles with a pion mass of 400 MeV [5, 6], labelled by $J^{P(C)}$. Results for charmonium are discussed first followed by those for D_s and D mesons.

3.1 Charmonium spectrum

Fig. 1 shows a comparison of the charmonium spectra where the masses are quoted with M_{η_c} subtracted, this is to reduce the systematic uncertainty that arises from the tuning of the charm quark mass. We assign states to a particular supermultiplet based on their operator state overlap values, see Ref. [9]. On the 240 MeV ensembles most of the states with non-exotic quantum numbers fit the $n^{2S+1}L_J$ pattern predicted by conventional quark models, these states are coloured in green. States coloured in red and blue do not fit the conventional pattern. Of these, there are four states with manifestly exotic quantum numbers: $0^{+-}, 1^{-+}, 2^{+-}(\times 2)$. These four states, as well as the excess states with non-exotic quantum numbers, have relatively large overlaps onto operators that are proportional to the spatial components of the gluonic field strength tensor F_{ij}. Following Ref. [15] these states are interpreted as hybrid mesons; the states highlighted in red form the lightest charmonium hybrid supermultiplet, and the states highlighted in blue form the first excited hybrid supermultiplet. The patterns of the hybrid supermultiplets appear consistent with a model of a quark-antiquark pair in S-wave (P-wave) coupled to a 1^{+-} gluonic excitation for the ground (first excited) supermultiplet.

In charmonium the masses of the lower lying states are generally consistent between the two ensembles. However, it is interesting that as the light quark mass is reduced we find a statistically significant increase in the mass splitting between η_c and J/ψ from 80.19 ± 0.13 MeV to 87.3 ± 0.3 MeV, where the quoted errors are statistical only. This is closer to the physical value of 113 MeV [16].

An increase in the masses of higher lying states on the $M_\pi \sim 240$ MeV ensemble is observed. As a consequence the splitting between the hybrids and lower-lying mesons increases slightly.

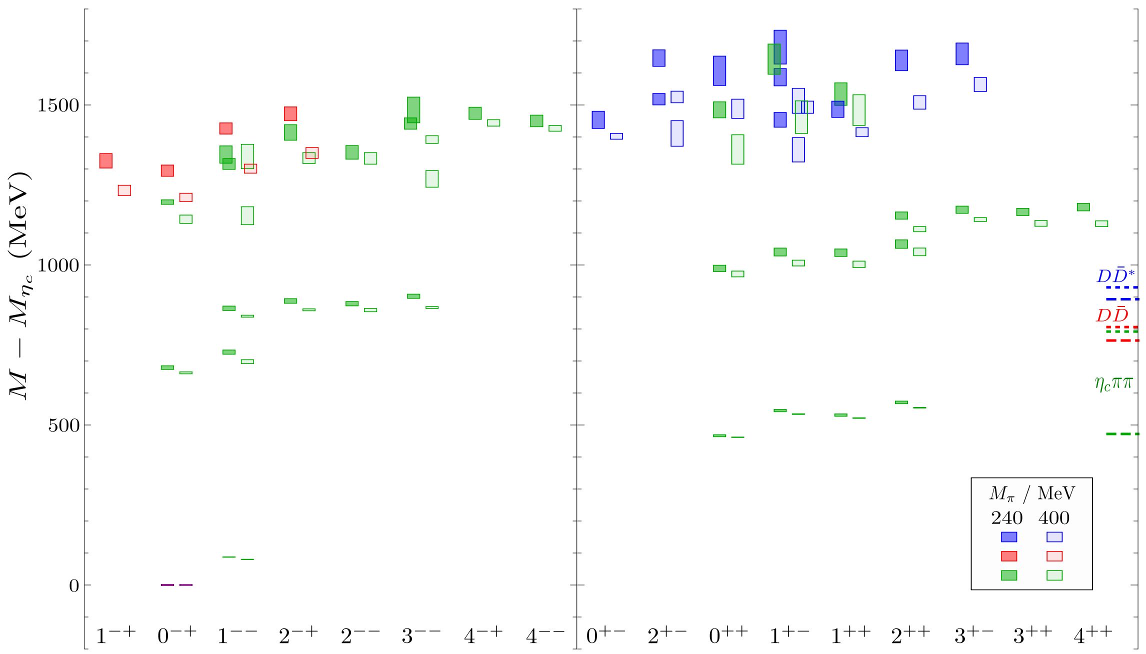

Figure 1: From Ref. [4], a comparison of charmonium spectra up to around 4.5 GeV labelled by J^{PC}; the left (right) panel shows the negative (positive) parity states. Within each J^{PC} channel the left column corresponds to states calculated with $M_\pi \sim 240$ MeV and the right column corresponds to states previously calculated on the $M_\pi \sim 400$ MeV ensemble [6]. The coloured boxes are the masses with the calculated η_c mass subtracted. The vertical size of the boxes represents the one-sigma statistical uncertainty on either side of the mean. States identified as hybrid mesons are coloured red and blue and grouped into, respectively, the lightest and first-excited supermultiplet. Dashed lines show some of the physically relevant low-lying thresholds using computed masses for $M_\pi \sim 240$ MeV (coarse dashing) and $M_\pi \sim 400$ MeV (fine dashing): green is $\eta_c\pi\pi$, red is $D\bar{D}$ and blue is $D\bar{D}^*$.

However, there is no change in their overall pattern. It should be noted that at higher energies the statistical uncertainties are much larger and that the unstable nature of states above threshold has not been considered.

3.2 D_s Meson spectrum

Fig. 2 shows a comparison of D_s spectra where the masses are quoted with $M_{\eta_c}/2$ subtracted. As with the charmonium spectrum, the D_s spectra can be interpreted in terms of an $n^{2S+1}L_J$ pattern, identifying complete S, P, D and F wave multiplets. In the negative parity sector four states are found, highlighted in red, that are identified as members of the lightest hybrid meson supermultiplet. Again, the pattern of the hybrid supermultiplet appears consistent with a model of a quark-antiquark pair coupled to a 1^{+-} gluonic excitation.

As can be seen in Fig. 2, the largest change in the D_s meson spectrum is for the lightest 0^+, a candidate for the $D_{s0}^*(2317)$. This state is expected to be heavily influenced by the nearby DK threshold to which it can couple in S wave. Interestingly, it has decreased in mass just enough to remain below the threshold which is in agreement with the current experimental situation. Once again, as the pion mass is reduced, there is a tendency for the hybrid states to increase in mass.

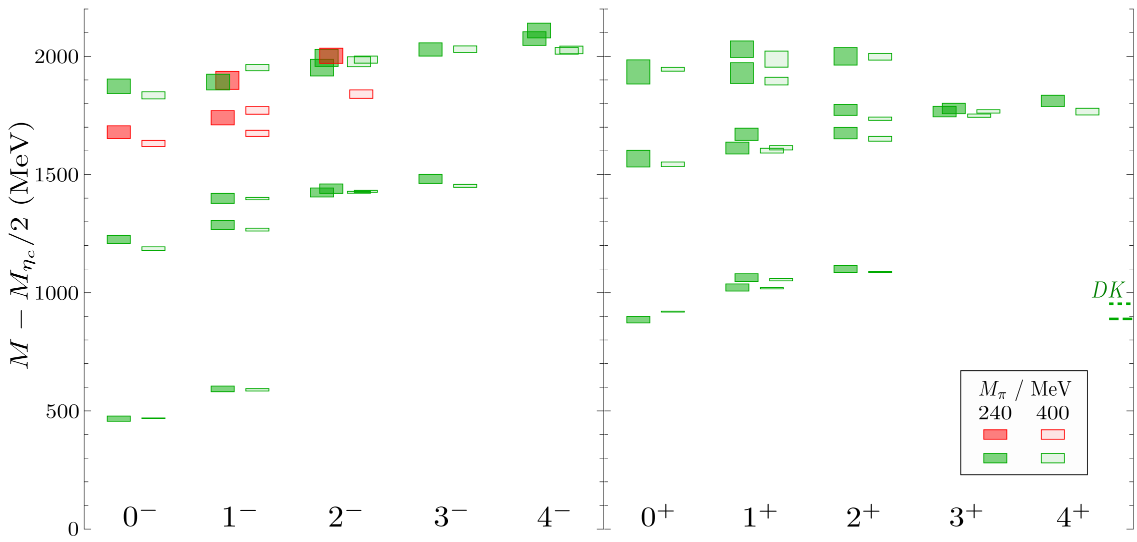

Figure 2: From Ref. [4], as Fig. 1 but for the D_s meson spectrum; states are labelled by J^P. The left (right) panel shows the negative (positive) parity states. The masses shown have half the calculated η_c mass subtracted. Red boxes show states identified as constituting the lightest hybrid supermultiplet. Dashed lines show the DK threshold using computed masses for $M_\pi \sim 240$ MeV (coarse dashing) and $M_\pi \sim 400$ MeV (fine dashing).

Hence, their splitting with low-lying conventional D_s mesons is increased but their overall pattern remains unchanged.

3.3 *D* meson spectrum

Fig. 3 shows a comparison of D spectra where the masses are quoted with $M_{\eta_c}/2$ subtracted. As for the D_s spectrum, complete S, P, D and F wave multiplets are identified. In the negative parity sector four hybrid states (highlighted in red) are found. These form the lightest supermultiplet and appear consistent with a model of a quark-antiquark pair coupled to a 1^{+-} gluonic excitation.

In the D meson spectrum the calculated meson masses are generally lower than on the 240 MeV ensemble. This is expected as a result of the valence quark content of D mesons. The most significant differences are seen for the lightest 0^+ and 1^+ states. These states couple in S wave to nearby thresholds, $D\pi$ and $D^*\pi$ respectively. Their relative position to these nearby thresholds is important, as this could be strongly influencing their behaviour. Interestingly we find that the second-lightest 1^+, which is also in the vicinity of the $D^*\pi$ threshold, does not change in mass significantly. The mass difference between the charm quark and the light quark is large enough such that the expectations of the heavy quark limit may be a reasonable guide. In this limit, one of the 1^+ states can decay to $D^*\pi$ in S wave only, whereas the other can only decay to $D^*\pi$ only in D wave; the latter would be expected to be influenced less by the position of the $D^*\pi$ threshold.

The general trend found in the charmonium and D_s spectrum was for hybrid mesons to become heavier as the pion mass is decreased. This change is somewhat less clear in the D meson spectrum due to the opposing trend for mesons to become lighter as the light-quark mass is decreased.

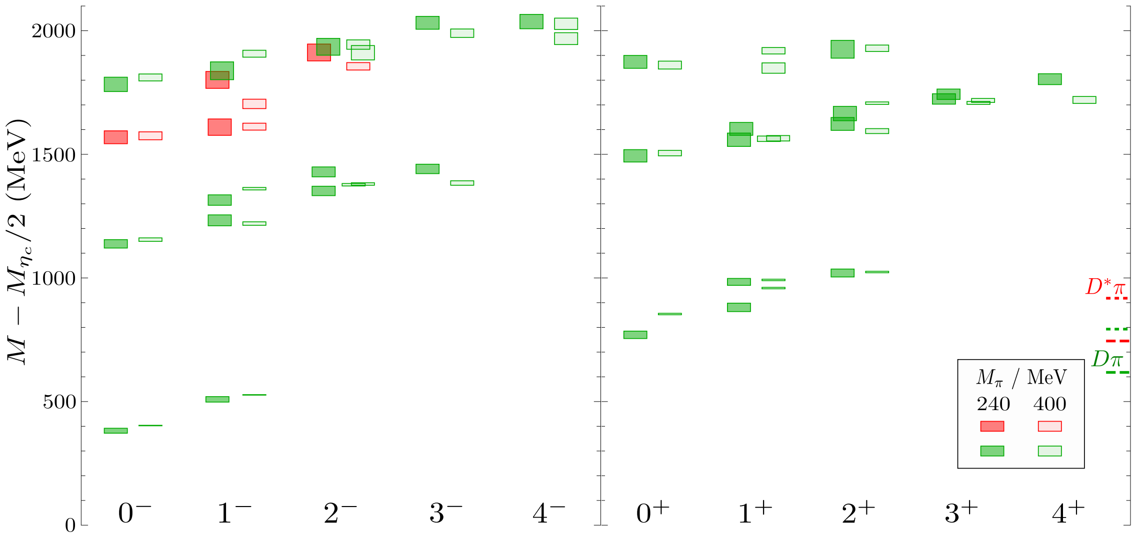

Figure 3: From Ref. [4], as Fig. 2 but for the D meson. Dashed lines show the $D\pi$ and $D^*\pi$ thresholds using computed masses for $M_\pi \sim 240$ MeV (coarse dashing) and $M_\pi \sim 400$ (fine dashing).

4. Summary

Hidden and open charm spectra at two values of the light quark mass were compared and it was found that there was no change in the overall qualitative pattern of identified states; even in the case of charmed mesons which contain a valence light quark, only small quantitative differences were found. States identified to be hybrid in nature appear to show a small but statistically significant increase in mass as the light quark mass is decreased, however their supermultiplet structure remained unchanged.

In this work we neglect the unstable nature of states above threshold. However, these states should be treated in a scattering framework, and we should consider the spectra only to be a guide to the pattern of resonances. See Refs. [17, 18, 19] for the current status of work in the charm sector by the Hadron Spectrum Collaboration.

5. Acknowledgements

We thank our colleagues within the Hadron Spectrum Collaboration. GC is supported by the Cambridge European Trust, the U.K. Science and Technology Facilities Council (STFC) and St John's College, Cambridge. COH acknowledges support from the School of Mathematics at Trinity College Dublin. GM acknowledges support the Herchel Smith Fund at the University of Cambridge. SMR acknowledges support from Science Foundation Ireland [RFP-PHY-3201]. CET acknowledges support from the STFC [grant ST/L000385/1]. DT is supported by the Irish Research Council Government of Ireland Postgraduate Scholarship Scheme [grant GOIPG/2014/65].

References

[1] S. L. Olsen, *PoS Bormio* **050** (2015) 26-30, [arXiv:1511.01589].

[2] E. S. Swanson, *AIP Conf. Proc.* **1735** (2016) 020013, [arXiv:1512.04853].

[3] E. Prencipe, *PoS Bormio* **044** (2015) 26-30, [arXiv:1510.03053].

[4] G. K. C. Cheung et al., *JHEP* **12** (2016) 089, [arXiv:1610.01073].

[5] G. Moir et al., *JHEP* **05** (2013) 021, [arXiv:1301.7670].

[6] L. Lui et al., *JHEP* **07** (2012) 126, [arXiv:1204.5425].

[7] R. G. Edwards, B. Joo and H. W. Lin, *Phys. Rev.* **D78** (2008) 054501, [arXiv:0803.3960].

[8] **Hadron Spectrum** Collaboration, H. W. Lin et al., *Phys. Rev.* **D79** (2009) 034502, [arXiv:0810.3588].

[9] J. J. Dudek et al., *Phys. Rev.* **D82** (2010) 034508, [arXiv:1004.4930].

[10] J. J. Dudek et al., *Phys. Rev. Lett.* **103** (2009) 262001, [arXiv:0909.0200].

[11] M. Peardon et al., *Phys. Rev.* **D80** (2009) 054506, [arXiv:0905.2160].

[12] C. Michael, *Nucl. Phys.* **B259** (1985) 58-76.

[13] M. Luscher and U. Wolff, *Nucl. Phys.* **B339** (1990) 222-252.

[14] J. J. Dudek, *Phys. Rev.* **D84** (2011) 074023, [arXiv:1106.5515].

[15] J. J. Dudek and E. Rrapaj, *Phys. Rev.* **D78** (2008) 094504, [arXiv:0809.2582].

[16] **Particle Data Group** Collaboration, *Chin. Phys.* **C38** (2014) 090001.

[17] G. Moir et al., *JHEP* **10** (2016) 011, [arXiv:1607.07093].

[18] G. Moir, (2016), [arXiv:1611.05822].

[19] D. J. Wilson, *PoS LATTICE2016* **016** (2016), [arXiv:1611.07281].