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Hadron Structure from the Feynman–Hellmann Theorem

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The determination of hadronic form factors at large momentum transfers has been a challenging problem in lattice QCD simulations. Here we show how the Feynman–Hellmann method may be extended to non-forward matrix elements to calculate hadronic form factors in lattice QCD at much higher momenta than previously accessible. We are able to determine the electromagnetic form factors of the pion and nucleon up to approximately 6 GeV^2, with results for G_E/G_M in the proton agreeing well with experimental results.

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1. Introduction

One of the great challenges of hadron physics is to build consistent pictures of the internal structures of strongly-interacting particles. An important aspect of this endeavour is the calculation of electromagnetic form factors, which describe the distribution of electromagnetic currents in hadrons.

Recoil polarisation experiments at Jefferson Lab show that the ratio of the electric and magnetic form factors in the nucleon, $\mu_p G_{Ep}/G_{Mp}$, decreases approximately linearly for $Q^2 \gtrsim 0.5$ GeV^2 (see e.g. [1, 2, 3, 4]). Experimental results at high-momentum scales are not yet precise enough to determine whether this trend continues and there is a zero crossing. Resolving the scaling of the form factors in this domain is one of the key physics goals of the upgraded CEBAF at Jefferson Lab.

The large-Q^2 behaviour of the pion electromagnetic form factor F_π is also challenging to investigate experimentally (see [5, 6, 7] for recent innovative advances). This information is important for understanding the transition from the soft to the hard regime in QCD (see [8] for a recent example). At present, the experimental data is not able to reliably discriminate different models describing the transition to the asymptotic domain [9].

Lattice calculations of hadronic form factors have typically focussed on the study of processes at low-momentum transfer (see e.g. [10, 11, 12, 13, 14, 15]), with only limited studies at large $Q^2 \gtrsim 3\,\mathrm{GeV}^2$ [16, 17]. These calculations are difficult because the signals fall with Q^2, and the signal-to-noise ratio deteriorates. This also makes it difficult to evaluate the degree of excited-state contamination [16, 18, 14, 19, 20].

In this work we demonstrate how high-momentum transfer in hadron form factors may be accessed on the lattice using an extension of the Feynman–Hellmann theorem to non-forward matrix elements. This builds upon the techniques developed for forward matrix elements [21, 22, 23, 24] (see also [25, 26, 27, 28, 29, 30, 31, 32] for similar related techniques). These methods allow one to access matrix elements from 2-point correlators, rather than a more complicated analysis of 3-point functions, which simplifies the elimination of excited-state contamination. The calculations are performed with Breit frame kinematics ($E(\vec{p}\,') = E(\vec{p})$) and hence one maximises the momentum transfer for any given state momentum, reducing the noise in the correlation function.

2. Feynman–Hellmann Methods

The Feynman–Hellmann method for the calculation of forward matrix elements is described in [22]. Here we describe only the subtleties involved in extending the technique to non-forward matrix elements. Suppose the QCD Lagrangian is modified in a lattice simulation such that

$$\mathscr{L}(y) \to \mathscr{L}(y) + \lambda \left(e^{+i\vec{q}\cdot\vec{y}} + e^{-i\vec{q}\cdot\vec{y}} \right) \mathscr{O}(y)\,, \tag{2.1}$$

where $\mathscr{O}$ is a quark-bilinear operator and λ is a freely-varying real parameter. It may be shown that the shift in the energy of a hadron state $H(\vec{p}\,')$ resulting from a shift in λ from $\lambda = 0$ is proportional to a matrix element of the operator $\mathscr{O}$,

$$\left.\frac{\partial E_H(\vec{p}\,')}{\partial \lambda}\right|_{\lambda=0} = \frac{\left\langle H(\vec{p}\,') \middle| \mathscr{O}(0) \middle| H(\vec{p}) \right\rangle}{\left\langle H(\vec{p}\,') \middle| H(\vec{p}\,') \right\rangle}\,, \tag{2.2}$$

where $\vec{p} = \vec{p}' \pm \vec{q}$. In order for this result to hold, $\vec{p}$ and $\vec{p}'$ must satisfy the Breit frame condition $E_H(\vec{p}) = E_H(\vec{p}')$. States not satisfying this requirement do not receive energy shifts at $\mathscr{O}(\lambda)$.

Following this procedure, we may calculate non-forward matrix elements for any particular operator $\mathscr{O}$ by performing hadron spectroscopy for multiple values of $\lambda \neq 0$. Connected quark contributions are calculated by inverting quark propagators according to the modified action corresponding to Eq. (2.1). Determining disconnected contributions requires the generation of new gauge ensembles [24].

3. Simulation Details

In this work, we use an ensemble of 1700 gauge field configurations with $2+1$ flavours of non-perturbatively $O(a)$-improved Wilson fermions and a lattice volume of $L^3 \times T = 32^3 \times 64$. The lattice spacing $a = 0.074(2)$ fm is set using a number of singlet quantities [33, 34, 35, 36]. The clover action used comprises the tree-level Symanzik improved gluon action together with a stout smeared fermion action, modified for the implementation of the FH method [22]. The hopping parameters $(\kappa_l, \kappa_s) = (0.120900, 0.120900)$ correspond to a pion mass of ~ 470 MeV. To study electromagnetic form factors, quark propagators are calculated with the modified Lagrangian

$$\mathscr{L}(y) \to \mathscr{L}(y) + \left(e^{+i\vec{q}\cdot\vec{y}} + e^{-i\vec{q}\cdot\vec{y}}\right) \bar{q}(y)\lambda \cdot \gamma q(y)\,, \tag{3.1}$$

for multiple values of $\vec{q}$, where either λ_2 or λ_4 take non-zero values of 1×10^{-4} or -1×10^{-5}. Note that we only use Breit-frame kinematics with $\vec{p}' = -\vec{p}$. This choice allows us to minimise $\vec{p}^2$ for each value of $\vec{q}^2$, and hence minimise the noise in the correlator. This choice of kinematics also results in nucleon energy shifts that are directly proportional to G_E and G_M.

4. Results

4.1 Electromagnetic Form Factors of the Nucleon

Individual quark flavour contributions to the Euclidean decompositon of the vector current matrix element of the nucleon are written in terms of the Dirac and Pauli (F_1^q and F_2^q) form factors,

$$\left\langle N(p',s') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N(p,s) \right\rangle = \bar{u}(p',s') \left[\gamma_\mu F_1^q(Q^2) + \sigma_{\mu\nu} \frac{q_\nu}{2M_N} F_2^q(Q^2) \right] u(p,s)\,, \tag{4.1}$$

where we denote the invariant 4-momentum transfer squared as $Q^2 = -q^2 = -(p'-p)^2$. The Sachs electromagnetic form factors are defined by

$$G_E^q = F_1^q - \frac{Q^2}{(2M)^2} F_2^q\,, \qquad G_M^q = F_1^q + F_2^q\,. \tag{4.2}$$

For Breit frame kinematics where $\vec{p}' = -\vec{p}$, the energy shifts resulting from insertion of the temporal and spatial components of the current are proportional to the electric and magnetic form factors respectively,

$$\left.\frac{\partial E_N}{\partial \lambda_4}\right|_{\lambda=0} \overset{\vec{p}'=-\vec{p}}{=} \frac{M_N}{E_N} G_E^q\,, \qquad \left.\frac{\partial E_N}{\partial \lambda_i}\right|_{\lambda=0} \overset{\vec{p}'=-\vec{p}}{=} \frac{[\hat{\vec{e}} \times \vec{q}]_i}{2E_N} G_M^q\,. \tag{4.3}$$

Here $\hat{\vec{e}}$ is the spin polarisation vector determined by the choice of polarisation direction of the nucleon.

Fig. 1 shows results for the proton electric and magnetic form factors neglecting disconnected contributions. In the low-Q^2 region we compare with results computed on the same ensembles using a variationally-improved 3-point function approach. Good agreement is observed in the region of comparable Q^2. The Feynman–Hellmann approach is seen to extract a clean signal to much higher momentum transfers than have previously been accessible. Fig. 2 displays the extraction

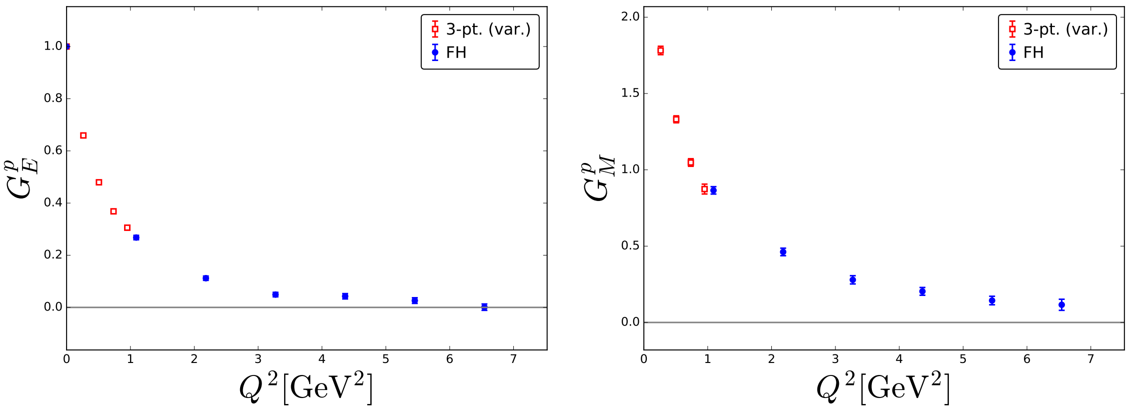

Figure 1: G_E and G_M for the proton from the Feynman–Hellmann method and from a variational analysis of three-point functions on the same ensemble (as described in [20]).

of the ratio G_E/G_M as a function of Q^2 from the Feynman–Hellmann technique, a variational approach and experimental data. The overall trend is seen to compare very well with the experimental data.

4.2 Electromagnetic Form Factor of the Pion

Individual quark flavour contributions to the pion form factor are defined by

$$\left\langle \pi(p') \,\middle|\, \bar{q}(0)\gamma_\mu q(0) \,\middle|\, \pi(p,s) \right\rangle = \frac{[p+p']_\mu}{2E_\pi} F_\pi^q(Q^2) \tag{4.4}$$

With the modified fermion action, pion energy shifts are given by

$$\left.\frac{\partial E_\pi}{\partial \lambda_4}\right|_{\lambda=0} \overset{\vec{p}'=-\vec{p}}{=} F_\pi^q\,, \qquad \left.\frac{\partial E_\pi}{\partial \lambda_i}\right|_{\lambda=0} \overset{\vec{p}'=-\vec{p}}{=} 0\,. \tag{4.5}$$

Following a similar analysis as that for the nucleon, we show the determination of the pion form factor in Fig. 2, along with a comparison to experimental data. The signal-to-noise ratio achieved gives confidence that future lattice simulations will be able to provide important insight into the transition between the perturbative and nonperturbative regimes.

5. Conclusion

In this work we have extended the Feynman–Hellmann technique to access non-forward matrix elements. We demonstrate that application of the technique provides a dramatic improvement in

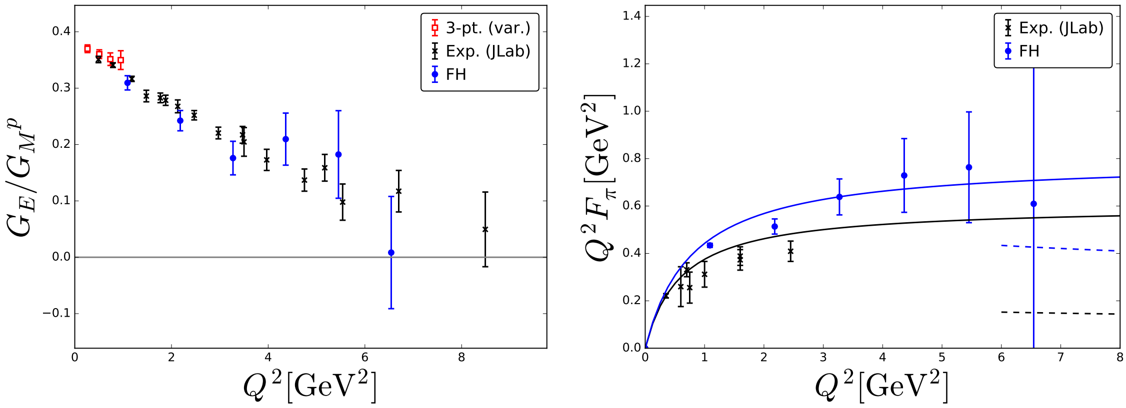

Figure 2: On the left, the ratio G_E/G_M for the proton from application of the Feynman–Hellmann method, from a variational analysis of three-point functions, and from experiment [37, 4, 3]. On the right, the scaled pion form factor Q^2F_π from the Feynman–Hellmann technique and from experiment [7]. The solid lines are the vector meson dominance at the relevant pion masses, and the dotted lines are the asymptotic values predicted by perturbative QCD (see [8] for a discussion of this value and its limitations).

the ability to extract nucleon and pion form factors at high momentum transfers. An additional improvement that we intend to pursue is the use of improved operators that couple more strongly to boosted hadron states, as proposed in [38].

Acknowledgments

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